

An exploration of critical thinking stages of junior high school students in solving contradictory mathematical problems

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ABSTRACT

Critical thinking is a fundamental competence in 21st-century education, particularly in mathematics, where students frequently encounter contradictory information that requires logical reasoning and reflective judgment. This study explores the stages of critical thinking among junior high school students when solving contradictory mathematical problems. A qualitative descriptive design was employed, involving two eighth-grade students from one of public junior high school in Malang regency, who were selected based on their skeptical responses to illogical mathematical tasks. Data were collected through open-ended tests and interviews, then analyzed to capture reasoning patterns and problem-solving strategies. The findings revealed three distinct stages of mathematical critical thinking: (1) Initial Stage (interpretation), where anomalies are sensed; (2) Tracing Stage (analysis), where contradictions are identified; and (3) Global View Stage (evaluation and inference), where holistic reasoning and alternative solutions are proposed. Subject 1 demonstrated conceptual awareness, cognitive flexibility, and evaluative rigor, while Subject 2 showed procedural accuracy but limited inferential precision. These findings suggest that contradictory problems can serve as effective instructional tools for balancing procedural and conceptual reasoning. Practical implications highlight the need for integrating contradictory problems into mathematics instruction to promote metacognitive reflection. Future research should expand participant diversity, employ longitudinal and experimental designs, and explore affective dispositions influencing students' critical engagement.

INTRODUCTION

Developing critical thinking skills has become a central objective in modern educational systems, particularly within the context of 21st-century learning demands. Critical thinking is a fundamental skill in 21st-century education that enables individuals to analyze, evaluate, and synthesize information systematically before making reasoned judgments (Facione, 2020; Malcolm, 2020). The rapid advancement of technology and the increasing complexity of real-world problems have further emphasized the need for critical thinking in various academic disciplines, including mathematics (Ali, 2025; Siregar et al., 2024). In the context of mathematics education, critical thinking plays a crucial role in problem-solving, as students frequently encounter complex and contradictory information that requires logical reasoning and evidence-based decision-making (Arifin, 2021; Lestari et al., 2024; Sachdeva & Eggen, 2021). Therefore, fostering students' critical thinking abilities is essential for equipping them to navigate mathematical challenges effectively and make well-founded decisions in both academic and real-life contexts.

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Understanding how students engage with mathematical problem-solving has become an essential concern in both educational research and classroom practice, particularly as curricula emphasize critical and analytical thinking. Mathematical problem-solving is not merely about obtaining correct answers but involves a comprehensive process of interpreting information, analyzing relationships, and verifying conclusions (Nurwita et al., 2022; Susanti & Wulandari, 2021). Particularly in mathematical problems that present contradictory data or inconsistencies, students are challenged to utilize higher-order thinking skills to identify, question, and reconcile conflicting elements (Darhim et al., 2020; Eviota & Liangco, 2020; Naim et al., 2025). However, empirical studies have consistently revealed that many junior high school students exhibit difficulties in this regard, often defaulting to rote memorization and procedural responses rather than engaging in reflective and analytical reasoning (Ebenezer Bonyah et al., 2023; Maryani et al., 2021). This recurring problem indicates a gap in the cultivation of critical thinking within mathematics instruction and highlights the urgent need for pedagogical approaches that can guide students in examining and resolving inconsistencies in mathematical contexts (Ali, 2025; Lestari et al., 2024; Safitri et al., 2024). Therefore, addressing students' challenges in dealing with contradictory information requires intentional instructional strategies that foster deeper cognitive engagement and support the development of critical thinking skills in mathematics.

The conceptualization of critical thinking has been a central focus in educational theory, with various scholars offering diverse frameworks to explain its underlying cognitive dimensions. Critical thinking is inherently multi-dimensional and involves cognitive processes such as interpretation, analysis, evaluation, inference, explanation, and self-regulation (Facione, 2020; Malcolm, 2020). While existing frameworks like Bloom's taxonomy and Polya's problem-solving stages provide a foundation for understanding these processes, they often address the skill in general terms and may overlook the specific cognitive challenges posed by contradictory mathematical information (Ali, 2025; Suharyat, 2022). Consequently, there is a growing need for models that not only encompass general critical thinking processes but also account for the unique reasoning demands encountered in complex mathematical problem-solving contexts.

The specific cognitive processes underlying students' responses to contradictions in mathematical problem-solving remains an underexplored area within critical thinking research. Although numerous studies have explored students' critical thinking in mathematical settings, there is a notable lack of research that specifically investigates how students cognitively navigate contradictions within mathematical problems (Naim et al., 2025; Sabiq et al., 2025). Most existing research focuses on overall performance in problem-solving rather than dissecting the discrete cognitive stages students experience when confronted with anomalies or inconsistencies in data (Eviota & Liangco, 2020; Fitriyah et al., 2022). The development of critical thinking is further influenced by several interrelated variables—cognitive ability, instructional design, and the learning environment (Lestari et al., 2024; Rivas et al., 2022). Among these, the instructional approach has a pivotal role, with research confirming that methods such as inquiry-based learning and problem-based learning can significantly enhance students' engagement in reflective and logical reasoning (Ali, 2025; Lestari et al., 2024; Safitri et al., 2024). Therefore, it is essential to conduct in-depth investigations into students' cognitive processes when encountering contradictory mathematical information, particularly within pedagogical contexts that aim to foster higher-order thinking.

Adolescence, particularly during the junior high school years, constitutes a pivotal stage in cognitive development that warrants close educational attention. According to Piagetian theory, this phase represents a developmental transition from concrete operational to formal operational thinking, making it critical for the cultivation of higher-order thinking skills (Adolph & Hospodar, 2024). Students at this level often require structured scaffolding to effectively develop skills such as evaluating conflicting information and formulating coherent solutions (Ali, 2025; Safitri et al., 2024). Research indicates that students who acquire the ability to think critically tend to show improvements in mathematical reasoning and efficiency in problem-solving (Katende, 2023; Katsamunsk & Rosenbaum, 2020; Siregar et al., 2024). Nevertheless, little is known about the precise mental stages they pass through when encountering mathematical contradictions—an area that deserves focused scholarly attention (Malcolm, 2020)(Mohammed et al., 2024).

To provide a more specific understanding of students' critical thinking in mathematical contexts, this study refines general stage of critical thinking into a domain-specific framework known as stage mathematical critical thinking. In this study, the researcher develops general critical thinking stages into mathematical critical thinking stages, which consist of the following phases: (1) Initial Stage –The ability to sense an anomaly in a problem but without the ability to trace the specific components causing the anomaly; (2) Tracing Stage – The ability to detect an anomaly in a problem and identify the components that contribute to the inconsistency; (3) Global View Stage – The ability to explain the problem from a global perspective and interpret it from multiple viewpoints. These stages provide a structured pathway for understanding how students progressively engage with and resolve contradictory mathematical problems using critical thinking.

This formulation is grounded in Facione's core critical thinking competencies and represents a novel contribution to the field, as such stages—especially in the context of contradictory mathematical problems—have not been explicitly examined in previous studies (Facione, 2020; Lestari et al., 2024). The three stages of critical thinking in this study have not been previously examined, making them one of the novelties of this research. The relevance and urgency of this study stem from both theoretical and practical dimensions. Theoretically, it enriches the understanding of cognitive stages in mathematical critical thinking, a topic underexplored in current literature. Practically, it offers a diagnostic lens for educators to identify students' cognitive positions and tailor instructional interventions accordingly. The study holds potential implications for curriculum development, teacher training, and assessment design in mathematics education.

Previous empirical studies reinforce the need for this exploration. For example, Putri et al, (2024) conducted a meta-analysis showing that junior high school students' critical thinking in science particularly physics—remains underdeveloped. Imayanti et al, (2021) observed students' inability to meet critical thinking benchmarks when solving mathematical problems related to relations and functions. Sa'diyah et al, (2024) emphasized the absence of learning strategies that fully engage students' cognitive potential, while Harahap et al, (2024) highlighted a general tendency among students to accept information passively without critical interrogation.

These findings converge on a central issue: students are not sufficiently equipped to engage in deep analytical thinking, particularly in resolving contradictory information in mathematics. This problem is not merely cognitive but also pedagogical, rooted in instructional practices that do not adequately support the development of critical thinking.

Accordingly, this study aims to explore the stages of critical thinking among junior high school students when solving contradictory mathematical problems. By identifying the specific cognitive stages students experience, this research seeks to inform educators on how to design instructional strategies that nurture critical thinking more effectively. The outcomes of this study are expected to contribute meaningfully to both theoretical discourses and practical implementations in mathematics education, ultimately advancing students' analytical capacity in solving complex, real-world problems (Ali, 2025; Suharyat et al., 2022 ; Siregar et al., 2024)

METHODS

Research design

This study employed a qualitative descriptive research design to explore the stages of critical thinking demonstrated by junior high school students when solving contradictory mathematical problems. The qualitative approach was selected to provide an in-depth understanding of students' cognitive processes, particularly their reasoning patterns, interpretative strategies, and analytical responses to inconsistencies within mathematical problems. This design is appropriate for capturing rich, context-bound insights into the phenomenon, aligning with the study's aim to develop a nuanced framework of mathematical critical thinking stages.

Participants and research context

The subjects of this study were two students from one of public junior high school in Malang regency. The selection of research subjects was based on the following criteria: (1) junior high school students aged 13 to 14 years old, (2) able to communicate their thoughts both verbally and in writing, and (3) skeptical of illogical mathematical problems. This study was conducted with eighth-grade junior high school students (Grade VIII). The selection of students at this grade level was based on

several considerations. First, students at this age are generally at the formal operational stage of cognitive development, allowing them to think more abstractly and thus more capable of generating critical responses. Second, students at this level have acquired foundational mathematical knowledge and experience, as they have completed elementary school education, which includes basic topics such as numbers and algebraic forms. Third, since Grade VIII is still part of the middle school level, the exploration of students' critical thinking stages at this level can serve as a foundation or reference for further educational stages.

In this study, 90 students from three classes at one of public junior high school in Malang Regency, aged 13–14 years and capable of effectively communicating their thoughts both orally and in writing, were selected based on the recommendations of their teachers. These students were administered a critical thinking test according to the schedule arranged by the teacher. From the 90 participants, students who demonstrated skepticism toward the validity of the contradictory mathematical problems were identified. Subsequently, two students were selected as they exhibited skeptical responses. These two students were then asked to provide written explanations regarding the reasons for their skepticism toward the contradictory mathematical problems. Based on these written explanations, interviews were conducted to validate the obtained data, which were then analyzed further.

From the data obtained, it was found that only two students consistently demonstrated a skeptical attitude toward the contradictory mathematical problems. This indicates that skepticism in evaluating the validity of mathematical information is relatively rare among the broader group of students. Although both students shared this skeptical disposition, their characteristics differed in several ways. The first student tended to approach the problems with a more analytical orientation, carefully identifying inconsistencies and explicitly questioning the logical basis of the given information. In contrast, the second student demonstrated a more intuitive skepticism, expressing doubt based on a sense of incongruity without providing detailed analytical reasoning at the outset. These differences highlight the diversity of critical thinking manifestations even among students who share similar skeptical attitudes, suggesting that individual cognitive styles and reasoning approaches influence how students engage with contradictory mathematical tasks.

Research instruments

The instrument used to collect data in this study is a critical thinking problem that presents contradictory information. This problem takes the form of a word problem that requires the engagement of higher-order thinking processes. Solving it requires deeper reasoning, as the procedures involved are not as straightforward or identical to those commonly taught in the classroom. In other words, the problem introduces a novel situation that students have not previously encountered in regular classroom instruction. An example of a critical thinking problem with contradictory information is presented as follows.

Ridwan has 48 marbles that he plans to distribute entirely among his playmates in different amounts. He gives $\frac{1}{2}$ of the total to Andi, $\frac{1}{4}$ to Boby, $\frac{1}{6}$ to Teguh, and $\frac{1}{8}$ to Dedi. Based on Ridwan's distribution, how many marbles did Andi, Boby, Teguh, and Dedi receive? Do you agree with Ridwan's method of distribution? Explain your reasoning!

In this study, only one problem was presented to analyze students' critical thinking abilities because the task itself was designed to be sufficiently complex and cognitively demanding. The contradictory nature of the information embedded in the problem required students to go beyond procedural knowledge and engage in higher-order thinking processes, such as interpretation, analysis, evaluation, and inference. A single well-constructed problem of this type is adequate to elicit critical thinking because it compels students to question the validity of the information, identify inconsistencies, and justify their reasoning explicitly. Furthermore, using one focused task avoids cognitive overload while ensuring that the students' responses can be examined in depth. Thus, although only one problem was used, its design was purposeful and rigorous, making it effective in uncovering the stages and characteristics of students' critical thinking.

Table 1
Stages of general critical thinking applied to mathematical critical thinking

Stages of Critical Thinking	General Critical Thinking	Components of Mathematical Critical Thinking
Initial	Recognizing anomalies in the problem but unable to identify the components causing the anomaly	Interpretation: Stating what is known and what is asked about the problem. Suspecting the problem but unable to pinpoint the suspected components, thus writing is limited to what is known and what is asked about the problem. Explaining the information available about the problem.
Tracing	Identifying unusual components of the problem but unable to place those components within the context of the problem	Analyz: Identifying unusual components of the problem Finding clues or steps to solve the problem.
Global-View	Explaining the problem from a global perspective or from different viewpoints	Evaluation: Assessing the truth of the belief about the anomalies in the problem. Assessing the accuracy of the steps or methods in solving the problem. Inference: Solving the mathematical problem by making a correct decision with logical reasoning through alternative thinking processes and verifying the solution steps.

Students' responses to this critical thinking problem will be analyzed based on three stages of critical thinking. The indicators of mathematical critical thinking were developed by the researcher based on Facione's (2011) critical thinking framework, as presented in Table 1. Based on this critical thinking framework, the critical thinking processes demonstrated by students in solving mathematical problems that present contradictory information will be described in Table 2.

Data collection procedure

Data collection in this study was carried out by administering a critical thinking test to students, consisting of three open-ended questions. These critical thinking questions were designed to explore students' critical thinking processes across the stages of initial understanding, tracing, and global view. During the problem-solving process, students were asked to elaborate on their answers along with the reasoning behind each step of their solution. Subsequently, in-depth interviews were conducted with the students to gain insights into how they arrived at their answers. The interviews allowed students the freedom to express their thoughts openly and in detail.

FINDINGS

The findings of this study are presented by analyzing students' responses according to the stages of critical thinking, beginning with the initial stage (interpretation), followed by the tracing stage (analysis), and culminating in the global view stage (evaluation and inference). The following section illustrates the responses of both subjects at each stage.

Table 2

Description of the critical thinking process in solving a problem with contradictory information

Stages of Thinking	Critical Indicator	Description
Initial	Interpretation:	This is demonstrated when students are able to thoroughly explain the information provided in the problem, state whether they agree with Ridwan's distribution, explain why the problem is contradictory, why the total number of marbles to be distributed does not match the number Ridwan possesses, identify the discrepancy between the number of marbles distributed and owned, determine whose portion is inaccurate and should be reduced, by how much it should be reduced, and how many marbles each person (Andi, Bobby, Teguh, and Dedi) should actually receive.
	Understanding the problem / identifying the core issue.	
	Stating what is known and what is being asked.	This is shown when students clearly and accurately write down all known information in the problem. Known information: Let Andi's portion = X_1 , Bobby's = X_2 , Teguh's = X_3 , Dedi's = X_4 , and Ridwan's total marbles = n . Then: $n = 48$, $X_1 = \frac{1}{2}$, $X_2 = \frac{1}{4}$, $X_3 = \frac{1}{6}$, $X_4 = \frac{1}{8}$. Questions posed in the problem: Based on Ridwan's distribution, how many marbles does each person receive? Do you agree with Ridwan's distribution? Explain.
Tracing	Identifying an inconsistency in the problem but unable to trace the specific component.	This is shown when students recognize that the problem presents contradictory information and question the fact that the total marbles to be distributed (50) does not match the marbles Ridwan owns (48), i.e., $48 \neq 50$.
	Analysis: Tracing the inconsistent components within the problem.	This is shown when students can identify and articulate the inconsistencies in the problem. To verify the miscalculation in Ridwan's distribution, they calculate each portion: Andi = $\frac{1}{2} \times 48 = 24$ marbles Bobby = $\frac{1}{4} \times 48 = 12$ marbles Teguh = $\frac{1}{6} \times 48 = 8$ marbles Dedi = $\frac{1}{8} \times 48 = 6$ marbles Total = $24 + 12 + 8 + 6 = 50$ marbles Or using fractions: $(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}) \times 48 = 25/24 \times 48 = 50$ marbles.

Table 2 (Continued)

Stages of Thinking	Critical Indicator	Description
Tracing	Identifying a strategy to solve the problem.	This is shown when students identify a method for resolving the issue by connecting the known and asked data. The discrepancy is: $50 - 48 = 2$ marbles. Ridwan can reduce 2 marbles from the share of Andi, Bobby, or Dedi (but not Teguh, to maintain different quantities). To correct the error, for instance, reduce Dedi's share by 2 marbles: Andi's revised share = $24 - 2 = 22$ marbles.
Global-View	Evaluation: Assessing the validity of the perceived inconsistency.	This is demonstrated when students evaluate the truth of their claim, showing confidence in identifying inconsistencies. Example: If Andi's share is reduced to 22 marbles $\rightarrow 22/48 = 11/24$.
	Evaluating the correctness of the problem-solving steps.	This is shown when students explain and justify their reasoning in solving the problem: Andi = $11/24 \times 48 = 22$ marbles Boby = $1/4 \times 48 = 12$ marbles Teguh = $1/6 \times 48 = 8$ marbles Dedi = $1/8 \times 48 = 6$ marbles Total = 48 marbles
	Inference: Proposing alternative solutions.	Students propose alternative solutions. If Bobby's share is reduced by 2 marbles: $12 - 2 = 10 \rightarrow 10/48 = 5/24$ Distribution: Andi = 24, Bobby = 10, Teguh = 8, Dedi = 6 $\rightarrow$ Total = 48 Fractions: $1/2 + 5/24 + 1/6 + 1/8 = 1$ (or $25/25$) $\rightarrow$ Total = $1 \times 48 = 48$ marbles If Dedi's share is reduced by 2 marbles: $6 - 2 = 4 \rightarrow 4/48 = 1/12$ Distribution: Andi = 24, Bobby = 12, Teguh = 8, Dedi = 4 $\rightarrow$ Total = 48 Fractions: $1/2 + 1/4 + 1/6 + 1/12 = 1$ (or $25/25$) $\rightarrow$ Total = 48 marbles
	Rechecking the steps and calculations.	This is shown when students review their problem-solving steps and calculations for accuracy, as confirmed through interviews.

Table 2 (Continued)

Stages of Thinking	Critical Indicator	Description
Global-View	Drawing conclusions.	This is shown when students accurately write conclusions, such as: - Disagree with Ridwan's distribution because 2 marbles are unaccounted for. - Disagree because the distributed marbles (50) do not match Ridwan's total (48). - Disagree because Andi should receive $11/24$ or 22 marbles. - Disagree because Bobby should receive $5/24$ or 10 marbles. - Disagree because Dedi should receive $1/12$ or 4 marbles.

Diketahui :

- Ridwan memiliki kelereng 48 butir yang akan dibagikan kepada temannya dengan jumlah yang berbeda.
- $\frac{1}{2}$ bagian kepada Andi
- $\frac{1}{4}$ bagian kepada Bobby
- $\frac{1}{6}$ bagian kepada Teguh
- $\frac{1}{8}$ bagian kepada Dedy

Ditanya :

Jika mengikuti pembagian yang dilakukan oleh Ridwan, berapaakah seharusnya jumlah kelereng yang diterima Andi, Bobby, Teguh, dan Dedy? Setujukah anda dengan pembagian yang dilakukan Ridwan? Jelaskan!

Jawaban :

Menghitung jumlah kelereng yang diterima masing-masing teman:

- Andi: $\frac{1}{2} \times 48 = 24$
- Bobby: $\frac{1}{4} \times 48 = 12$
- Teguh: $\frac{1}{6} \times 48 = 8$
- Dedy: $\frac{1}{8} \times 48 = 6$

Karena jumlah kelereng yang dibagikan Ridwan tidak sama dengan kelereng yang dimilikinya

Total kelereng yang dibagikan adalah 50, sedangkan Ridwan hanya punya 48. Ada yang aneh pada soal ini.

Translation of figure 1

Given:

Ridwan has 48 marbles that he intends to distribute entirely among his friends in varying amounts. Ridwan allocates $\frac{1}{2}$ to Andi, $\frac{1}{4}$ to Bobby, $\frac{1}{6}$ to Teguh, and $\frac{1}{8}$ to Dedi.

Question:

According to Ridwan's distribution, how many marbles does each of Andi, Bobby, Teguh, and Dedi receive? Do you agree with Ridwan's distribution? Please explain.

Answer:

Calculating the number of marbles received by each friend:

Andi: $\frac{1}{2} \times 48 = 24$

Bobby: $\frac{1}{4} \times 48 = 12$

Teguh: $\frac{1}{6} \times 48 = 8$

Dedy: $\frac{1}{8} \times 48 = 6$

Why does the total number of marbles distributed by Ridwan not equal the number of marbles he possesses?

The total number of marbles distributed is 50, while Ridwan only has 48. There seems to be an inconsistency in this problem.

Figure 1. The first subject's answer sheet during the initial stage

Response of the first subject Initial stage (interpretation)

At the Initial stage, Students explain the information presented in the problem, articulate what is known and what is being asked, and express suspicions regarding the issue. However, they are unable to identify the suspected components, resulting in their written responses being limited to what is known about the problem. This can be observed in the students' answer sheets shown in Figure 1.

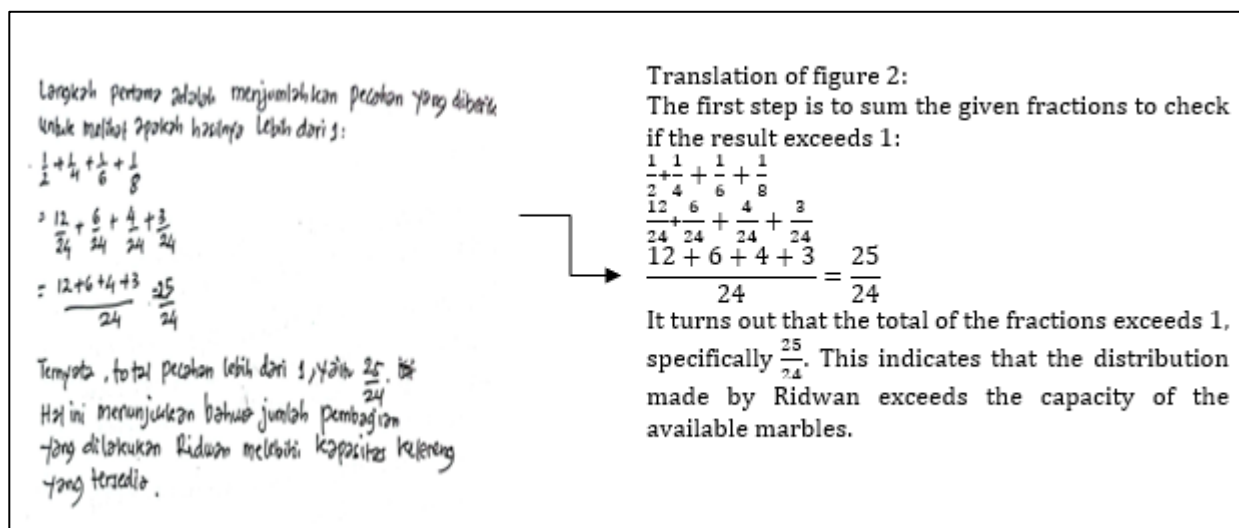


Figure 2. The first subject's answer sheet during the tracing stage

Figure 2 shows that the student is able to express and identify inconsistencies in the problem. This is supported by the results of the interview with the subject as follows.

- Researcher : How do you determine whether this distribution is correct or not?
 Subject-1 : I will calculate the number of marbles received by each friend to see if the distribution is valid
- Researcher : Can you show me your calculations?
 Subject-1 : Andi: $1/2 \times 48 = 24$
 Bobby: $1/4 \times 48 = 12$
 Teguh: $1/6 \times 48 = 8$
 Dedy: $1/8 \times 48 = 6$
 When summed: $24+12+8+6=50$. Ridwan only has 48 marbles, but the total given amounts to 50. This indicates a discrepancy in the distribution

Tracing stage (analysis)

At this stage, the students also identified clues regarding the methods and steps to solve the problem, as illustrated in Figure 2. Based on the student's answer sheet in Figure 2, a deeper exploration of the critical thinking process was conducted to uncover how the student identified the methods and steps in problem-solving through the following interview.

- Researcher : How do you ensure that there is an error in this distribution?
 Subject-1 : I sum the given fractions to check if the result exceeds 1..

Equalizing the denominators using the least common multiple of 24:

$$\frac{12}{24} + \frac{6}{24} + \frac{4}{24} + \frac{3}{24}$$

$$\frac{12+6+4+3}{24} = \frac{25}{24}$$

The result is greater than 1, specifically $\frac{25}{24}$, which means Ridwan is attempting to distribute more than the amount he possesses.

Global view stage (evaluation and inference)

In the Global View stage, Students are able to evaluate problems by assessing the validity of their doubts regarding discrepancies in the problems, as well as evaluating the accuracy of the methods or steps taken to solve those problems. At this stage, students also demonstrate the ability to infer solutions to problems by resolving mathematical issues and making sound decisions based on reasonable justifications through alternative thinking processes and rechecking their solution steps. This is evidenced by the students' response sheets shown in Figure 3.

Berdasarkan perhitungan diatas, saya yakin ada kesalahan dalam pembagian Ridwan. Total pecahan yang digunakan adalah lebih dari 1, yaitu $\frac{25}{24}$. ini berarti Ridwan mencoba membagikan lebih banyak dari jumlah yang dimilikinya.

Untuk memperbaiki pembagian ini, Ridwan harus membagi kelerengnya menggunakan pecahan yang totalnya tepat 1 agar seluruh kelereng terbagi habis.

Alternatif pembagian yang benar:

Misalnya, jika Ridwan ingin membagi secara adil, ia bisa menggunakan pecahan berikut:

Jika bagian Dedy yang dikurangi, maka $6 - 2 = 4$ butir.

Pembagiannya adalah $\frac{4}{48} = \frac{1}{12}$.

Sehingga:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{6+3+2+1}{12} = \frac{12}{12} = 1$$

Dengan cara ini total kelereng yang dibagikan tetap 48, sesuai dengan jumlah yang dimiliki Ridwan.

Menurut saya, saya tidak setuju dengan pembagian yang dilakukan Ridwan. Pembagian ini tidak bisa dilakukan karena jumlah kelereng Ridwan tidak cukup untuk memenuhi total distribusi, kurang 2 butir.

Translation of figure 3:

Based on the calculations above, I believe there is an error in Ridwan's distribution. The total fractions used exceed 1, specifically $\frac{25}{24}$. This indicates that Ridwan is attempting to distribute more than the total amount he possesses. To rectify this distribution, Ridwan should allocate his marbles using fractions that total exactly 1 to ensure that all marbles are allocated completely.

An alternative correct distribution could be as follows: If Ridwan wishes to distribute fairly, he can use the following fractions: If Dedy's portion is reduced, then $6 - 2 = 4$ marbles. The distribution would be $\frac{4}{48} = \frac{1}{12}$. Thus,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{6+3+2+1}{12} = \frac{12}{12} = 1$$

The number of marbles that Ridwan's friends would receive is:

Andi: $\frac{1}{2} \times 48 = 24$
 Bobby: $\frac{1}{4} \times 48 = 12$
 Teguh: $\frac{1}{6} \times 48 = 8$
 Dedy: $\frac{1}{12} \times 48 = 4$

In this manner, the total number of marbles distributed remains 48, which corresponds to the amount Ridwan has. In my opinion, I disagree with Ridwan's method of distribution. This distribution is not feasible because Ridwan does not have enough marbles to meet the total demand, falling short by 2 marbles.

Figure 3. The first subject's answer sheet during the global view stage

Based on the answer sheet in [Figure 3](#), students' critical thinking process will be further explored through interviews as follows.

- Researcher** : After identifying discrepancies in the distribution, what is your opinion on how Ridwan distributed his marbles?
- Subject-1** : This distribution is not feasible because Ridwan does not have enough marbles to meet the total demand
- Researcher** : How could you correct this distribution?
- Subject-1** : Ridwan should divide his marbles using fractions that total exactly 1 so that all the marbles can be completely distributed. For example, a better distribution alternative could be as follows: If Ridwan wishes to distribute fairly, he can use the following fractions: If Dedy's share is reduced, then $6 - 2 = 4$ marbles. The allocation would be $\frac{4}{48} = \frac{1}{12}$.

So,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{6+3+2+1}{12} = \frac{12}{12} = 1$$

The number of marbles that Ridwan's friends would receive is:

Andi: $\frac{1}{2} \times 48 = 24$

Bobby: $\frac{1}{4} \times 48 = 12$

Teguh: $\frac{1}{6} \times 48 = 8$

Dedy: $\frac{1}{12} \times 48 = 4$

In this manner, the total number of marbles distributed remains 48, which corresponds to the amount Ridwan has.

$$\frac{12 + 6 + 4 + 3}{24} = \frac{25}{24}$$

Diketahui:
 Ridwan memiliki kelereng 48 butir yang akan dibagikan kepada temannya dengan jumlah yang berbeda.
 $\frac{1}{2}$ bagian kepada Andi
 $\frac{1}{4}$ bagian kepada Bobby
 $\frac{1}{6}$ bagian kepada Teguh
 $\frac{1}{8}$ bagian kepada Dedy

Ditanya:
 Jika mengikuti pembagian yang dilakukan oleh Ridwan, berapakah seharusnya jumlah kelereng yang diterima Andi, Bobby, Teguh, dan Dedy? Setujukah Anda dengan pembagian yang dilakukan Ridwan? Jelaskan!

Jawab:
 Total pembagian yang akan dibagikan Ridwan:
 $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{12+6+4+3}{24} = \frac{25}{24}$ bagian.
 Maka, $\frac{25}{24} \times 48 = 50$ kelereng.
 Ternyata, jumlah kelereng yang akan dibagikan Ridwan (50) lebih dari jumlah kelereng yang dimilikinya yaitu (48).
 Berarti ada kesalahan pembagian yang dilakukan Ridwan

Translation of figure 4
Given:
 Ridwan has 48 marbles that he intends to distribute entirely among his friends in varying amounts. Ridwan allocates $\frac{1}{2}$ to Andi, $\frac{1}{4}$ to Bobby, $\frac{1}{6}$ to Teguh, and $\frac{1}{8}$ to Dedi.
Question:
 According to Ridwan's distribution, how many marbles does each of Andi, Bobby, Teguh, and Dedi receive? Do you agree with Ridwan's distribution? Please explain.
Answer:
 Calculating the number of marbles received by each friend:
 Andi: $\frac{1}{2} \times 48 = 24$
 Bobby: $\frac{1}{4} \times 48 = 12$
 Teguh: $\frac{1}{6} \times 48 = 8$
 Dedy: $\frac{1}{8} \times 48 = 6$
 Why does the total number of marbles distributed by Ridwan not equal the number of marbles he possesses?
 The total number of marbles distributed is 50, while Ridwan only has 48. There seems to be an inconsistency in this problem.

Figure 4. The second subject's answer sheet during the initial stage

Langkah pertama adalah menjumlahkan pecahan yang diberikan untuk melihat apakah hasilnya lebih dari 1:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$$

$$= \frac{12}{24} + \frac{6}{24} + \frac{4}{24} + \frac{3}{24}$$

$$= \frac{12+6+4+3}{24} = \frac{25}{24}$$

Ternyata, total pecahan lebih dari 1, yaitu $\frac{25}{24}$. Hal ini menunjukkan bahwa jumlah pembagian yang dilakukan Ridwan melebihi kapasitas kelereng yang tersedia.

Translation of figure 5:
 The first step is to sum the given fractions to check if the result exceeds 1:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$$

$$= \frac{12}{24} + \frac{6}{24} + \frac{4}{24} + \frac{3}{24}$$

$$= \frac{12+6+4+3}{24} = \frac{25}{24}$$

It turns out that the total of the fractions exceeds 1, specifically $\frac{25}{24}$. This indicates that the distribution made by Ridwan exceeds the capacity of the available marbles.

Figure 5. The second subject's answer sheet during the tracing stage

Global view stage (evaluation and inference)

In this stage, students evaluate the problem by assessing the validity of their belief in the existence of irregularities and evaluating the correctness of the methods or steps involved in solving the problem. Students articulated their belief in the presence of inconsistencies within the problem and expressed confidence in the appropriateness of the steps taken to solve it. However, during the inference stage, they drew conclusions based on less accurate reasoning. This can be observed in the students' response sheet shown in Figure 6.

Menjumlahkan pecahan: $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$
 $= \frac{12+6+4+3}{24}$
 $= \frac{25}{24}$

Dari hasil tersebut lebih dari 1 atau lebih dari 100% dari total kelereng

Alternatif Solusi:

2. Agar pembagian benar, jumlah pecahan yang digunakan harus tepat 1 (100% dari total kelereng)

3. Ridwan bisa membagikan dengan pecahan yang jumlahnya tidak melebihi 1, misalnya dengan mengurangi jumlah kelereng Teguh sebanyak 2 butir.

Sehingga $8-2 = 6$ butir $\Rightarrow \frac{6}{48} = \frac{1}{8}$

Maka, jumlah kelereng yang seharusnya di terima teman Ridwan adalah:

Andi: $\frac{1}{2} \times 48 = 24$
 Bobby: $\frac{1}{4} \times 48 = 12$
 Teguh: $\frac{1}{8} \times 48 = 6$
 Dedy: $\frac{1}{8} \times 48 = 6$

Total: $24+12+6+6 = 48$

jadi kesimpulan, saya tidak setuju karena seharusnya Ridwan membagikan kelereng sebanyak 48 bukan 50, dan harus ada yang dikurangi jumlah kelereng yaitu Teguh.

Translation of figure 6:
 Summing the fractions:
 $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$
 $\frac{12+6+4+3}{24} = \frac{25}{24}$

From this result, it is evident that the distribution exceeds 1 or 100% of the total number of marbles.

Alternative solution:
 To ensure correct distribution, the sum of the fractions should equal exactly 1 (100%) of the total number of marbles.

Ridwan can distribute the marbles with fractions that do not exceed 1, for example, by reducing Teguh's number of marbles by 2. Thus, $8 - 2 = 6$ marbles. This can then be converted into a fraction: $6/48 = 1/8$.

Therefore, the number of marbles that each friend should receive is:
 Andi: $\frac{1}{2} \times 48 = 24$
 Bobby: $\frac{1}{4} \times 48 = 12$
 Teguh: $\frac{1}{8} \times 48 = 6$
 Dedy: $\frac{1}{8} \times 48 = 6$
 The total is: $24+12+6+6=48$

Corresponds to the number of marbles Ridwan has.

In conclusion, I agree, because Ridwan should distribute exactly 48 marbles, not 50, and the total number of marbles received by Teguh should be reduced.

Figure 6. The second subject's answer sheet during the global view stage

From the students' response sheets in Figure 6, the thought process involved in problem-solving will be examined more deeply through the following interview.

Researcher : After identifying this irregularity, how do you evaluate the steps you have taken to solve the problem?

Subject-2 : I rechecked my calculations to ensure there were no errors. I found that the fractions Ridwan distributed indeed added up to more than the total number of marbles he had, which made the distribution incorrect.

Researcher : In your opinion, how should the distribution be done correctly?

Subject-2 : To ensure the total equals 1 or 100%, I need to replace one of the fractions with a smaller value. For example: Andi would still receive $\frac{1}{2}$.

Boby would still receive $\frac{1}{4}$.

Dedy would still receive $\frac{1}{6}$

For Teguh, I need to find a fraction that, when added to the others, does not exceed 1. I will try recalculating: 1.

for example, by reducing Teguh's number of marbles by 2. Thus, $8 - 2 = 6$ marbles. This can then be converted into a fraction: $6/48 = 1/8$.

Therefore, the number of marbles that each friend should receive is:

Andi: $\frac{1}{2} \times 48 = 24$

Boby: $\frac{1}{4} \times 48 = 12$

Teguh: $\frac{1}{8} \times 48 = 6$

Dedy: $\frac{1}{8} \times 48 = 6$

The total is: $24 + 12 + 6 + 6 = 48$, and

$$\frac{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}}{\frac{12+6+3+3}{24}} = \frac{24}{24} = 1$$

which corresponds to the number of marbles Ridwan has.

Researcher : Are you confident in your answer?

Subject-2 : I am confident, God willing.

Researcher : So, what do you believe is the final conclusion?

Subject-2 : I can conclude that Ridwan's initial distribution was incorrect because the total exceeded the actual amount available. If he wants to distribute correctly, Ridwan needs

to adjust the fractions he uses so that the total equals 1 by reducing Teguh's number of marbles by 2.

Based on the answer sheet and interview of the second subject, it is seen that students ignore the absolute conditions in the problem, especially the division conditions involving different amounts, so that errors occur in the steps to solve the problem. Overall, it can be seen in the student's answer for example, by reducing Teguh's number of marbles by 2. Thus, $8 - 2 = 6$ marbles. This can then be converted into a fraction: $6/48 = 1/8$, even though the division is the same as Dedy's, which is $1/8$ (not in accordance with the conditions requested in the problem with a different number of divisions).

DISCUSSION

The comparative analysis of the two subjects highlights distinct trajectories in their critical thinking development across the stages of interpretation, analysis, evaluation, and inference. At the Initial Stage (Interpretation), both subjects demonstrated an ability to extract key information from the problem and expressed suspicion toward the plausibility of the distribution. Subject 1 exhibited stronger conceptual awareness by identifying that the fractional distribution appeared excessive before engaging in calculation, whereas Subject 2 relied primarily on procedural computation to validate suspicions. This contrast reflects differences in the depth of interpretation, where Subject 1 showed more advanced reflective judgment, while Subject 2 operated within a procedural orientation. Recent studies have emphasized that effective interpretation requires both recognition of surface features and anticipation of underlying inconsistencies (Li & Schoenfeld, 2019; Yulia & Salirawati, 2023).

In the Tracing Stage (Analysis), both subjects accurately performed computations to demonstrate that the total distribution exceeded the available quantity ($25/24 > 1$). However, Subject 1 integrated numerical and verbal reasoning, demonstrating cognitive flexibility in representing the problem, while Subject 2 confined the analysis to algorithmic procedures. This distinction suggests that Subject 1 engaged in deeper structural analysis, whereas Subject 2 remained at the level of procedural verification. Prior research indicates that such differences are critical, as analytical reasoning grounded in multiple representations is associated with stronger transfer of critical thinking skills (Rochaminah et al., 2025; Savaş et al., 2024). At the Global View Stage (Evaluation and Inference), both subjects recognized that the original distribution was flawed. Subject 1 evaluated the problem holistically and proposed an alternative solution ($1/2, 1/4, 1/6, 1/12$) that satisfied all constraints, thereby demonstrating logical coherence, evaluative rigor, and sound inferential judgment. In contrast, Subject 2 adjusted the distribution (assigning $1/8$ to both Teguh and Dedi), which corrected the numerical inconsistency but violated the problem condition requiring different shares. This indicates that Subject 2's evaluation was only partially accurate and that inferential reasoning lacked attention to contextual constraints. Such findings resonate with recent scholarship showing that students often succeed in detecting contradictions but struggle to integrate all problem conditions into coherent solutions (Chirove, 2023; Fauzi, 2025).

Overall, both subjects demonstrated engagement in critical thinking, yet their characteristics diverged significantly: Subject 1 exhibited a coherent progression across interpretation, analysis, evaluation, and inference, reflecting both conceptual and procedural fluency. Subject 2, while procedurally competent, revealed limitations in evaluative judgment and inferential precision. These findings align with recent research emphasizing that critical thinking development requires not only computational fluency but also metacognitive reflection to ensure logical consistency and constraint integration (Anggo et al., 2021; Nobutoshi, 2023).

The strengths and weaknesses of the two subjects are further distinguished and made more evident in the following Table 3. The comparison presented in Table 3 highlights the complementary yet contrasting profiles of the two subjects. Subject 1 demonstrated stronger conceptual awareness and evaluative rigor, while Subject 2 showed procedural fluency but struggled to integrate contextual constraints into coherent solutions. These findings resonate with recent studies emphasizing that students often exhibit imbalances between procedural and conceptual dimensions of critical thinking (Li & Schoenfeld, 2019; Fauzi, 2025). Moreover, the strengths of Subject 1 in integrating multiple

Table 3
Comparison of strengths and weaknesses of both subjects

Aspect	Subject 1	Subject 2
Strengths	▪ Recognized inconsistency in distribution early, even before detailed calculation (strong interpretive skill). ▪ Integrated numerical and verbal reasoning, showing cognitive flexibility. ▪ Proposed a valid alternative solution ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$) that satisfied all constraints. ▪ Displayed coherent progression across interpretation, analysis, evaluation, and inference.	• Successfully extracted relevant problem information. • Accurately performed computations, verifying over-distribution ($25/24 > 1$). • Persistently attempted correction strategies. • Demonstrated procedural accuracy in calculations.
Weaknesses	▪ Limited articulation of which components were problematic in the initial stage. ▪ Metacognitive reflection was not explicitly articulated, focusing mainly on mathematical reasoning.	• Relied heavily on procedural computation without deeper conceptual reasoning. • Confined analysis to algorithmic steps, lacking multiple representations. • Proposed a numerically valid but logically inconsistent correction (two identical fractions for different friends). • Limited evaluative judgment and inferential precision, failing to fully integrate problem constraints.

representations align with research suggesting that cognitive flexibility enhances the transferability of critical thinking across problem contexts (Rochaminah et al., 2025; Savaş et al., 2024). Conversely, the weaknesses observed in Subject 2 reflect patterns reported in more recent scholarship, where students tend to focus narrowly on algorithmic accuracy while overlooking logical coherence and problem-specific conditions (Khairunnisa et al., 2022; Putri et al., 2023). These findings underscore the importance of fostering both computational fluency and metacognitive reflection, as supported by Anggo et al. (2021), to ensure that critical thinking development is balanced and sustainable.

CONCLUSION

This study explored the critical thinking stages of two junior high school students in solving contradictory mathematical problems, focusing on interpretation, analysis, evaluation, and inference. The findings revealed both similarities and differences in their approaches. At the interpretation stage, both subjects successfully identified key information and expressed suspicion regarding the plausibility of the distribution, yet Subject 1 demonstrated stronger conceptual awareness by recognizing inconsistencies prior to calculation, while Subject 2 relied more on procedural computation. During the analysis stage, both subjects detected over-distribution; however, Subject 1 integrated multiple representations and verbal reasoning, whereas Subject 2 confined the analysis to algorithmic verification. At the evaluation and inference stages, both identified flaws in the original distribution, but Subject 1 proposed a logically coherent alternative that satisfied all constraints, while Subject 2 suggested a numerically correct yet logically inconsistent correction. Overall, Subject 1 displayed a coherent progression across all stages, reflecting both conceptual and procedural fluency, whereas Subject 2 showed procedural accuracy but limited evaluative judgment and inferential precision.

The practical implication of these findings is that contradictory problems can serve as effective instructional tools to balance procedural and conceptual dimensions of critical thinking. Teachers can employ such problems not only to test computational accuracy but also to encourage students to reflect on the coherence of their reasoning, question underlying assumptions, and propose

alternative solutions. By integrating contradictory problems into regular instruction, educators can foster metacognitive reflection and enhance students' ability to reconcile surface-level computations with deeper conceptual understanding.

For future research, several directions are recommended. First, expanding the sample size and including more diverse participants would provide broader insights into the variability of students' critical thinking. Second, longitudinal studies are needed to trace how exposure to contradictory problems contributes to the sustained development of critical thinking over time. Third, experimental research should investigate the effectiveness of instructional interventions, such as scaffolding strategies, collaborative problem-solving, and digital learning environments, in supporting students' ability to integrate conceptual and procedural reasoning. Fourth, cross-cultural and cross-curricular comparisons could illuminate contextual factors shaping students' responses to contradictory problems. Finally, examining the relationship between cognitive processes and affective dispositions, such as curiosity, persistence, and openness to ambiguity, could enrich understanding of how critical thinking skills are fostered in mathematical learning.

Collectively, these findings and implications highlight the pedagogical value of contradictory mathematical problems in fostering balanced and sustainable critical thinking, while also pointing toward fertile avenues for further research to strengthen both theory and practice in mathematics education.

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AUTHORS' DECLARATION

Authors' contributions

A: was responsible for conceptualization, research design, and drafting the initial manuscript P: contributed to data collection, data analysis, and interpretation of findings. RR: provided critical review, theoretical insights, and refinement of the manuscript. All authors read and approved the final version of the manuscript.

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