

## A constructivist analysis of Grade 8 learners' errors and misconceptions in simplifying mathematical algebraic expressions

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### ABSTRACT

Algebra is an important branch of mathematics which applies to many fields related to mathematics. However, many studies show algebra as posing problems even to the most gifted students. This phenomenon, therefore, necessitates more studies to be conducted in this area. As such, the study explored the types of errors that Grade 8 learners committed in simplifying algebraic expressions and the misconceptions that might have given rise to such errors. Ninety-five Grade 8 learners were selected as the subjects of the study at one high school in Lesotho. Within the framework of the Qualitative case study design, the study used tasks and interviews for data collection. The thematic approach to data analysis within the framework of the constructivist theory was adopted. The study identified most errors committed by the learners as persistent. Overgeneralizing the rules of prior knowledge to new knowledge, particularly in different contexts, was the most frequent cause of the errors. In addition to this was the misunderstanding and misinterpretation of correct meanings in the given context. Some of the identified errors overlapped with those in the reviewed literature while others did not.

## INTRODUCTION

A constructivist theory maintains that students construct their own knowledge through self-modification of cognitive structures (Luneta & Makonye, 2013; Iddrisu et al., 2017). In this view, a student is to accommodate a novel piece of information (Iddrisu et al., 2017). It is during the accommodation process that the knowledge is refined and reorganized. Further central to the theory is that students develop misconceptions during the accommodation process. As such, they could be seen as not coming to class as empty vessels or blank slates. Rather, they could be viewed as bringing with them to class ideas from interacting with the environment, the feature which may either resonate or be inconsistent with the accepted mathematical and scientific concepts presented during instruction. Such ideas, though inconsistent with some accepted norms by the mathematical community, are of interest in this study.

Misconceptions are unavoidable stages in knowledge acquisition (Fumador & Agyei, 2018; Irawati et al., 2018). There is, therefore, a need for further research on and for better understanding of misconceptions and their role in learning. While human errors could be seen as part of human nature, any persistence of such errors signals an individual conception which is inconsistent with that of the mathematical or scientific community. Many studies have been conducted on errors and misconceptions in algebra (the topic of the reported study) in mathematics education. Mulungye et

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al.'s (2016), Pournara et. al.'s (2016), Iddrisu et. al.'s (2017), and Sarımanoğlu's (2019) studies examined the various errors committed by students in algebra and the misconceptions that gave rise to such errors. The studies thus aimed at not only identifying the origin of such errors, but also at suggesting remedy for classroom teaching. Makonye (2016), Fumador and Agyei (2018), Gardee and Brodie (2015), and Irawati et. al. (2018) investigated the extent to which pedagogical intervention could help learners to minimize their errors and misconceptions in algebra. In the context of Lesotho, the studies conducted by Moru and Qhobela (2013) and Moru et. al. (2014) on errors and misconceptions set out to investigate sets and differential calculus, respectively. These studies found the teachers' pedagogical content knowledge with regard to identifying errors, diagnosing their source, and suggesting a remedial action to address the misconceptions that gave rise to them.

According to the Examinations Council of Lesotho (ECOL) examiners' reports on the high school-leaving examinations, the students' performance in algebra was not satisfactory (ECOL, 2018; 2019). Students who are incompetent in algebra cannot do well in areas of mathematics such as calculus, analysis, geometry, and trigonometry. This is because algebra is the cornerstone in dealing with concepts within these areas. In addition, these branches of mathematics are also important in such disciplines as science, technology, economics, and engineering. This shows that a lack of algebraic skills and knowledge is of utmost importance. Iddrisu et. al.'s (2017) study has also shown that regardless of the efforts made in addressing the stated difficulties, algebra still poses problems even to the most "gifted" students. This view is supported by Marpa (2019) who contends that learners consider algebra to be a difficult branch of mathematics. Thus the study sought to explore students' errors in simplifying algebraic expressions and their possible sources, thereby suggesting teachers' remedial actions for minimizing the problem.

The prescribed textbook for Grade 8 Learners in Lesotho high schools defines an algebraic expression as follows: "An algebraic expression contains terms variables, coefficients and constants" (Makara & Ntau, 2019, p. 92). Adding to this definition, we would see an algebraic expression as a collection of numbers, variables and signs (positive or negative) connected by any of the four basic operations (addition, subtraction, multiplication and division). Such an expression does not have an equal sign; otherwise, it would be taken as an algebraic equation. As defined in the prescribed textbook, the constructs associated with algebraic expressions include the following: terms, variables, coefficients and constants. A typical example of an algebraic expression is  $-3x^2 + 4ax + 5$ . The given expression has three terms ( $-3x^2$ ,  $4ax$ , and 5), two variables ( $a$  and  $x$ ), two coefficients (-3 and 4) and one constant (5). The terms may either be like or unlike. For instance, all the terms that appear in the given expression are unlike whereas terms like  $-3x^2$  and  $2x^2$  or  $2ax$  and  $3xa$  are like terms. The first two terms are the coefficients of the variable  $x$  raised to the same power while the latter terms have variables  $ax$  and  $xa$  which carry the same meaning or interpretation because of the commutative property that holds between  $a$  multiplied by  $x$  and  $x$  multiplied by  $a$ . Other algebraic expressions may be written in the rational form as in the case of  $\frac{3x+2a-3}{ab}$ , where  $ab$  is the common denominator and the simplification should consider this. The given examples show that the definition provided in the prescribed text book for the learners lack some of the key elements of an algebraic expression. The issue of the signed terms is left out and the nature of the operations that form part of the algebraic expressions. It was also necessary to put emphasis on differentiating between an algebraic expression and an algebraic equation as the two are often taken to mean one and the same thing by learners. This we have witnessed through our teaching experience.

Most studies (e.g. Gardee & Brodie, 2015; Makonye, 2016; Fumador & Agyei, 2018; Irawati et al., 2018) conducted elsewhere, focused on the pedagogical intervention to minimize learners' errors and misconceptions in algebraic expressions. Some employed descriptive and inferential statistics (e.g. Iddrisu et al., 2017). Pournara et. al. (2016) tracked a cohort from Grade 8 to Grade 11 on the extent to which they commit errors. Sarımanoğlu (2019) used the context of algebraic equations which differ from algebraic expressions. A few studies on errors and misconceptions conducted in Lesotho were on sets and differential calculus. The conducted studies in Lesotho differ with the reported one in two ways: (i) the studies were analysed through the lens of pedagogical content knowledge on error analysis, and (ii) the subjects of the study were the teachers and not the learners. In the reported study the learners were the subjects of the study and the theory that underpinned the study is the constructivist view of learning. Since each topic in mathematics is unique, this paucity

necessitates an investigation of errors and misconceptions in algebra in this particular context. The main purpose of the reported study therefore, was to investigate the errors committed by Grade 8 learners in Lesotho when simplifying the algebraic expressions together with tracing the misconceptions that might have given rise to such errors. The research questions emanating from this purpose are:

1. What errors do Grade 8 learners' display when solving algebraic expressions?
2. What type of misconceptions seem to have given rise to these errors?
3. What are the possible causes of such misconceptions?

Noting errors as unavoidable stages in learning, we believe that the findings of the study will sensitize many teachers to the importance of error analysis. The findings will also capacitate and/or enable teachers to organize instruction in such a way as to attend to any misconceptions arising from the identified errors. Since, in some cases, learners' errors differ from one context to another, the findings of the study will most probably contribute to existing empirical body of knowledge, particularly mathematics education literature.

### Literature review

The literature on errors and misconceptions has been reviewed from both theoretical and empirical perspectives. In this case, these two perspectives which are discussed, in turn below, could be considered complementary.

#### *Errors and misconceptions: theoretical perspective*

An error is regarded as a mistake committed in the process of solving mathematical problems algorithmically, procedurally or by any other method (Mulungye et al., 2016). Errors may also be defined as mistakes made by learners, which can occur for a number of reasons ranging from a data entry or calculation error to a lack of conceptual understanding (Holmes et al., 2013). Through experience as teachers we have identified learners' errors at any step in the implemented method or when an incorrect answer is given. The incorrect answer may result either from the proper method incorrectly carried out (procedural) or an application of an incorrect step or procedure to a question that cannot be solved using such a procedure or method (conceptual). As Makonye and Fakude (2016) posit, errors can either be slips or persistent. Slips are human errors that are not systematic but sporadic. They are said to be a result of carelessness and are easily detected and corrected. On this basis, slips are not a sign of conceptual misunderstanding; such slips can be made not only by novices, but also by experts (Gardee & Brodie, 2015). Frequent or persistent errors which cannot be corrected through typical instruction are caused by misconceptions; however, such errors need supplementary intervention in order for learners to acquire correct strategies (Booth et al., 2014; Makonye, 2016; Makonye & Fakude, 2016). Irawati et. al. (2018) assert that misconceptions, which are unavoidable and give rise to persistent or frequent errors, are beliefs or ideas that learners possess regarding the phenomena that are scientifically unreliable. Ojose (2015), on the other hand, defines misconceptions as "misunderstandings and misinterpretations based on incorrect meanings" (p.30). Holmes et. al. (2013) define a mathematical misconception as part of a learner's framework that is not consistent with that of the mathematical community which leads such a learner to providing incorrect answers. Central to the above descriptions is resonance of both errors and misconceptions, thus showing misconceptions as being inconsistent with the scientific meaning of concepts and traceable to the persistent resultant errors.

#### *Errors and misconceptions in algebraic expressions: empirical perspective*

Sarımanoğlu (2019) conducted a study in Turkey focusing on Grade 7's errors and misconceptions in dealing with algebraic expressions. The study revealed that learners took no notice of the negative sign during the manipulation of algebraic expressions. Similarly, Pournara et. al. (2016) argue that learners fail to pay attention to signs in dealing with algebraic expressions. As reported in Pournara et. al.'s (2016) study, participants saw the negative sign as representing only the subtraction. These studies show that learners only consider the signs in between the expressions, without considering the sign that categorise the terms as either negative or positive. The studies conducted by Mulungye et. al. (2016) and A'yun and Lukito (2018) reported that a *radical sign error* was committed in the second degree radical addition. Here, the learners were given the task in which

they simplified  $\sqrt{x^2 + y^2}$  to  $x + y$ . Thus, they applied the rule of simplification of radicals as though the two terms were multiplied by each other instead of being added together.

In Mulungye et al. (2016), learners were also asked to simplify  $3x + 5$ . The answer obtained by some learners after simplification was  $8x$ . The same question was posed in the study by Pournara et al. (2016) in which the same result was obtained. These findings show that this was a frequently occurring error among the participants. Mulungye et al. (2016), therefore, asserted that learners perceived the signs “+” or “-” “as an invitation to do something, which the learners continued to do. Learners thought that the answer should be free from operator symbols, hence conjoining (*conjoin error*). Concurring with the view, Makonye (2016) noted learners for seeing “+” and “-” as an instructional directive to do something. This finding seems to highlight their arithmetic understanding that  $5 + 4$  yields 9, with the rule being generalised to algebraic expressions where it does not apply (Makonye, 2016).

Change to an equation error (or an *equation formation error*) was committed by learners as shown by Makonye’s (2016) study. In simplifying the rational algebraic expression,  $\frac{3x+6}{6}$ , some learners came up with answers like  $x = 2$ ,  $x = 4$  and  $x = 3$  which were a result of some simplification errors amongst which was the interference of schema for solving equations. The rule for simplifying algebraic equations was mistakenly generalised to the rules of simplifying algebraic expressions to which it is not applicable. Moreover, Pournara et al. (2016) reported participants’ simplification of  $2a + 5a$  to  $7a^2$ . In the same vein, an error was observed in Mulungye et al.’s (2016) study where learners simplified  $3x + 3x$  and obtained  $6x^2$ . Learners, in these studies, multiplied the variables instead of adding the like terms in the algebraic expressions. As such, the learners misapplied the rule of exponents by committing an *exponent error*. In the case of Iddrisu et al. (2017), learners committed the *reversal error* when forming the algebraic expression from a sentence. For instance, the problem read: “subtract  $3a$  from 7”, the subtrahend is  $3a$  and the minuend is 7. In this particular study, the learners carried out the operations in the reverse order by matching the letter in the given words order by writing the expression  $3a - 7$  as an alternative to  $7 - 3a$ . The same phenomenon of reversal error was observed by Aydin-Guc and Aygun (2021).

The studies conducted by Dodzo (2016) and Mulungye et al. (2016) showed learners as having *difficulties dealing with the variables* in the simplification of algebraic expressions. For example, Dodzo (2016) found some learners as simplifying  $2x + 5$  to 7, and thus ignoring the variable,  $x$ . This finding could therefore indicate the learners’ incompetence in a variable in one of the terms. Another related study by Mulungye et al. (2016) noted students for simplifying the expression  $3x + 5$  to  $8x$ , suggesting that the students were not aware that  $3x$  and 5 are unlike terms. Although the expressions are of the same form in the two studies, the errors displayed about variables are different. Gardee and Brodie’s (2015) study showed that learners were convinced that a letter has a specific fixed value (*substituting a numeric value for letters error*). In the same study, learners assigned values to the variables which were not in the question. In addition, learners went further to say a number order is as good as an alphabetical order. For example, since letter “d” is the fourth letter in the alphabet, it has the value of 4 in algebraic problems. In Pournara et al. (2016), learners used two strategies in committing errors: (i) *Equal splits*: splitting the known quantity equally depending on the number of letters on the left and then giving the new letter the same value as the others, for example  $e = f = 4$  so  $g$  also has a value of 4, hence  $e + f + g = 12$ . (ii) *Assigning a value of 1*: the new letter is given a value of 1, because its value is unknown, it can be any value, with the simplest value being chosen and giving  $e + f + g = 8 + 1 = 9$ .

### Theoretical framework

This study is underpinned by the Piagetian constructivist theory. Olusegun (2015) describes constructivism as a learning theory found in psychology which explains how people might acquire knowledge. According to Olusegun (2015), this theory suggests that humans construct knowledge and meaning from their experiences. Furthermore, constructivism views learners as actively constructing knowledge rather than passively receiving it from the environment (Davis et al., 2020).

The constructivist view also sees learners as using their prior knowledge as a foundation for building new knowledge. In this view, learners do not come to class as empty vessels; rather, they bring with them prior knowledge from previous classes and other interactions with the environment.

Further related directly to the process of knowledge acquisition is the idea of schema. According to Luneta and Makonye (2013), schemas are valuable intellectual tools in human memory that allow for the generalisation, synthesis, storage and retrieval of similar experiences. Schemas are developed or refined through the complementary processes of assimilation and accommodation (Zhiqing, 2015). The process by which individuals incorporate new experience into an already existing schema is termed assimilation (Bhattacharjee, 2015). When a new object is assimilated into an old schema, the schema gets refined (Zhiqing, 2015). Accommodation is a process of restructuring or modifying the existing schema to incorporate new information (Bhattacharjee, 2015; Zhiqing, 2015). When a learner fails to connect the old and new knowledge, this leads to the formation of misconceptions as a result of misinterpretation and poor linkage of assimilation and accommodation (Luneta & Makonye, 2013; Ojose, 2015).

## **METHODS**

This section of the paper focuses on the following: the research design, the research setting, population and sampling, data collection techniques, validity and data analysis techniques.

### **Research design**

A qualitative case study design was employed for this study. This is because a case study is a type of qualitative research in which in-depth data are gathered concerning a single individual, programme, or event for the purpose of learning more about it (Starman, 2013; Njie & Asimiran, 2014). This is done on the natural setting of a phenomenon under study (Njie & Asimiran, 2014). Multiple methods of data collection are also employed to better understand the phenomenon under study (Njie & Asimiran, 2014; Shareia, 2016). On this premise, the study involved an in-depth data collection methods and analysis of a group (observed as a case) of Grade 8 learners in a school environment. The group was studied in relation to the type of errors committed in simplifying algebraic expressions. In addition, the type of misconceptions that might have given rise to these errors was also the focus of the study. The phenomenon was investigated for the purpose of suggesting ways of remedying and improving learning of algebraic expressions. Multiple methods of data collection were used. The findings were expected to give a way forward as to the type of studies that could be conducted to contribute further in studying the same phenomenon.

### **The research setting**

The study setting is one high school in the semi-urban area in Lesotho. This school is located half way (16 kilometres from each side) between a town and a township, namely Teyateyaneng and Mapoteng. The school consists of classes from Grade 8 to Grade 11. It was selected purposively because it is closer to the second co-author, hence it was easily accessible and had no time and financial constraints. It is a co-educational school which admits learners from the entire neighbouring villages.

### **Population and sampling**

The population comprised 120 Grade 8 learners, 95 (females and 36 males) of whom constituted the sample for the study. The learners' ages ranged from 13 to 15 years. One hundred and twenty pieces of paper numbered 1 to 120 were placed in a box. The learners were asked to pick one piece of paper from the box without looking on. Learners who chose pieces of paper from 1 to 95 were identified as the sample of the study. The number was reduced from 120 to 95 for the following reasons: (i) making the printing of the tasks cheaper and (ii) having a manageable group for data analysis.

### **Data collection**

The data were collected from tasks on simplifying algebraic expressions and interviews. The tasks were set in such a way that they had the potential to reveal the students' misconceptions from the errors that were likely to be committed. The choice of the tasks was guided by the type of errors reviewed in the literature together with some variations where there was a need to do so according to the level of education of the learners. Two out of the six set tasks tested the learners' understanding of the technical terms used in the algebraic expressions. The purpose of setting such types of tasks was to see if a lack of understanding of some terms could lead to committing some errors. Twelve learners were selected for the interviews on the basis of the variation of the errors

displayed in their written responses to the tasks. The learners responded to the tasks under the supervision of the second co-author on 10<sup>th</sup> June, 2021. The time given for the completion of the tasks was 1 hour 40 minutes. All the target learners completed the tasks within the specified time. The interviews were conducted a week later from the 17<sup>th</sup> June, 2021 to 8<sup>th</sup> July, 2021. On average the interviews lasted for about 1 hour. The one week taken before the conduct of the interviews was to allow the researchers to go through the learners' responses to the tasks so as to make an informed choice of the interviewees.

### Validity

In order to ensure the credible data in answering the research questions the study took two main actions. Firstly, the research instruments were scrutinized by a group of both mathematics and science educators in the Department of Science Education at the National University of Lesotho. This is a normal practice for any research study conducted in fulfilment or partial fulfilment of the requirements for the Masters programme. Secondly, the data collection was administered by the second co-author by monitoring and ensuring the learners' individual work, rather than discussions when responding to the written tasks.

### Data analysis

The major mode of analysis used was thematic from the learners' tasks. The interview data were used to clarify the learners' responses to the tasks. Thematic analysis is described by Ibrahim (2012) as the process of locating the thinking pattern of the participants and the pattern of action showed. In the light of these, the researchers took the learners' tasks one- by-one and analyzed them noting their error patterns on their solutions. The displayed errors were shaded with a colour and abbreviated, using the first letters of the type of error. For example, a *conjoin error* was abbreviated as CE and marked with a blue colour and an *exponent error* was labelled EE and was highlighted with a green colour, just to mention a few. This was to ease the process of counting the number of learners who were involved in committing each type of error in data analysis. The misconceptions that gave rise to the errors were traced back to the students' tasks and interview responses by using the researchers' knowledge of mathematics within the framework of the constructivist theory.

## FINDINGS

During the analysis the researchers identified nine persistent errors made by the learners, six of which cohere with the findings of the previous studies as reviewed in the literature. These are a *conjoin error*, an *exponent error*, a *reversal error*, *learners' difficulties with variables*, *substituting numeric values for letters*, and an *equation formation error*. The other two errors unique to the study are a *commutative like term error*, and an *imposed radical sign error*. Individual errors are now presented together with the diagnosis of their causal misconceptions within the framework of the constructivist theory.

### Conjoin error and misconception(s)

The questions where a conjoin error was committed and a number of learners per question are shown in Table 1. The conjoin error is the unnecessary addition or subtraction of unlike terms (Pournara et al., 2016). This type of error is seemingly caused by a lack of understanding the concept of like and unlike terms. Learners simplified  $2x + 3y$  to  $5xy$  and  $3x + 5$  to  $8x$ . Forty-six (48.4%) out of 95 of the learners did the same error two or more times from the written tasks. While 14 (14.7%) out of 95 committed the same error once. Learner 44 committed a conjoin error in Q3 (i), Q4 (v), Q4 (vi), Q4 (vii) and Q4 (viii). Below is what transpired in the interview with learner 44 about Q4 (v). Figure 1 shows the original work of L44.

- R : Can you tell me the answer that you have and how you came up with this answer.  
 L44 :  $9ab$ . I got 9 after adding 5 and 4 since the sign in between is plus and I put "a" and "b" at the end to get  $9ab$ .  
 R : Ok. What came into your mind to have solved it in this way?  
 L44 : The question said simplify so I added the two terms sir, we cannot leave it as  $5a + 4b$  since it is not simplified and the question said simplify.

**Table 1**  
Learners' responses and the frequency of conjoin error

Question	Answer	Number of learners	Percentage
Simplify where possible			
Q3 (i) Add 4 to $3n$	$7n$	54	56.8%
Q4 (v) $5a + 4b$	$9ab$	45	47.4%
Q4 (vi) $3x + 5$	$8x$	45	47.4%
Q4 (vii) $2x + 3y$	$5xy$	49	51.6%
Q4 (xi) $9v - 6w$	$3vw$	48	50.5%
Simplify the given algebraic expressions			
Q6 (i) If $a + b = 6$ then $a + b + c = ?$	$6c$	11	11.6%
Q6 (ii) ii. If $e + f = 8$ then $e + f + g = ?$	$8g$	11	11.6%

Handwritten working and answer for Q4 (v):

$$5a + 4b$$

$$5 + 4 = 9$$

$$= 9ab$$

Answer ... 9ab ...

**Figure 1.** The working and answer of L44 when responding to Q4 (v) on a conjoin error.

To the learner the word 'simplify' seems to mean having to come up with a single term at the end regardless of whether the terms are like or unlike. In arithmetic, one cannot leave the answer as  $5 + 4$  but as  $9$  after using the addition operation. In this case, the learners assimilated addition of an algebraic expression schema into an inappropriate addition of numbers schema. This is confirmed when L44 says "... we cannot leave it as  $5a + 4b$  since it is not simplified and the question said simplify". This misconception could also be attributed to the language of origin, that is, a cross-linguistic influence or language transfer. In Sesotho *ho kopanya*, meaning 'to add' could be interpreted as 'to combine' or 'bring together'. Therefore, the Sesotho language worldview might have been mapped onto or transferred into mathematical applications, particularly in the rules of simplifying algebraic expressions.

### Exponent error and misconception(s)

Table 2 shows the questions where exponent error was made and number of learners per question who committed it. Forty-five (47.4%) out of 95 of the learners committed the exponent error persistently. Learner 5 made an exponent error in Q4 (i), Q4 (ii), Q4 (x) and Q4 (xi). Figure 2 shows the working of L5 to Question 4(ix). During the interview the learner explained her thinking as follows:

- R : What answer do you have to this question? And can you tell me how you simplified to get to this answer.
- L5 : I have  $d^2$ . I added  $d$  plus  $d$  and said  $d^{1+1}$  is  $d^2$
- R : Please explain again.
- L5 : Sir, I said  $d$  plus  $d$  is  $d^2$ , because when the bases are the same we add the powers.

The learner says "when the bases are the same we add the powers". Learners mistook the rules for addition of algebraic exponents to that of multiplication of exponents. They retrieved the incorrect exponent schema to answer this question about algebraic expressions.

### Reversal error and misconception(s)

Table 3 shows the questions in which the reversal error was committed and the number of learners per question.

**Table 2**  
Learners' responses and the frequency of exponent error

Question	Answer	Number of learners	Percentage
Simplify where possible			
Q4 (i) $a + 7a$	$7a^2$	35	36.8%
Q4 (ii) $-9a + 6a$	$-15a^2$	35	36.8%
Q4 (ix) $d + d$	$d^2$	60	63.2%
Q4 (x) $y^4 + y^4$	$y^8$	45	47.4%
Q4 (xi) $2k^2 + k^2$	$3k^4$	40	42.1%

Handwritten work for Q4 (ix):  
 $d + d$   
 $d \times d$   
 $= d^{1+1} = d^2$   
 Answer .....  $d^2$

**Figure 2.** L5's workings and the answer demonstrating an exponent error on Q 4 (ix)

**Table 3**  
Learners' responses and the frequency reflecting the reversal error

Question	Answer	Number of learners	Percentage
Write each phrase as an algebraic expression and simplify			
Q3 (ii) $2x$ less than 3	$2x - 3$	48	50.5%
Q3 (iii) Subtract $3a$ from 5	$3a - 5$	48	50.5%
Q3 (iv) take $2x$ from $6x$	$2x - 6x$	40	42.1%

Handwritten work for Q3 (ii):  
 $2x$  less than 3  
 $2x - 3$   
 Answer .....  $2x - 3$

**Figure 3.** L2's workings and answer to Q3 (ii) illustrating a reversal error.

This type of error is likely to be resulting from the language used as in the prepositional verb “take... from...” instead of “subtract ... from” and “less than” which have the same frequency. “Take ... from” may be a verb phrase that is used in everyday language, hence a little bit easier to deal with. Eight learners were aware of the intended meaning here. Forty-eight (50.5%) learners made this error persistently in the first two expressions Q3(ii) and Q3(iii).

The interview was conducted with learner 2 who happened to make reversal error in all the questions in Table 3. Accounting for the given answer to Q3 (ii) in Figure 3, the following is the conversation with L2.

- R : What answer do you have for this question and tell me how you came up with your answer.  
 L2 : My answer is  $2x - 3$  because the question says  $2x$  less than 3 so less than is minus in algebraic expressions.  
 R : Are you saying  $2x$  less than 3 is written as  $2x - 3$ .  
 L2 : Yes Sir

The learner matched the order of words as they appear in the question; “...less than means minus”. Whatever comes first is written first together with the minus sign in place of “less than”. According to constructivism, this shows that the schemas for the phrases used were not coherent. When counting numbers in arithmetic, the order is crucial. This conception seems to be applied to the irrelevant context.



**Table 4**

Learners' responses showing the learners' difficulties with variables and their frequency			
Question	Answer	Number of learners	Percentage
Simplify where possible			
Q4 (v) $5a + 4b$	$9ab$	48	50.5%
Q4 (vi) $3x + 5$	$8x$	45	47.4%
Q4 (viii) $9v - 6w$	$3vw$	40	42.1%
Q5(ii) $8q - 5q - 3q$	$0q$	45	47.4%
Express in algebraic form and simplify			
Q6 (iv) Find the perimeter of the triangle with the sides $3n, 6, 4$ all sides are in $cm$	$13n\ cm$	38	40%

**Figure 4.** L12's workings on Q 5 (ii) on errors difficulties with variables.

### Learners' difficulties with variables and misconception(s)

Table 4 shows the results of simplification in which the learners ignored the significance of the variables which were part of the terms of the algebraic expressions. Learner 12 made this error across all the five questions. Below is an extract from learner 12's interview in Question 5 (ii). The original work of the learner when responding to this question is reflected in Figure 4.

- R : Can you please tell me the answer that you have and explain how you came up with it.  
 L12 : Sir, I took 8, subtracted 5 and subtracted 3 and I got 0. As a result, I put q in front of zero to get 0  
 R : Where is the letter q you are writing in front of zero from?  
 L12 : I brought it down here from the question.

The language used displays that variables are ignored during the simplification process. The learner said, "I put q" thereby explaining how the variable comes to be placed next to the number. Their use of the phrase "in front" could be seen as showing their reading from right to left, or it could be a result of the use of everyday language where two people who are walking to the right, the one in the position of q and other mentioned variables are in front or ahead. Learner 12 said, "I brought down q", thus illustrating that the variable q was ignored and attended to last. The interview revealed that learner typically reflects an operational view of addition from arithmetic not considering the variables and the sign. This means that the learner simplified by first attending to addition of numbers. In this error, learners have a misconception of interference of arithmetic rules with those of algebraic expressions. In arithmetic the subtraction of integer schema was developed first and interfered with the new algebraic expression schema. This led to a misconception as the learner assimilated the algebraic simplification schema to an inappropriate arithmetic schema of addition or subtraction. This probably indicates that the restructuring of the schema during the accommodation process was not successful.

### Substituting numeric values for letters and misconception(s)

Learners turned to substitute numeric values for letters. For example, where "a" is given a value 1 is substituted and 2 for "b" and in the case of "c" 3 is used. The other type of substitution of numeric values for a letter took a different form as will be explained after the presentation of the results in a tabular form. Table 5 gives the number of learners and the questions in which substituting numeric values for letters was done.

**Table 5**  
Learners' responses and frequency of substituting numeric values for letters

Question	Answer	Number of learners	Percentage
Simplify where possible			
Q4 (i) $a + 7a$	8	17	17.9%
Q4 (ii) $-9a + 6a$	-15	14	14.7%
Q4 (iii) $-5r - 4r$	-r	10	10.5%
Simplify the given algebraic expressions			
Q6 (i) If $a + b = 6$ then $a + b + c = ?$	9	43	45.3%
Q6 (ii) If $e + f = 8$ then $e + f + g = ?$	12	40	42.1%
Q6 (iii) If $m + n = 4$ then $m + n + 10 = ?$	14	40	42.1%

$a + 7a$   
 $1 + 7 \times 1$   
 $= 1 + 7 = 8$   
 Answer ..... 8

**Figure 5A.** Learner 93's working for Question 4 (i) of substituting numeric values for letters.

If  $a + b = 6$  find  $a + b + c = ?$   
 $3 + 3 = 6$   
 $a + b + c$   
 $3 + 3 + 3 = 9$   
 Answer ..... 9

**Figure 5B.** The workings and answer for L24 on Q 6 (i) showing substitution of a numeric value for a letter.

In Q6 (i), (ii) and (iii) learners applied what Pournara et. al. (2016) call equal splits where the letters are assigned the same value. In this question learners were given  $a + b = 6$  and asked to find  $a + b + c = ?$ . Learners decided to give each letter "a" number 3 so  $a = b = c = 3$  and  $a + b + c = 9$ . This was done by 43 (45.3%) out of 95 of the learners. Equal split happened in Q6 (i) 43 (45.3%) and Q6 (ii), also Forty (42.1%) out of 95 assigned the letters equal value. A number that is slightly less than 43. Fifty-two (54.7%) out of 95 learners committed the error persistently. Learner 93 was among the learners who committed this type of error (see Figure 5A). The interview discussion with learner 93 on Q4 (i) ensued as follows:

- R : Can you tell me the answer that you have and explain how you got this answer?
- L93 : My answer is 8. I took 1 plus 7 times 1 because "a" is 1, so where I see "a" I substituted 1.
- R : Why do you put a number one where you see "a"? And where is one from?
- L93 : Sir "a" is equal to one; in class we were still doing them like this. The example in class was like this and we substituted 1 for "a" and 2 for "b".

The learner believes that the number order is similar to that of the alphabetical order. The interview with learners24 shows that the different letters were assigned the same value in Q4 (i to iii) and Q6 (i and ii) on Q6 (i) with slightly varying reasoning (See Figure 5B for Question 6(i)). The explanation for such a working is as follows:

- R : Can you tell me the answer that you have and explain how you got the answer?
- L24 : Sir, if  $a + b = 6$  this means that  $a = 3$  and  $b = 3$ . So for  $a + b + c$  we must add sir 3 plus 3 plus 3 and the answer is 9.
- R : Does this mean you worked question 6 (ii) this way?
- L24 : Yes Sir

The learner assigned the value 3 to each of the variables "a", "b" and "c". This result is consistent with that of Pournara et. al. (2016) where the learners had a misconception that variables cannot represent more than one value. This misconception also shows that the learner could not leave the answer as  $6 + c$  as the c had to be assigned a value for the simplification to be complete.

**Table 6**

Learners' responses presenting an equation formation error and their frequency			
Question	Answer	Number of learners	Percentage
Write the phrase as an algebraic expression and simplify Q3 (iii) Subtract $3a$ from $5$	$a = \frac{5}{3}$	18	18.9%
Simplify where possible Q4 (vi) $3x + 5$	$x = \frac{5}{3}$	12	12.6%
Q5 (i) $5b + c - b$	$b = \frac{-c}{4}$	11	11.6%
Write in algebraic form and simplify Q6 (v) Find the perimeter of the triangle with the sides $3n, 6, 4$ all sides are in <i>cm</i> .	$n = \frac{10}{3}$	24	25.3%

Handwritten work for Q4 (vi) showing the expression  $3x + 5$ , an incorrect equation  $3x = 5$ , and the solution  $x = \frac{5}{3}$ . The final answer is written as  $x = \frac{5}{3}$ .

**Figure 6.** L9 on Q 4 (vi), revealing an equation formation error.

### Equation formation error and misconception(s)

Learners changed the expression into an equation by splitting the expression into two parts and solving for the variable of their choice. The variable solved for was mostly found on the left hand side of the equation formed. At times, the learners equated the whole expression to zero and solved (see Figure 6).

In simplifying  $3x + 5$  where the first term is  $3x$  and the second term is  $5$ , the learner inserted the equal sign in between the two terms and solved the equation to get  $x = \frac{5}{3}$ . In Q3 (iii) the equal sign was inserted in between  $3a$  and  $5$  and the final answer was  $a = \frac{5}{3}$ . The second case is an expression with more than 3 terms. The equal sign was placed such that there was only one variable on the left hand side of the formed equation. For example, Q5 (i) learners wrote their answer as  $b = \frac{-c}{4}$ , having solved for " $b$ ". Twenty-two (23.2%) learners out of 95 of the learners made this error persistently. Table 6 gives the summary of the quantitative results.

The discussion with learner 9 progressed as follows:

- R : Look at your answer and explain how you obtained it.  
 L9 : I said  $3x = 5$  and divided by 3 both side and got " $x$ " equals to five over three.  
 R : Ok! Where is the equal sign from and where is the positive sign.  
 L9 : Sir. This is  $3x = +5$  and I have to solve for " $x$ ".

The working shows that the equation-solving schema interfered with the algebraic expression schema. The learners in this case seem to confuse the algebraic expressions with the algebraic equations. In algebraic equations, the equal sign is part of the expression which is not the case in the given expressions. The rules for solving algebraic equations seemed to have interfered with the simple algebraic expressions schema rules.

### Other errors

This section presents the other two errors which were not part of the errors reviewed in the literature. These are a *commutative like term error* and an *imposed radical sign error*.

### Commutative like term error and misconception(s)

This is a case whereby a learner cannot recognize the like terms when the order of the variables are exchanged, for example,  $mn$  and  $nm$ . Table 7 shows the questions and the number of learners who made this type of error.

**Table 7**  
Learners' responses showing commutative like term errors and their frequency

Question	Answer	Number of learners	Percentage
Simplify the given expressions			
Q5(iv) $5mn - 5 + 8nm + 6 - 4m$	$5mn + 8nm - 4m + 1$	35	36.8%
Q5 (v) $5str - 2pq + rst - 5pq$	$5str + rst - 7pq$	34	35.8%

Handwritten work for Q5(iv) showing a like-term error. The student starts with  $5mn - 5 + 8nm + 6 - 4m$ , then incorrectly groups terms as  $5mn + 8nm - 5 + 6 - 4m$ , and finally as  $5mn + 8nm - 1 + 4m$ . The final answer is given as  $5mn + 8nm - 1 + 4m$ .

**Figure 7.** L66's workings and answer to Q 5 (iv) showing a like-term error.

**Table 8**  
Learners' responses showing radical sign error and frequency

Question	Answer	Number of learners	Percentage
Simplify where possible			
Q4 (ix) $y^4 + y^4$	$y^2 + y^2$	30	31.6%
Q4 (xi) $2k^2 + k^2$	$2k + k$	14	14.7%
Q4 (xiv) $y^2 + x^2$	$y + x$	31	32.6%

Handwritten work for Q4 (xiv) showing a radical sign error. The student writes the expression  $y^2 + x^2$  under a square root symbol, then simplifies it to  $y + x$ . The final answer is given as  $y + x$ .

**Figure 8.** L66's workings and answer to Q 4 (xiv) on an imposed radical sign error.

The following is an extract from the interview on Q5 (iv) with learner 66 who committed the error in the two questions shown on the table. In seeking clarification in the interview for Figure 7, the discussion transpired as follows:

- R : Are you saying there are no further like terms in your answer?  
 L66 : Yes sir ...yes there are no like terms in the answer now. I would say  $8nm$  and  $5mn$  but they are not because " $nm$ " and " $mn$ " are unlike terms and the first one start with " $m$ " and the other one with " $n$ ".  
 R : Do you mean that the different positions  $m$  and  $n$  occupy make them different?  
 L66 : Yes sir they are different, in class we were only asked like  $4ab$  and  $2ab$  only.

The learner seemed to have a schema for like terms with the variables that appear in the same order. This apparently created some difficulties for the learner to reorganize or restructure the schema during the accommodation process of the like terms with interchangeable positions of the variables. This clearly shows that the learners have no proper conception of the commutative property of multiplication. Multiplying  $m$  by  $n$  gives the same result as multiplying  $n$  by  $m$  regardless of the position of each variables. This is not easy to conceptualize as it does not cohere with the behavior or properties of real numbers. For example, the number 43 cannot be equated to 34 as 43 does not mean 4 times 3. If the position of digits in real numbers cannot be interchanged, certainly this does not make sense to students as it is inconsistent with what they have known to be true in their earlier mathematics education.

This is a case where learners applied the radical sign to simplify the expression where a power is involved. This error is called an *imposed radical sign error* because the radical sign was used by the learners to simplify the algebraic expressions when it was not part of the expressions.

In Q4 (ix) 30 (31.6%) out of 95 of the learners gave their answer as  $y^2 + x^2$  and some as  $y^2x^2$  when simplifying  $y^4 + x^4$ . As shown in the table, 14 (14.7%) out of 95 of the learners have their answers as  $2k + k$  to  $2k^2 + k^2$ . Thirty-one (32.6%) out of 95 of the learners applied the radical sign to the expression in Q4 (xiv) to get  $yx$ , while others left their answer as  $y + x$  after applying the radical sign. Thirty learners (31.8%) out of 95 of the learners persistently made an imposed radical sign error. Learner 93 explained his working in [Figure 8](#) as follows:

- R : *Why did you apply the square root in this question?*  
 L93 : *Sir  $y^2$  and  $x^2$  are unlike terns so the only way to simplify them is by applying the radical sign. And in class we used this for example, when asked about 25 and we said  $25 = 5^2$  and we applied the sign like  $25 = 5^2 = \sqrt{5^2} = 5$ .*

The learner applied the radical sign where there was a second-degree radical term. Learners used the Pythagoras' theorem, where the hypotenuse H is found as follows  $H^2 = A^2 + O^2$  and a radical sign is applied to both sides  $\sqrt{H^2} = \sqrt{A^2 + O^2}$ .

## DISCUSSION

The study has shown that committing errors in responding to tasks is an unavoidable stage in learning. The type of errors that were experienced were mostly similar to the ones encountered in the studies conducted in other contexts. Such errors include conjoin error, an exponent error, a reversal error, learners' difficulties with variables, substituting numeric values for letters, and an equation formation error. These are the errors that the study shared with the results of other studies. There are however a few errors that were unique to the study they are a commutative like term error, and an imposed radical sign error. A conjoin error was part of the findings from the work of Makonye (2016) and Mulungye et. al. (2016). Their view is that the plus sign in between the terms is an instruction to do something. The learner then continued to add even though the terms were unlike. In the context of the reported study this could also be attributed to the language of origin where "ho kopanya" means to combine, whose implication is that an answer should always be given as a single term regardless of whether the terms are like or unlike. The results of the exponent error are consistent with the studies by Mulungye et. al. (2016) and Pournara et. al. (2016) where learners reportedly misapplied the rules of exponents to simplify algebraic expressions. Both in those studies and the reported one, the learners' prior knowledge about the multiplication of exponents seemed to contribute to the misapplication of the rule where the operation of exponents was addition.

In Iddrisu et. al.'s (2017) and Aydin-Guc and Aygun's (2021) studies, learners also committed the reversal error when forming the algebraic expression from a sentence or phrase. According to constructivism, this shows that the schemas for the phrases used were not coherent. When counting numbers in arithmetic, the order is crucial. This conception seems to be applied to the irrelevant context. In the studies of Dodzo (2016) and Mulungye et. al. (2016) learners also had problems with variables in simplifying algebraic expressions. Although this finding overlaps with that of the reported study, in this study the same terms whose variables were interchanged also became a problem. Besides the findings by Pournara et. al. (2016), the study of Gardee and Brodie (2015) also revealed that learners were convinced that a letter is attached to a particular value, which was experienced from learners in the reported study. In the previous studies (e.g. Mulungye et al., 2016; A'yun & Lukito, 2018) where the radical sign error was committed, the radical sign was part of the given tasks whereas in the reported study, the radical sign was introduced by the learners, hence an imposed radical sign error. This shows that the application of the radical sign is probably not conceptually understood by the learners in the identified contexts.

All the identified errors seem to have originated from learners' prior learning, mostly because of some interference as in applying the rules of the old knowledge to the new knowledge in different contexts (Olusegun, 2015; Makonye, 2016). Such misapplication of the rules demonstrated that there was a problem of assimilating new knowledge into appropriate schemas (Ojose, 2015). This could be a sign that the proper restructuring of the old and the new knowledge had failed during the accommodation process (Luneta & Makonye, 2013; Ojose, 2015). From the results of the study it is evident that teachers should develop the skill of error analysis. During error analysis, teachers should identify, diagnose and remedy the errors learners commit (Moru & Qhobela, 2013; Moru et al., 2014).

Learners should also be allowed to actively participate in their own learning, that is, as part of the process (Davis et al., 2020). The remedial action that could be taken is that after diagnosing the source of the errors, the important elements of the concept that are lacking during the diagnosis should be remedied through teachers' intervention (Booth et al., 2014; Makonye, 2016; Makonye & Fakude, 2016). For example, if an error results from the misconception of overgeneralisation, teachers should draw the learners' attention to the contexts in which certain procedures or methods are applicable and where they are not applicable by giving a number of suitable examples (Makonye, 2016). Class exercises and tests should also be given to students frequently in assisting the process of remediation. These should include different contexts in which the rules are applicable and where they are not. Students should also be given the opportunity to reflect on the origin of certain conceptions from their own perspectives, which is a sign of actively involving them in knowledge acquisition (Davis et al., 2020).

## CONCLUSIONS

Drawing on the constructivist theory, authors' knowledge of mathematics and related literature, this study has examined error analysis in mathematics, with a focus on Grade 8 learners at the target school in Lesotho. The study has traced such errors, especially knowledge of mathematical concepts within the algebraic expressions in relation to their hierarchical order and interrelationships. In identifying such errors and causal misconceptions, the study has further revealed how the constructivist theory, as well as accommodation and assimilation processes of knowledge acquisition, explain the misconceptions which have brought about such errors. Equally significant is that the restructuring of schemas and absorption of new knowledge into old knowledge lies mainly in the above-mentioned processes. Having identified the errors as well as their causal misconceptions, the paper would recommend a remedial action intended for minimizing such errors, and, of course, possible misconceptions. The findings of this study have contributed to the existing body of literature, thus highlighting some errors which are unique to this particular group of Grade 8 learners. The findings could persuade mathematics educators to undertake more studies of a similar kind in other related contexts. Any such studies would help to further shed light on the phenomenon of errors as probably committed by learners in different contexts for various reasons. Further studies may not only include the idea of error analysis, but they may also indicate how knowledge is constructed. Studies on knowledge construction could complement the ones on errors and misconceptions as these concepts go hand-in-hand.

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## BIBLIOGRAPHY

- A'yun, Q. & Lukito, A. (2018). Misconceptions of senior high school students in radicals. *Journal Ilmiah Pendidikan Matematika*, 7(2). 1-6.
- Aydin-Guc, F., & Aygun, D. (2021). Errors and misconceptions of eighth-grade students regarding operations with algebraic expressions. *International Online Journal of Education and Teaching (IOJET)*, 2021(2), 2148–2225. <https://orcid.org/0000-0002-3922-017X>
- Bhattacharjee, J. (2015). Constructivist approach to learning – an effective approach of teaching learning. *International Research Journal of Interdisciplinary & Multidisciplinary Studies*, 7969(65), 65–74. <http://www.irjims.com>.
- Booth, J. L., Barbieri, C., Eyer, F., & Paré-blagoev, E. J. (2014). Persistent and pernicious errors in algebraic problem solving. *Journal of Problem Solving*, 7 (Special Issue), 10–23. DOI:10.7771/1932-6246.1161.
- Davis, S. K., Edwards, R. L., & Hadwin, A. F. (2020). Using prior knowledge and student engagement to understand student performance in an undergraduate learning-to-learn course. *International Journal for the Scholarship of Teaching and Learning Volume*, 14(2), 1–12. <https://digitalcommons.georgiasouthern.edu/ij-sotl>.
- Dodzo, W. (2016). *Secondary school studies errors and misconceptions in Algebra with specific reference to a school in Mashonaland*. Unpublished Master Thesis, Faculty of Science Education, Bindura University of Science Education, Zimbabwe.
- Examinations Council of Lesotho. (2018). Mathematics examiners' reports, syllabus 0178/4. Maseru: Lesotho.

- Examinations Council of Lesotho. (2019). Mathematics examiners' reports, syllabus 0178/3. Maseru: Lesotho.
- Fumador, E. S. & Agyei, D. D. (2018). Students' errors and misconceptions in algebra: Exploring the impact of remedy using diagnostic conflict and conventional teaching approaches. *International Journal of Education, Learning and Development*, 6(10), 1–15. [www.eajournals.org](http://www.eajournals.org). Online ISSN: ISSN 2054-6300.
- Gardee, A., & Brodie, K. (2015). A teacher's engagement with learner errors in her Grade 9 mathematics classroom. *Pythagoras*, 1–9. <https://doi.org/10.4102/pythagoras.v36i2.293>
- Holmes, V., Miedema, C., & Haugen, N. (2013). Data-driven intervention: Correcting mathematics students' misconceptions, not mistakes. *The Mathematics Educator*, 23(1), 24–44.
- Ibrahim, M. (2012). Thematic analysis : A critical review of its process and evaluation. *West East Journal of Social Sciences*, 1(1), 39–47.
- Iddrisu, M. M., Abukari, A. & Boakye, S. (2017). Some common misconstructions and misinterpretations in basic algebra: a case of students of university for development studies and Navrongo senior high school in Ghana. *British Journal of Education*. 5(9), 22-35.
- Irawati, C., Zubainur, M., & Ali, M. R. (2018). Cognitive conflict strategy to minimize students' misconception on the topic of addition of algebraic expressions. *IOP Conf. Series: Journal of Physics: Conf. Series* **1088** 012084 [doi:10.1088/1742-6596/1088/1/012084](https://doi.org/10.1088/1742-6596/1088/1/012084)
- Luneta, K. & Makonye, J. P. (2013). Learners' mathematical errors in introductory differentiation : A theoretical framework. *US-China Education Review*, 3(12), 914–923. ISSN 2161-623X.
- Makara, M., & Ntau, J. M. (2019). *On point Mathematics*. Cambridge University Press. ISBN 078-1-108-72984-0.
- Makonye, J. P. (2016). Understanding of Grade 10 learner errors and misconceptions in elementary algebra. *Journal of Educational Studies*, 15(1), 288–313. <https://www.researchgate.net/publication/302598848>
- Makonye, J. P. & Fakude, J. (2016). A study of errors and misconceptions in the learning of addition and subtraction of directed numbers in Grade 8. *SAGE Open*, 6(4), 1-10. <https://doi.org/10.1177/2158244016671375>
- Marpa, E. P. (2019). Common errors in algebraic expressions : A quantitative-qualitative analysis. *International Journal on Social and Education Sciences Volume*, 1(2), 63–72. ISSN: 2688-7061 (Online).
- Moru, E. K., & Qhobela, M. (2013). Social science students' concept images and concept definitions of anti-derivatives. *Pythagoras*, 40(1), 1–12. <https://doi.org/10.4102/PYTHAGORAS.V40I1.484>
- Moru, E. K., Qhobela, M., Wetsi, P., Nchejane, J. (2014). Teacher knowledge of error analysis in differential calculus. *Pythagoras*, 1–10. <https://doi.org/10.4102/pythagoras.v35i2.263>
- Mulungye, M. M., O'Connor, M., & Ndethiu, S. (2016). Sources of student errors and misconceptions in algebra and effectiveness of classroom practice remediation in Machakos County-Kenya. *Journal of Education and Practice*, 7(10), 31–33. ISSN 2222-1735.
- Njie, B & Asimiran, S. (2014) Case study as a choice in qualitative methodology. *Journal of Research and Method in Education*, 4(3), 35-40. e-ISSN: 2320-7388, p-ISSN: 2320-737X.
- Ojose, B. (2015). Students' misconceptions in mathematics: analysis of remedies and what research says. *Ohio Journal of School Mathematics*, 72, 30–34.
- Olusegun, S. (2015). Constructivism learning theory: A paradigm for teaching and learning. *IOSR Journal of Research and Method in Education (IOSR-JRME)*, 5(6), 66–70. <https://doi.org/10.9790/7388-05616670>
- Pournara, C., Hodgen, J., Sanders, Y., Adler, J. (2016). Learners' errors in secondary algebra : Insights from tracking a cohort from Grade 9 to Grade 11 on a diagnostic algebra test. *Pythagoras*, 37(1), a334. <http://dx.doi.org/10.4102/pythagoras.v37i1.334>
- Sarımanoğlu, N. U. (2019). The investigation of middle school students' misconceptions about algebraic equations. *Studies in Educational Research and Development*, 3(1), 1–20.
- Shareia, B. F. (2016). Qualitative and quantitative case study research method of social science: Accounting perspective. *International Journal of Economics and Management Engineering*, 10(12), 3839–3844.
- Starman, A. B. (2013). The case study as a type of qualitative research. *Journal of Contemporary Educational Studies*, 1, 28–43.
- Zhiqing, Z. (2015). Assimilation, accommodation and equilibration: A schema-based perspective on translation as process and as product. *International Forum of Teaching and Studies*, 11 (1), 84–89.