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Exploring Pre-Service Teachers' Understanding of Join and Separate Word Problems in the Foundation Phase (Grades 1-3)

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Abstract

This study explores pre-service teachers' understanding of different types of join and separate word problems, focusing on their ability to construct word problems from given number sentences. It is a descriptive qualitative study. The participants were second-year pre-service foundation phase teachers taking mathematics learning courses at a public university. Data were collected through number sentence tasks and analysed using a qualitative content analysis procedure. The findings of this study revealed that pre-service teachers demonstrated challenges in correctly formulating word problems from change-unknown and start-unknown scenarios. The findings further reveal that pre-service teachers conflate additive and subtractive reasoning. The findings suggest that pre-service teachers rely on rote memory rather than a deeper understanding of mathematical relations. In addition, the study revealed common misconceptions, particularly in framing "change-unknown" and "result-unknown" problems, where pre-service teachers struggled to model real-life contexts accurately. Findings suggest the need for teacher education programs to focus more explicitly on developing a deep understanding of word problem structure. Initial teacher education programs should include word problem formulation and problem-solving tasks in all five mathematics learning areas, thereby underlining the importance of this research.

INTRODUCTION

Background of the Study

Word problems have always been a feature of school mathematics, dating back to antiquity. The use of word problems in school mathematics has been recorded in ancient Chinese and Indian literature, and up to today, word problems are a permanent feature of primary and secondary school mathematics Swetz (2009). Word problems help learners develop problem-solving skills, application skills, and creative thinking skills and assist in the development of new mathematical concepts and skills (Verschaffel et al., 2000). It is trite that pre-service mathematics teachers understand the different classifications of word problems they are expected to teach in the Foundation Phase(FP).

Policymakers and curriculum planners in South Africa have emphasised the need to teach word problems, particularly in the Foundation Phase (Grades 1-3). The Curriculum Assessment Policy Statement (CAPS) encourages the teaching of word problems and offers suggestions on how learners should solve them DBE (2015). CAPS aims to equip learners with foundational numeracy skills. This focus aligns with the goals of the UNESCO Education Framework 2030, which seeks to promote lifelong learning and quality education for all(UNESCO,2015). Despite the policy directions, the Diagnostic Report for the Annual National Assessment (ANAs) for the FP in mathematics results reveal that learners perform poorly in word problems and suggests a need to focus on word problems from an early age, Department of Basic Education DBE (2012; 2014; 2015). This poor performance in word problems is not unique to South Africa and correlates with global trends where learners find word problems challenging (Verschaffel et al., 2015).

Teachers play a significant role in addressing this challenge. Research shows that teachers' content and pedagogical knowledge significantly increase learners' outcomes in mathematics (Shulman 1986; Pajares 1992). Primary school students' difficulty in solving word problems is a global challenge, not unique to South Africa. The general trend in South Africa and the world is that pre-service teachers must be better equipped to teach word problems. Teacher training institutions increasingly focus on equipping pre-service teachers with the skills and knowledge to teach word problems effectively.

Understanding pre-service teachers' “foundational knowledge” Rowland et al. (2009) and views on teaching word problems is imperative. This study explores pre-service teachers' content knowledge and beliefs about mathematics, especially their understanding of various types of join and separating problems. Word problems are crucial in honing learners' problem-solving skills Sepeng & Webb (2012). Investigating how pre-service teachers approach these join and separate problems is critical. The crucial role that word problems play in promoting realistic modelling and problem-solving skills has dominated the literature for the past decades (Verschaffel et al., 2010). Besides enhancing these skills, word problems promote the development of critical thinking skills and bridge the gap between abstract mathematics and real-world applications. By engaging with word problems, learners become deeply familiar with mathematical concepts and apply them in meaningful and practical contexts.

Learners' understanding of and retention of critical ideas is enhanced through engaging with word problems. Tan (2021) state that word problems assist learners in connecting with real-life situations; hence, a deeper understanding of the material is cultivated. When learners experience real-world applications, they are motivated, and learning outcomes improve Kaiser (2007). In a study on pre-service teachers. There is a need to integrate word problems into the mathematical curriculum to engage learners in mathematical practices, thus making mathematics more meaningful and applicable to daily life. All this literature emphasises the importance of word problems promoting mathematical skills, critical thinking, engagement, and the application of mathematics in eel-life contexts. These studies underpin the need to incorporate diverse and meaningful word problems into mathematics instruction to promote learners' learning effectively.

Research in Hungary indicates that cultural and instructional practices influence how learners solve word problems Csikos & Szitanyi (2020). Since mathematics is a human activity, word problems must be anchored within the idea that problem-solving is a human activity in the mathematics classroom (Lave 1992; Verschaffel et al., 2000; 2014). These insights can inform and enhance local practices in South Africa. Teacher training programs must prioritise upskilling pre-service teachers' proficiency in teaching word problems. Pre-service teachers play a crucial role in shaping learners' mathematical understanding and problem-solving skills, so it is crucial to understand their understanding of teaching word problems. We focused on teachers' content knowledge, the knowledge they bring to the classroom and their content knowledge about mathematics. In addition, in our work to support Foundation Phase teachers in teaching mathematics, we have observed mathematics lessons where learners are taught to solve word problems using solution steps as early as second grade. To reimagine how and why word problems are taught, it is crucial to determine how pre-service teachers think about word problems.

Research suggests many teachers would eliminate word problems in public schools' first and second grades Hungarian Academy of Sciences (2016). Verschaffel et al. (2010) have defined word problems as verbal descriptions of problem situations in which one or more questions are posed, the answer to which can be obtained by applying mathematical operations to numerical data available in the problem statement. By this definition, word problems can range from routine classroom tasks to more complex, realistic word problems. In the classroom context, word problems are often used in a way described by Palm (2006) since many of them are just ordinary school math problems disguised in an extracurricular pictorial context (p. 42). Some simple strategies learners use to solve word problems can be traced to teaching practices and cultural traditions in textbooks Csikos & Szitanyi (2020). Learners are taught to find two or more numbers in the text and select and execute arithmetic operation(s). The numerical results are the answer (Verschaffel et al., 1997).

Problem of The Study

The poor performance of learners in mathematics is well documented in South Africa. The Curriculum Assessment Policy Statement (CAPS) encourages the teaching of word problems from grade 1. However, indications from the Diagnostic Report for the Annual National Assessment (ANAs) show that learners continue to perform poorly in solving word problems. Given this background, it becomes imperative to investigate how pre-service teachers understand and approach the teaching of word problems, particularly joining and separating word problems. Pre-service teachers are expected to bring foundational knowledge into the classroom that will enhance the development of problem-solving skills in their learners. It is essential to understand how they solve word problems, particularly join and separate word problems, since pre-service teachers will soon be teaching primary school mathematics and addressing the myriad of challenges in mathematics education.

Research's State of the Art

Research on word problems has been a subject of inquiry by psychologists and educationists alike (Daroczy et al. 2015; Thevenot & Barrouillet 2015; Verschaffel et al. 2000; Depaepe et al. 2013a). Until the 1970s, research focused on conceptual understanding, algorithmic skills, and abstract thinking abilities, which influenced how students approached mathematical problems Goldin & McClintock (1979). However, the research shifted to cognitive and thinking processes because of the information-processing approach (Verschaffel et al. 2015). Researchers employed verbal protocols, interviews, reaction times, eye movements, and neuropsychological measurements (Lave 1992; Verschaffel et al. 2000; 2014). Since the 1990s, the shift has turned to socio-cultural theories and ethno-mathematics (Verschaffel et al. 2015). New insights show that classical information processing models are inadequate in dealing with learners' solving word problems (Verschaffel et al., 2000, 2014). Since mathematics is a human activity, word problems must be anchored within the idea that problem-solving is a human activity in the mathematics classroom (Lave 1992; Verschaffel et al., 2000; 2014).

Few studies have been conducted on word problems among pre-service teachers Chapman (2017). Research on how Pre-service teachers deal with contextual factors is limited. Research is limited on how pre-service teachers deal with contextual factors of word problems, i.e., the extent to which realistic considerations are considered. Verschaffel et al. (1997) found that pre-service teachers lack realistic considerations when solving word problem tasks. The lack of realistic considerations manifested in pre-service teachers' solutions to problems and their evaluation of possible solution types Csikos & Szitanyi (2020). Researchers have examined children's learning of addition and subtraction concepts and skills from several perspectives (Carpenter et al., 1982). Carpenter et al. (1999) examined how children think mathematically, developing a scheme to classify problem-solving skills applicable to addition and subtraction. The scheme is based on the numerical relationships inherent within these contexts in which teaching and learning occur.

Carpenter et al. (1999) identify four basic classes of word problem types in these contexts: join and separate word problems, which children tend to solve using action. Part-part-whole word problems and comparison of word problems involve relating elements to the problem. Join word problems involve adding elements to a given set, and separate word problems involve removing elements from a given set. Part-part-whole problems require the study of the relationship between a set and its two subsets, comparing problems and encouraging comparison between two different problem sets.

Join and separate word problems use actions, making the problem types easier to understand and solve. Within each class of word problems are individual problem types distinguished by whichever element of the problem is unknown; for example, join problems have three elements: start, change, and result. If one presented the problem "Thabo has 6 sweets, and Mpho gave her 9 more sweets, how many sweets does Thabo have now?" that would be a join (result unknown) problem. The problem could be varied and presented as: "Thabo has 6 sweets. How many more sweets does he need to have 15 sweets altogether?". That would be a join (change unknown) problem. Table 1 lists and explains 11 problem types organised by class and unknown problem elements.

Another way to differentiate between problem types is to think about the number of sentences that can be used to represent them. Semantic propositions are listed in the order that follows the inherent meaning of the problem, such as 6+.? =15. In contrast, arithmetic propositions list the unknowns on one side of the problem separated by an equal sign Van de Walle (2007), for example, (15 - 6 =?). In the above examples, multiple semantically specific problems can be created by varying the structure of the problem, although most of the exact words appear in each problem.

In Table 1, we included both number sentence types described for each problem type. Some pairs are similar, while in some cases, they differ in the order of the numbers or the operations used. Besides problem type, another way these tasks can become complex depends on the magnitude of the number. Bahr & de Garcia (2008) point out that when a child seems confident at solving a problem, an increase in the size of one or both numbers usually presents a new challenge. Bringing the numbers down allows learners to have access to the problem. The two levels of difficulty based on numerical complexity are associated with all the problem types. The more accessible level includes single-digit numbers. For example, "Shawn had 4 toy cars. Lizeka gave him 3 more toy cars. How many does Lizeka have altogether?" or "Gertrude had 7 toy cars. She gave 4 toy cars to Sibusiso. How many toy cars does Gertrude have left?"

The more difficult problems involve a two-digit number Bahr & de Garcia (2008). For example, Donny had 4 toy cars. Lynn gave him 3 more toy cars. How many toy cars does she have?" or "Lindiwe had 12 toy cars. She gave 4 toy cars to Sibusiso. How many toy cars does she have left?" Although both problems involve facts of 18 or fewer, including numbers greater than 10, it represents a noticeable increment in numerical complexity that is significant for a young child.

Table 1. Classification of word problems with number sentences (Adapted from Carpenter et al., 1999, p.12).

Problem Type	Problem		
Join	*Result Unknown* Mike had 4 toy cars. Lynn gave him 8 more toy cars, so he now has a total of 12 toy cars. 4 + 8 =?	*Change Unknown* Sue has 4 toy cars. How many more does she need to have 12 toy cars altogether? 4 +? = 12 12 – 4=?	*Start Unknown* Lerato had some toy cars. Lunga gave her more toy cars, and she now has 12 toy cars altogether. How many toy cars did Lerato start with? ? + 4 = 12 12 – 4=?
Separate	*Result Unknown* Michelle had 12 toy cars. She gave 4 toy cars to Lynn. How many toy cars does Michelle have left? 12–4=? 12 – 4 =?	*Change Unknown* Sue had 12 toy cars. She gave some to Gary. Now she has 4 toy cars left? How many toy cars did she give to Gary? 12 - ? = 4 12 – 4 =?	*Start Unknown* Lerato had some toy cars? She gave 4 to Philip. Now she has 8 toy cars left. How many toy cars did Betty start with? ? – 4 = 8 4 + 8 =?
Part-Part-Whole	*Whole Unknown* Michelle has 4 red toy cars and 8 yellow toy cars. How many toy cars does she have? 4 + 8 =? 4+8=?	*Part Unknown* Sue has 12 toy cars; 4 are red, and the rest are yellow. How many yellow toy cars does Sue have? 12 = 4 +? 12 - 4 =?	
Compare	*Difference Unknown* Michelle has 4 toy cars, and Lynn has 12 toy cars. How many more toy cars does Lynn have than Michelle? 4 +? = 12 12 – 4 =?	*Compare Quantity Unknown* Sue has 4 toy cars. Gary has 8 more toy cars than Sue. How many toy cars does Gary have? 4 + 8 =? 4 + 8 =?	*Referent Unknown* Lerato has 12 toy cars. She has 4 more than Philip. How many toy cars does Philip have? 12 = 4 +? 12 – 4 =?

Gap Study & Objective

Numerous studies have investigated word problem-solving in the primary school context. Despite recognising the importance of word problems, there still needs to be a gap in understanding how pre-service teachers understand and approach the teaching of join and separate word problems, especially in the Foundation Phase (Grades 1-3). Many studies that have been conducted have paid attention to linguistic and computational task features on learner outcomes as well as cognitive processes involved in solving word problems. There is a paucity of research on pre-service teachers' foundational knowledge and instructional strategies for solving join and separate word problems. Teachers play a critical role in honing learners' problem-solving skills and understanding mathematics at an early age.

METHOD

Type and Design

The study employed a qualitative descriptive approach to gain insights into how foundation pre-service teachers understand types of word problems Sidiq & Choiri (2019). Qualitative descriptive research effectively captures participants' experiences and employs terminology that aligns with the original research questions in the original research questions (Bradshaw et al., 2017). Data were analysed using qualitative content analysis(QCA), focusing on pre-service teachers' responses to computational addition and subtraction problems and how they could be represented in word format. Each pre-service teacher's answers were examined based on the content displayed, precisely their responses to number sentence construction tasks. QCA was chosen for its ability to provide a nuanced interpretation of contextual meaning, making it a fitting choice within the naturalistic paradigm Hsieh & Shannon (2008).

A critical component of the research was the collection and analysis of data. The content analysis method examined the responses of pre-service teachers (PSTs) to assigned tasks. The researchers and mathematics lecturers in the course aimed to establish the pre-service teachers' (PTS) knowledge of word problem construction and identify the types of errors they made. The primary data consisted of the PST's responses to the assigned tasks. The foundation phase program mandates the pre-service teachers complete mathematics courses from the first to third years, covering topics such as Number Operations and Relationships, Patterns, functions, and algebra; shape and space (Geometry); measurement; and Data handling. The research was conducted in August after the PTSs had completed instruction in Numbers, operations and relationships, patterns, functions, and algebra.

Data and Data Sources

Data consisted of written responses from pre-service teachers to assigned tasks involving the construction and solving of join and separate word problems. Responses were collected from pre-service teachers in a Foundation Phase mathematics course. The tasks included creating number sentences and solving various types of join and separate problems, reflecting their understanding and instructional approaches. Contextual information about the content covered, including numbers, operations and relationships, patterns, functions, algebra, geometry, measurement and data handling, was obtained from course syllabi and instructional materials. These materials assisted in framing the educational background and the scope of knowledge expected from the pre-service teachers.

Data collection technique

Pre-service teachers were given six addition and subtraction computational tasks to assess their understanding of join and separate word problem construction and solution strategies. The tasks included constructing word problems from join and separate number sentences. The tasks were administered during regular class sessions to ensure an authentic educational setting. The focus groups facilitated collective reflection among pre-service teachers about their experiences and challenges in teaching word problems. The focus groups provided a platform for pre-service teachers to share insights and collaboratively explore solutions to common issues. Course materials and syllabi were analysed to contextualise the educational content covered in mathematics courses. This analysis helped to identify the scope and depth of word problem instruction provided to pre-service teachers.

Data analysis

The data were analysed using Qualitative Content Analysis (QCA). Pre-service teachers' responses were analysed to identify and categorise the content displayed in their responses, primarily focusing on their understanding and execution of join and separate problems. The analysis aimed to discover patterns, themes, and misconceptions in pre-service teachers' responses. The researchers read through all written responses and transcriptions of the focus group interviews multiple times to familiarize themselves with the data. Each response was systematically coded for critical concepts, terms, and phrases related to the join types and separate word problems. Segments of text representing distinct ideas or pieces of information were assigned codes. The codes were then grouped into broader categories representing common themes or patterns in the data. The themes that emerged were using inductive open coding of "sentences in learners' replies" Corbin & Straus (2015) the researchers compared every number sentence with the pre-service teachers' word problem. The procedure resulted in a group of sentences elaborating on each theme. Miles & Huberman (1994) suggested independently coding 25% of statements to enhance interrater reliability. Using random sampling from the 364-word problems, we- identified 97- (30%) word problems.

Responses were categorised based on whether they correctly identified the type of problem (result unknown, start unknown, change unknown) and the accuracy of the mathematical reasoning. Furthermore, the categories were analysed to develop overarching themes that encapsulated the pre-service teachers' knowledge and misconceptions about join and separate problems. Themes were derived to explore common errors, successful strategies and prevalent misconceptions. Themes were then interpreted within the context of literature in mathematics problem-solving. Findings were compared with established theories and previous studies about the pre-service teachers' and educational needs. To ensure the reliability and validity of the analysis, triangulation was used by comparing the written responses with focus group discussions. Peer review from colleagues in the mathematics department was sought.

RESULTS

The exercise should have produced 324 responses (6 questions multiplied by 64 student teachers) if all the Pre-service teachers had answered all the questions. We, however, managed to get 249 responses. Each variant of join and separate problem were analysed regarding the 'knowledge quartet' dimension - foundational (Rowland et al., 2009). What follows are thick descriptions and interpretive analyses of the student teachers' works focusing on how their (Mathematics Content Knowledge) MCK played out in their word problems derived from the number sentences (Table 2, Table 3).

Table 2. Summary of quantified Pre-Service Teachers' responses

Computational Problems	Problem Type	Correct Responses (%)	Common Errors/ Misconceptions
		JOIN	
9 + 5 = ?	Join Result Unknown (JRU)	75%	-Out of context
7+? = 13	Join Change Unknown (JCU)	67.2%	- Ambiguous scenarios - Reversing known and unknown quantities
? + 6= 11	Join Start Unknown (JSU)	67.2%	- Convoluted framing of start quantity - Unknown quantities positioned incorrectly
		SEPARATE	
7- 5=?	Separate Result Unknown (SRU)	76.6%	- Presented as join problems - Irrelevant context
12- ? = 7	Separate Change Unknown (SCU)	37.5%	- Unknown quantity misplaced

? – 5 = 12	Separate Start Unknown (SSU)	6.5%	- Context does not match subtraction - Difficulty framing start quantity - Resulting in negative numbers unintentionally

Table 3. Examples of Pre-Service Teachers' Word Problem Constructions

Computational Problem	Problem Type	Sample Correct Response	Sample Incorrect Response
		JOIN	
9+ 5= ?	Result Unknown (RU)	*"James has 9 oranges. Jim has 5 oranges. How many oranges do they have in total?"*	*"Nine more than five is?"* *"Nine add five is what?"* (Ambiguous statement, Repetition that lacks context)
7+? = 13	Change Unknown (CU)	*"Mary has 7 hats. How many more does she need to have 13 hats?"*	*"The teacher has 13 balls. She gives 7 to the girls. How many balls are left?"* (Logical but wrong context, misleading)
? + 6= 11	Start Unknown (SU)	*"How many cows does one need to add in a kraal with 6 cows to make them 11?"*	*"5 runners got gold medals. 6 runners managed to get silver medals in the runners' competition."*(Misinterpretation of the problem type, illogical inconsistency, Incorrect mathematical representation).
		SEPARATE	
7- 5 =	Result Unknown (RU)	*"7 birds were standing on a branch. 5 birds flew away. How many birds were left?"*	*"In a tournament that we had at school there were 7 kids and 5 of them did not come to school."* (Inaccurate Representation of subtraction, ambiguity in the problem, missing clarity in the word problem statement).
12- ?= 7	Change Unknown (CU)	*"I have 12 chickens. I need to give away some chickens until I am left with 7. How many chickens must I give away?"*	*"My grandfather had 12 cows. 5 died from an unknown virus."* (Misalignment of the problem type, Missing target equation, Clarity and Mathematical Context)
? – 5= 17	Start Unknown (RU)	*"Thando had some sweets. He gave Lulu 5 sweets. He was left with 17*	*"We had 17 chickens. 5 went missing. How many chickens are left?"*

	sweets. How many sweets did Thando start with?	(Problem Type Mismatch, Understanding of the problem structure)

Analysis of Join Result Unknown Problem: 9 + 5 =?

Three quantities are involved in the join action: an initial or starting amount, a change amount (the added or joined portion), and the resulting amount (the total amount after the action is complete). Any one of these can be unknown in a problem. The task was a Join Result Unknown (JRU) number sentence.

Below, we reproduce an excerpt from one of the PSTs in Figure 1. The excerpt highlights an ambiguous and repetitive word problem.

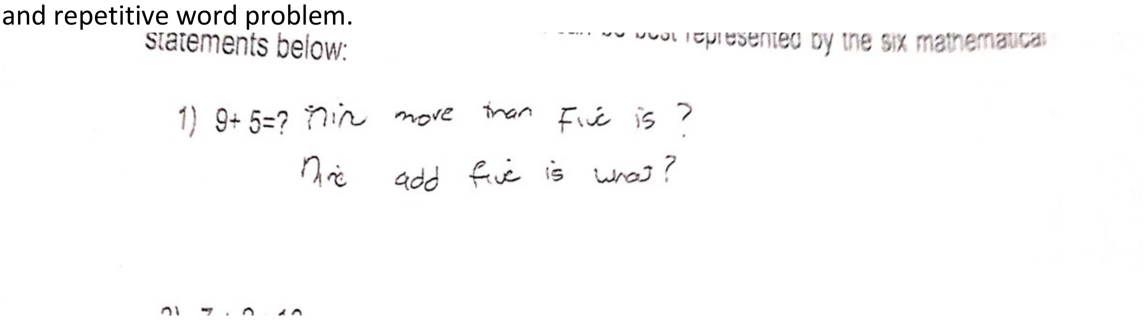

Figure 1. An example of an ambiguous repetitive statement lacking context.

Figure 1. show that pre-service teachers were asked to write the computation: 9 + 5 =? into a word problem. The PST identified that the problem involves addition, which is the correct operation for the offered computation problem (9+5=?). The PST understands the basic operation required to solve the problem. However, the construction of the word problem could be more precise. "*Nine more than five is"* depicts a comparative operation rather than a straightforward addition problem. While the statement "*Nine more than five "*could be interpreted as adding 9 to 5, which aligns with the mathematical operation, the wording is atypical for a word problem. There is repetition in the form of a restatement of the arithmetic operation rather than a contextually rich word problem. A critical component of word problems is their applicability to real-life situations. The response lacks real-world context since word problems are designed to place mathematical concepts in real-life situations.

While the PST indicates a basic understanding of the addition operation, the response fails to construct a meaningful word problem that locates the operation within a real-life context, translating the computation of the unknown operation into a scenario the learner can relate to and comprehend. A strong focus on clarity and context and transforming the mathematical operation into a world scenario would be beneficial.

A good word problem allows learners to engage with the problem that arises from the words, visuals, and numbers to explain how they solved it. This is especially critical for second-language learners who require much work with language in problem types such as those shown in Table 1.

Analysis of Join Change Unknown Problem: 7+? = 13.

The PST's response to the "Join Change Unknown" problem was framed as: "*The teacher has* 13 *balls. She gives 7 to the girls. Now, how many balls are left?"*. Below, we reproduce the excerpt of the PST's response.

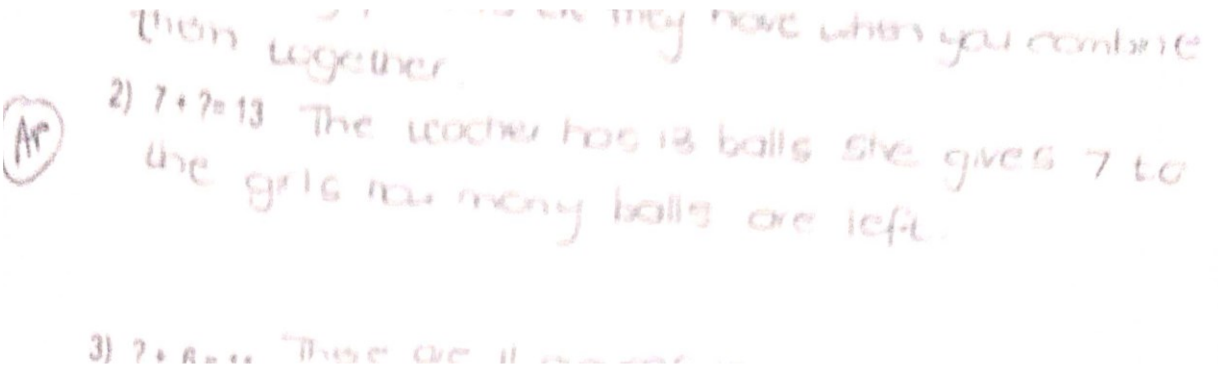

Figure 2. An example of an illogical, wrong context that is misleading

Figure 2. show the PST's response suggests a subtraction scenario where a known amount reduces a total quantity to find the remainder. Also, the response incorrectly represents the mathematical operation required by the problem. The original computation problem 7+? = 13 is a "Join Change Unknown" problem. The issue involves finding the missing addend needed to reach a total when one part is known. The response reveals a misalignment with the given computation problem. There is a contextual mismatch because the computational problem involves a situation where a total (13 balls) is reduced by a certain amount (7 balls), leaving the remaining number of balls unknown. However, this does not align with the "Join Change Unknown" problem, where the task determines how many more are needed to reach a total.

Since the PST's response suggests a subtraction operation (13-7=?), which fundamentally differs from the addition operation required by the original computation problem, this reveals a misunderstanding of the problem types and the associated mathematical operation. There is, therefore, confusion between addition and subtraction operations. While the PST provided a logical scenario, the scenario does not match the mathematical context of the original computational problem. The question "How many balls are left?" fits a subtraction scenario. However, it is inappropriate for an addition context where the objective is to determine how many more are needed to reach a total. There is potential for misleading learners if this is used in teaching since it will reinforce an incorrect understanding of how to solve "Join Change Unknown" problems. There is a need for PSTs to further understand problem types and their corresponding mathematical structure.

Analysis of Join Start Unknown Problem: ?+ 6 = 11

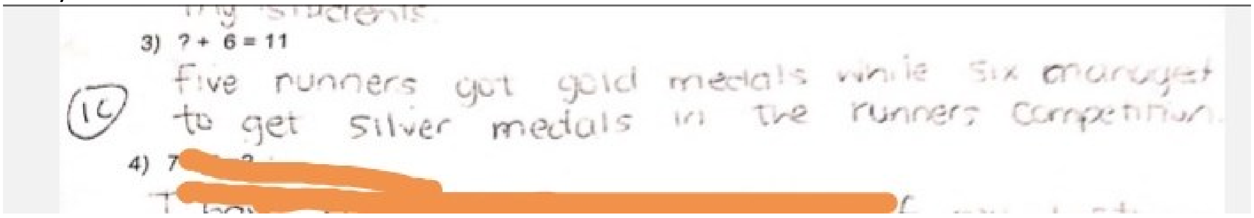

Figure 3. An example of a misinterpretation of the problem type, illogical inconsistency, and incorrect mathematical representation of the word problem

Figure 3. show the start unknown problem type required the PST to find the starting quantity when increased by 6 to a total of 11. However, the response does not address this structure. Instead of characterising a situation where a particular number (the unknown) is added to 6 to reach 11, the PST described two separate, unrelated quantities: gold medals and silver medals. The response misrepresented the problem type. There is a logical inconsistency in that the scenario presented seems to suggest that the gold and silver medals are separate categories, suggesting no addition to them. There is no indication in the response that the 5 gold medals and 6 silver medals add up to a single

total, which is what the problem requires. Of the 64 participants who responded to this question, 67.2% responded with the correct problem type structure.

The response shows an incorrect mathematical representation. It does not correspond with the number sentence: ? + 6+= 11. The scenario should show some items, e.g., athletes and medals. Objects start with an unknown quantity, and then 6 more are added, leading to 11. For instance,” The school awarded some gold medals. 6 medals were awarded later. In total 11 medals were awarded. How many gold medals were awarded at first?”. The response highlights an underlying problem with understanding the structure of join-start unknown problems. This response provides important insights into the difficulty experienced by PSTs in conceptualising a scenario where an unknown initial quantity, when combined with a known addition quantity, results in a given quantity when combined with a known addition quantity.

Analysis of a Separate Result Unknown problem type: 7- 5=?

Turning to separate result unknown problem structure, we provided an excerpt of a separate result unknown problem.

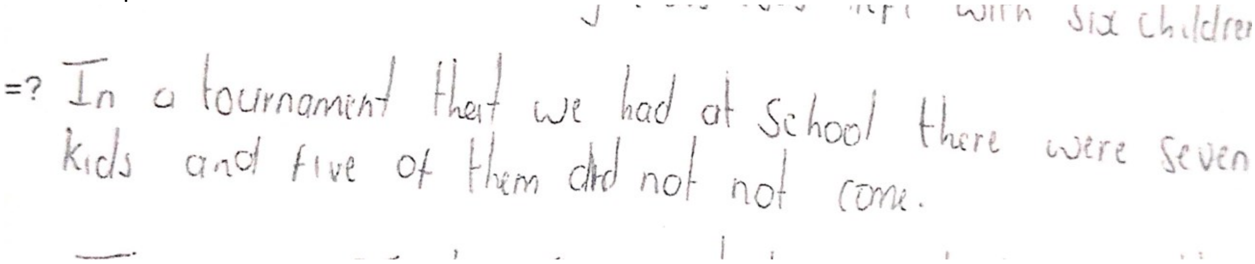

Figure 4. Inaccurate Representation of subtraction, ambiguity in the problem, and missing clarity in the word problem statement

Figure 4. show the word problem construction describes a subtraction scenario; however, the operation 7-5=? It is not accurately reflected. While the problem type suggests a scenario where 5 kids did not attend, the word problem does not state what is being subtracted from what. The connection between the number of kids who did not come and the remaining number is not made explicit. While the statement: “5 of them did not come” could imply that 5 is being subtracted from 7, the context is opaque about the final question or what is unknown (e.g., how many kids did not attend or are present. The construction of the problem leaves room for multiple interpretations. For example, someone might wonder whether the problem is asking for the number of kids who did not attend, which is 9s 2, or simply stating that 5 out of 7 kids did not attend without asking a specific question related to subtraction. Ideally, the word problem should end with a question that aligns with the subtraction operation, such as, “How many kids came to the tournament?”

The problem construction lacks a clear conclusion or direct question. A more effective construction would be:” There were 7 kids expected to attend the tournament. 5 kids did not come. how many kids attended the tournament?” This directly connects with the subtraction operation and the expected solution of 2.

Analysis of Separate Change Unknown problem type” 12-? =7.

Nest we focus on separate change unknown problem types. Below, we present an excerpt from one of the responses.

Figure 5. show there appears to be a misalignment of the problem type in the word construction offered. The number sentence 12-?=5 problem is a Separate unknown problem type. The initial quantity is known (12), the result after the subtraction is known (7 cows), but the quantity that was subtracted (died) is unknown. The word problem offered by the PST explicitly states the subtracted quantity. This turns the problem into a ‘Separate Result Unknown problem’, corresponding to 12-5=?.

The equation 12-7=? This should lead to a word problem in which the number of cows that die is unknown, not the number that remains. For instance,” Grandfather had 12 cows. Some cows died from an unknown virus. Now, he remains with 7 cows. How many cows died?”
Cows dying is a realistic context and provides a concrete scenario; however, critical mathematical detail (the number of cows that died) is missing, detracting from the problem's clarity and educational mathematical context.

5) 12 - ? = 7
My grandfather had twelve cows and five of his cows died from unknown virus
6) ? - 5 = 17

Figure 5. Misalignment of the problem type, Missing target equation, Clarity and Mathematical Context.

This error highlights a common issue in understanding problem types. The PST seems to have misunderstood the difference between “Change Unknown” and “Result Unknown” problems, which is essential for effective mathematics instruction.
Analysis of problem type Separate Start Unknown: ? – 5=17.
We present the last excerpt of word construction on the separate start unknown problem; the following is the construction offered by the PST,” We had 17 chickens. 5 went missing. How many chickens are left?”. We provide the excerpt below:

We had 17 chickens and 5 went missing how
chickens are left.

Figure 6. Problem type mismatch and understanding of the problem structure

Figure 6. show the original equation represents a “Separate Start Unknown” problem. The objective is determining the starting quantity (unknown) when a certain number is subtracted, leaving an unknown result. The word problem construction offers is typically a Separate Result Unknown problem (17-5=?). The initial number is known, and the result is sought after subtraction. Hence, the problem does not reflect the original equation‘s structure.

To be in tandem with the original problem, “? -5=17”, the word problem should describe a scenario where the original number of chickens is unknown, and after 5 chickens went missing, 17 are left. An appropriate word problem might be: “We had some chickens. 5 went missing. Now we have 17 left. How many chickens did we start with?”. The PST seems to misunderstand the structure of a Separate Start Unknown problem. The PST recognised that the subtraction context misrepresented which part of the equation should be unknown. The PST's formulation does not correctly represent the Separate Start Unknown problem type associated with the equation: ? – 5= 17. Instead, the problem is framed as a separate result unknown problem, showing a gap in understanding the differences between these problem types

DISCUSSIONS

The study aimed to explore pre-service teachers' understanding of join and separate word problems, focusing on their ability to construct them in word format. The analysis revealed several common patterns and misconceptions in how pre-service teachers approach word problems, particularly those involving unknowns. The results of this study indicate that pre-service teachers demonstrated challenges in correctly formulating word problems for change-unknown and start-unknown scenarios. The findings observed in this study mirror those of the previous studies Carpenter et al. (1999), where pre-service teachers demonstrated challenges in correctly formulating word problems for change-unknown and start-unknown scenarios. However, unlike previous studies Verschaffel et al. (2010), a significant portion of participants showed an overreliance on additive reasoning when solving separate word problems, indicating a gap in conceptual understanding. In accordance with the present results, previous studies have demonstrated that pre-service teachers tend to conflate additive and subtractive reasoning when constructing word problems (Verschaffel et al.,.2010). Our findings contrast with those of Sepeng & Webb (2012), who observed fewer errors in separating word problems, indicating potential differences in teacher training contexts.

Word problems are foundational in mathematics education as they help to bridge the gap between real-world applications and abstract mathematical concepts (Verschaffel et al., 2010). Word problems serve as a vehicle for promoting realistic mathematics, modelling, and problem-solving skills. The simplistic framing observed in the PTSs responses suggests a reliance on rote memory rather than a deeper understanding of mathematical relationships. However, the simplistic characterization of problems by some PTSs in this study, such as "Ben has nine bananas and five apples; how many bananas and apples does Ben have in total? implies a reliance on rote memory rather than a deeper understanding of mathematical relationships. This finding is consistent with findings from Csikos & Szitanyi (2020), which emphasise the role of teaching practices and cultural contexts in shaping pre-service teachers' problem-solving strategies.

The responses to join change unknown problems indicate a tendency to focus on superficial narratives rather than exploring the mathematical essence of the problem. Using the term 'sharing' in the separate result unknown problem indicated a potential confusion between addition and subtraction contexts, which can mislead learners. Although this characterization represents the situation, it lacks the complexity necessary to engage pre-service teachers in critical thinking about the underlying mathematical operations (Bradshaw et al., 2017).

This finding aligns with Carpenter et al. (1999), who emphasised the importance of precise language in transmitting mathematical ideas effectively. Blending operations in problem narratives may reflect broader issues in mathematical language and its implications for problem-solving. It is trite to consider the pedagogical implications of these findings to address the challenges PSTs face. The National Council for Accreditation of Teacher Education (2012) states that knowledge of both content and pedagogy is critical for effective instruction. Teacher education programs need to prioritise teachers' content knowledge and pedagogical strategies that enable PSTs to construct and interpret word problems meaningfully. Reflective practices and collaborative discussions with PSTs regarding word problem constructions could foster a deeper understanding and appreciation of mathematical concepts.

Teacher education programs must provide opportunities for PSTs to engage more with various complex word problems to avoid passing on misconceptions to learners in their future classrooms. The misconceptions displayed by PSTs, especially in constructing separate problems, suggest that PSTs could not have a solid foundation of inverse operations, leading to challenges in teaching problem-solving in primary schools. It appears that the tendency to misconstrue start-unknown and change-unknown problems might arise from insufficient exposure to complex word problem types during teacher training, especially in handling problem situations. Targeted instruction in word problem classification and construction is needed within teacher preparation programs. PSTs need to be equipped with strong content knowledge in different problem types to enable them to teach their learners problem-solving strategies more effectively. To minimize misconceptions, the curriculum could be strengthened by integrating more practice with complex, non-routine word problems.

Therefore, PSTs must employ various strategies to engage learners with different word problems. Teachers must incorporate real-life contexts and encourage learners to think critically about the problem structure to help learners develop a deeper understanding of mathematical operations and problem-solving. This study focused on pre-service teachers' written responses to word problems without capturing the PSTs 'reasoning through interviews or reflective writing. Future research could benefit from incorporating these additional data collection methods to understand better the thought processes behind problem construction. In addition, future research could investigate how PSTs' understanding of word problems evolves as they gain more teaching experience. Also, employing longitudinal studies could explore whether targeted interventions in teacher education programs lead to improved problem-solving instruction in the classroom. This investigation highlights crucial gaps in pre-service teachers' understanding of word problems and provides a foundation for improving mathematics education.

CONCLUSION

This study explored pre-service teachers' understanding and construction of join and separate word problems, shedding light on their ability to translate mathematical number sentences into meaningful problem contexts. The present results are significant in at least two major respects. Firstly, while PSTs generally demonstrated an understanding of basic problem structures, many continue to rely on superficial narratives and sometimes exhibit misconceptions in their use of mathematical language and context. Secondly, the investigation undergirds the importance of targeted interventions that zoom into deepening PTSs' understanding of the underlying mathematical structures of word problems. Hence, ensuring that PTS know the cognitive demands of constructing and solving such problems is critical to improving their future instructional practices. Since PSTs demonstrated a foundational understanding of join and separate problems, there is a clear need for more focused training on constructing mathematically rigorous and contextually meaningful word problems. Enhancing PST's pedagogical content knowledge in word problems will lead to better learning outcomes for their future students, fostering a more robust understanding of mathematics at the foundational level.

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