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Assessing cognitive obstacles in learning number concepts: Insights from preservice mathematics teachers

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ABSTRACT
Persistent difficulties in learning abstract algebraic concepts—particularly among preservice mathematics teachers—continue to hinder students' mathematical development. While prior studies have documented general misconceptions, few have grounded their analysis in comprehensive learning theories. Addressing this gap, the present study adopts the APOS (Action, Process, Object, Schema) theoretical framework to examine the cognitive obstacles encountered in understanding logarithmic, matrix, and quadratic function concepts. This qualitative study employed a descriptive case study design involving six preservice mathematics teachers with varying levels of mathematical ability (high, moderate, and low). Data were collected through written responses, semi-structured interviews, classroom observations, and cognitive mapping. The findings revealed that most participants were at the action stage, relying on procedural steps without deep conceptual understanding. Key cognitive obstacles included errors in applying logarithmic properties, difficulties integrating logarithms with matrices, and an inability to perceive systems of equations as unified entities. Group discussions proved effective in helping participants transition through the learning stages. Collaborative interactions enabled participants to identify errors, correct misconceptions, and strengthen conceptual understanding through reflection and validation. Furthermore, the use of visual tools, graphical representations, and real-world contexts supported deeper conceptual integration. This study underscores the importance of implementing APOS-based instructional strategies, including group discussions, exploratory exercises, and problem-based learning, to facilitate transitions between stages. The implications of these findings highlight the need for developing APOS-based diagnostic tools and innovative instructional designs to address cognitive obstacles effectively.

INTRODUCTION

Algebra serves as a foundational component of mathematics education (Hartono, 2025; Jamil et al., 2025), acting as a critical bridge between arithmetic and higher-level mathematical thinking (Jung et al., 2024). Research consistently highlights the challenges students face in mastering fundamental algebraic concepts, which are pivotal for advanced mathematics (Hyland & O'Shea, 2022; Umiralkhanov et al., 2024). Despite high levels of achievement in some areas of school mathematics, many students—including high-achieving undergraduates—struggle with essential topics such as solving equations and inequalities, indicating gaps in both conceptual understanding and procedural fluency (Copur-Gencturk & Doleck, 2021; López-Martín et al., 2022; Sari et al., 2024).

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Addressing these challenges is critical, as the continuity of algebra instruction across secondary and higher education levels plays a significant role in ensuring student adaptation and long-term success in university mathematics (Alsina et al., 2024; Faustino & Sales, 2024; Ontiveros et al., 2025; Umiralkhanov et al., 2024). Furthermore, the effective transfer of mathematical skills to other domains, such as physics and engineering, highlights the interdisciplinary value of strong mathematical foundations. These efforts are supported by research suggesting that conceptual understanding is crucial in mediating and enhancing learning in algebra fields (Agustyaningrum et al., 2020; Mera et al., 2022). In addition, current research findings emphasize the urgency of improving mathematical readiness at the tertiary level to better equip students for both academic and professional challenges (Nhat et al., 2024; Padernal & Tupas, 2024; Pasigon, 2024).

Nevertheless, existing research consistently reveals that students across educational levels continue to experience persistent difficulties in understanding algebraic concepts (Pape et al., 2022; Thomaidis & Tzanakis, 2022), including misconceptions in symbolic manipulation (Mutambara & Bansilal, 2022), limited conceptual comprehension (Wilkins & Norton, 2018), and challenges in translating among various representations (symbolic, graphical, and verbal) (Soneira et al., 2021). Moreover, a deeper exploration into students' difficulties reveals that cognitive obstacles are not limited to general mathematics but extend to specific areas such as algebra and calculus, where misconceptions frequently emerge (Fitriani et al., 2021; Fitriawan et al., 2022). For instance, students often struggle with interpreting abstract mathematical concepts, translating real-world problems into mathematical models, and articulating their reasoning clearly (Cinar et al., 2023; Pinter & Siddiqui, 2024; Žakelj et al., 2024). These issues highlight the need for educational approaches addressing cognitive and communicative obstacles in mathematics education. While some students demonstrate proficiency in fundamental mathematical principles (Farida et al., 2019; Schillinger, 2021; Yerizon et al., 2019), others encounter persistent difficulties in domains such as geometry (Volikova et al., 2019), particularly in extracting information from complex problems and applying appropriate calculation methods (Hasan, 2020; Roulstone et al., 2024). The prevalence of these challenges across diverse mathematical domains underscores the importance of adopting targeted, evidence-based interventions that bridge conceptual and procedural gaps.

One promising theoretical framework for addressing these issues is APOS (Action, Process, Object, Schema) theory, which offers a constructivist approach to understanding how students develop mathematical concepts (Moru, 2020; Bilondi & Radmehr, 2023; Tatira, 2021). APOS theory posits that learning mathematics involves transitioning through four mental structures—action, process, object, and schema—each building upon the previous. Studies applying APOS theory to topics such as derivatives, binomial expansions, and trigonometric functions have demonstrated its effectiveness in diagnosing and addressing cognitive obstacles (Nga et al., 2023; Tuktamyshov & Gorskaya, 2024). Additionally, the integration of the activity, classroom discussion, and exercise learning cycle within APOS-based instruction has been shown to enhance students' academic performance (Arnawa et al., 2021; Nga et al., 2023) and attitudes toward mathematics (Pham & Nguyen, 2023; Tuktamyshov & Gorskaya, 2024). This underscores the potential of APOS theory to identify students' misconceptions and provide a structured pathway for their conceptual development.

In the context of teacher education, prospective mathematics teachers face unique challenges in conceptual and pedagogical development. Studies in Indonesia reveal that many prospective teachers rely heavily on procedural approaches and memorization (Istanto et al., 2024; Ndiung & Menggo, 2024; Romadhon et al., 2024; Suwardika et al., 2024), often struggling to develop and convey a deeper conceptual understanding of topics such as linear equations and systems (Dewi et al., 2021; Martín Molina et al., 2024; Zayyadi et al., 2019). Moreover, their instructional practices frequently emphasize procedural fluency over conceptual exploration, limiting their ability to foster critical thinking and problem-solving skills in students (Kania et al., 2023; Siswono et al., 2019; Usmayati & Gürbüz, 2024). These findings highlight an urgent need to strengthen content knowledge and PCK in teacher preparation programs, emphasizing scaffolding strategies and metacognitive approaches to enhance teaching effectiveness.

Given the foundational role of algebra in mathematics education, this study focuses on the cognitive obstacles prospective mathematics teachers encounter in learning number concepts, particularly through the lens of APOS theory. Two key research questions guide this investigation: (1) What cognitive obstacles do prospective mathematics teachers face in developing algebraic concepts? Furthermore, (2) How do they integrate their mental construction of numbers with the introduction to genetic decomposition? By examining these aspects, this study aims to provide new insights into the cognitive processes underlying the learning of algebraic concepts among prospective mathematics teachers. Furthermore, it seeks to inform the design of instructional strategies that support their conceptual and pedagogical development, ultimately enhancing their ability to teach mathematics effectively.

Given the interpretive orientation of this qualitative inquiry and the theoretical grounding in APOS (Action–Process–Object–Schema) theory, the following working hypotheses are formulated to guide the investigation:

1. Prospective mathematics teachers are anticipated to experience specific cognitive obstacles in constructing algebraic concepts related to number systems. These difficulties are expected to emerge predominantly during the early developmental phases of the APOS framework—namely, the Action and Process stages—where procedural fluency and conceptual integration are often insufficiently developed.
2. The implementation of instructional strategies grounded in genetic decomposition is hypothesised to enhance the mental construction of algebraic knowledge among pre-service teachers.

METHODS

This study adopted an interpretive research paradigm to investigate the cognitive obstacles preservice mathematics teachers encounter while learning, particularly algebra, within a basic mathematics course. Adigun et al. (2025) asserted that the interpretive paradigm was chosen because it aims to understand individuals' subjective experiences and interpret meaning through qualitative inquiry, aligning with the study's focus on the mental constructions of mathematical concepts. The methodological framework follows a qualitative approach, which is well-suited for exploring complex phenomena within their real-life context. This approach allows for a deeper understanding of the research subject by examining the participants' meanings, experiences, and perspectives.

Specifically, the study employs a single-case study methodology, providing a comprehensive, detailed analysis of a particular case, event, or individual (Jiang et al., 2024; Mwakililo et al., 2025). The objective is to gain insights into the unique context and intricacies of the case, uncovering patterns, underlying causes, or nuances that may not be immediately apparent in more generalised research methods. By focusing on a single case, the research can explore a specific phenomenon within a defined setting, revealing deeper insights that might otherwise be overlooked in broader contexts.

Participant

A purposive sampling technique was employed to select participants for this study, which was conducted at a private university located in Majalengka, West Java, Indonesia. Initially, three first-year preservice mathematics teachers enrolled in a Basic Mathematics course within the Bachelor of Education program in mathematics were selected. To enhance the depth and validity of the findings, the sample size was expanded to six to ensure greater diversity and triangulation of findings. Participants were selected based on the following refined criteria:

1. Active engagement in learning algebraic concepts,
2. Representation of diverse academic abilities (high and medium-performance levels),
3. Willingness to participate in extensive and iterative data collection processes, including follow-up interactions.

The expanded participant pool allowed for greater variability in experiences and cognitive patterns, strengthening the transferability of the study's conclusions while maintaining a focus on in-depth exploration.

Data collection

A multi-method data collection strategy was implemented, grounded in APOS theory. Instructional activities followed the activities class, discussion, and exercises learning cycle, designed to foster deep engagement and active learning according to (Mutambara & Tsakeni, 2022). The data collection methods included:

1. Collaborative problem-solving sessions, where participants worked in pairs or groups to solve algebraic problems. These sessions emphasised reflective discussions to uncover cognitive obstacles and learning strategies.
2. Diagnostic worksheets with scaffolding techniques focused on algebraic structures and number concepts. Worksheets incorporated conceptual, procedural, and application-level tasks to analyze participants' understanding comprehensively.
3. Semi-structured interviews, conducted in multiple rounds to capture participants' evolving cognitive processes. Questions were refined based on participants' written responses and classroom observations. Follow-up interviews probed deeper into specific cognitive challenges.
4. Non-participant classroom observations, with detailed field notes capturing interactions, peer discussions, and instructional interventions. Observations served to identify behavioral indicators of conceptual difficulty and provided contextual data.
5. Reflective journals, where participants documented their thought processes, challenges, and problem-solving approaches after completing tasks.

These methods were designed to ensure triangulation and validity. They capture cognitive obstacles from multiple perspectives while aligning with the APOS framework.

Data analysis

The four stages of learning a mathematical concept, as proposed in APOS theory, are based on the foundational work of Dubinsky (1997), which provides a clear framework for understanding the genetic decomposition of complex numbers. Building upon this, the researchers developed the following genetic decomposition for numbers, which serves as a tool for analyzing the cognitive processes involved in mastering such concepts.

Thematic analysis procedure

Thematic analysis in this study followed a structured coding protocol grounded in the APOS theory (Dubinsky & Wilson, 2013), aimed at identifying students' cognitive constructions and obstacles in learning algebraic concepts. The analysis was conducted in four sequential stages:

Initial coding

Raw data—including interview transcripts, written responses, and classroom observation notes—were segmented into components corresponding to the APOS framework: Action, Process, Object, and Schema. This step enabled researchers to map cognitive responses to specific theoretical constructs.

Identification of cognitive obstacles

Within each APOS component, instances of misconceptions, incomplete mental constructions, and procedural errors were identified. These obstacles were coded and categorized to reveal areas where participants struggled in forming coherent mathematical understanding.

Cross-case analysis

A detailed cross-case analysis was performed to compare cognitive patterns across the six participants. This involved examining the consistency and variation in how each participant navigated through the APOS stages. The analysis revealed recurring cognitive difficulties, especially in the Action and Process stages, such as rigid reliance on algorithms, difficulty transitioning from procedures to mental processes, and weak abstraction of algebraic structures. However, distinct context-specific challenges were also identified. For instance, participants with stronger procedural fluency sometimes lacked representational flexibility, while others demonstrated conceptual insight but struggled with symbolic manipulation.

These comparisons provided insight into both shared learning trajectories and individualized obstacles, allowing for a nuanced understanding of pre-service teachers' cognitive development.

Table 1
Genetic decomposition and analysis

Stage	Description (Aligned with Number Concepts and Introductory Algebra)
Action	Individuals begin to manipulate algebraic expressions involving number operations (e.g., integer operations, basic properties of operations, identities) by applying memorised procedures without necessarily understanding underlying structures.
Process	Individuals internalise the procedures and begin to conceptualise operations (e.g., commutativity, associativity, distributivity) as dynamic processes. At this stage, they can mentally transform algebraic expressions and begin to identify and generalise patterns.
Object	Individuals encapsulate processes into mental objects. For instance, they can perceive an algebraic expression as a manipulable entity, recognise inverse operations, and understand equivalence of expressions beyond procedural execution.
Schema	Individuals construct a coherent mental structure linking various number concepts and algebraic operations. They can flexibly navigate between representations (symbolic, verbal, and tabular), reason abstractly about relationships among concepts, and apply these concepts in unfamiliar contexts.

Genetic decomposition analysis

Participants' learning trajectories were analyzed to trace the development and transformation of introductory number and algebraic concepts through the lens of APOS theory. This genetic decomposition illuminated how participants constructed mental actions, progressed to internal processes, and eventually formed structured schemas.

This analysis served as a theoretical model to interpret how participants overcame (or failed to overcome) specific cognitive barriers. Table 1 presents the refined genetic decomposition aligned with basic algebraic concepts and the APOS framework.

To ensure reliability and credibility, the coding scheme and interpretive themes were reviewed in peer debriefing sessions with two independent researchers. Discrepancies were discussed and resolved through consensus. Additionally, triangulation of data sources—including interviews, worksheets, classroom observations, and reflective journals—enhanced the trustworthiness of the findings.

Ethical consideration

Ethical approval for the study was granted by the institutional review board, ensuring compliance with established ethical research standards. Informed consent was obtained from all participants, and their confidentiality was rigorously safeguarded. Pseudonyms were employed throughout the study, and all audio and video recordings were securely stored on encrypted devices. Participants were kept informed about the study's progress at regular intervals, promoting transparency and fostering trust.

FINDINGS

This research explores the cognitive obstacles participants face in learning algebra, focusing on the concepts of logarithms, matrices, and quadratic functions. Data collected through various sources—including written answers, observations of group interactions, semi-structured interviews, diagnostic tasks, and cognitive mapping using APOS theory—revealed significant difficulties related to procedural understanding and limitations in abstract thinking, as shown in Table 2.

Cognitive obstacles in mathematical proof

Many participants experienced difficulty simplifying logarithmic expressions in the logarithm and matrix questions. Most of the answers showed confusion in applying the basic rules of logarithms or in connecting the concept of logarithms with matrix operations. When asked to determine the determinant of the logarithm matrix, many participants could not solve the problem correctly. Their answers were more procedural, without a deep understanding of the basic principles of logarithms and how these concepts are applied in a matrix context (Ndlovu & Brijlall, 2015). This suggests that

Table 2
Overview of cognitive obstacles across APOS stages

APOS Stage	Cognitive Obstacle Description	Example of Student Difficulty	Pedagogical Implication
Action	Mechanical manipulation without conceptual understanding	Incorrect simplification of log expressions; procedural matrix multiplication	Reinforcement of algebraic properties through guided practice
Process	Partial conceptualisation; difficulties integrating steps	Errors in elimination steps for solving systems	Visual aids and step-by-step scaffolded instruction
Object	Difficulty perceiving mathematical structures as wholes	Misinterpretation of determinant meaning and matrix-log relations	Conceptual linking between abstract entities
Schema	Inability to generalise and integrate concepts	Difficulty connecting quadratic forms to graph and real-world context	Integration of multiple representations and problem contexts

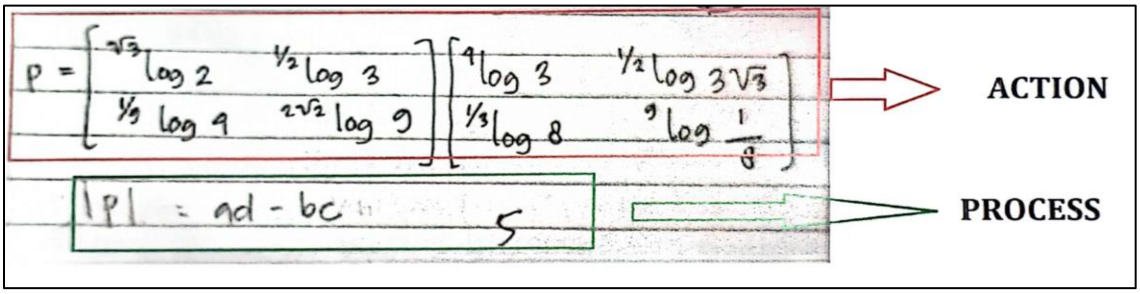

Figure 1. Participant's response showing procedural focus and misconceptions

most participants were operating at the action stage in APOS theory, where they focused more on the mechanical steps than understanding the underlying concepts.

Similar findings have been documented in recent studies. For instance, (Aksu et al., 2022) found that pre-service mathematics teachers often rely on memorised procedures without demonstrating conceptual understanding, especially in algebraic contexts. Likewise, Mustafa and Derya (2016) reported that many university students could carry out symbolic manipulation correctly while failing to interpret the underlying mathematical structures, indicating a predominance of action-level reasoning. These results underscore the persistent gap between procedural fluency and conceptual understanding among prospective mathematics teachers, particularly during the early phases of cognitive development in algebraic thinking.

Figure 1 shows the problem involves multiplying two matrices containing logarithmic expressions, which requires not only algebraic manipulation but also a strong conceptual understanding of the properties of logarithms and matrix operations. In the action stage, students apply basic procedural steps such as substituting logarithm values and performing basic matrix operations. However, errors in calculations or misapplication of logarithm properties often indicate a reliance on memorised procedures rather than an understanding of the underlying concepts. In the process stage, students move from performing individual operations to understanding the relationships between matrix elements. However, challenges may arise due to misunderstandings regarding the rules of logarithms or the procedure for matrix multiplication, highlighting the need for a deeper understanding of the interrelationships of these operations. Thus, the Object and Schema stages cannot be performed.

The following is a discussion that highlights the cognitive challenges faced by students to provide a clearer picture of the difficulties they experience in understanding mathematical proofs and how teachers help them overcome these obstacles;

Teacher: Before we proceed, let us start with this simple problem. How do we simplify log a(xy)? Can anyone explain?
Student 1: For me, I divide loga(x) and loga(y).
Teacher: Ah, that is a common mistake. Remember, the correct property is log a(xy)= log a(x) + log a(y). So we cannot divide them. Can anyone show us the correct way?
Student 2: So, we add log a(x) and log a(y)? That sounds reasonable, but what if we have to multiply in logarithms?
Teacher: Yes, that is right. Remember this as a rule that applies when we work with logarithms. Now, what if we try to calculate the determinant of this matrix?
Student 3: I am confused, sir. Should we calculate these logarithms one by one first or go straight to the matrix operation?
Teacher: Let's simplify the elements of the logarithm. Let's do the steps one by one.
Student 1: And then?
Teacher: After we simplify, the matrix becomes easier to calculate. Does anyone have difficulty with simplifying logarithms?
Student 1: I am still a bit confused, ma'am, about how to visualise the relationship between logarithms and these matrix operations. Why do we need to convert logarithms to ordinary numbers?
Teacher: That is a great question! It is important because we want to simplify the calculations to focus on the bigger steps. Remember, logarithms help us express numbers in a form that's easier to process in some cases.

Research on mathematical errors among students reveals common challenges in matrix operations, algebraic calculations, and problem-solving. Factors contributing to these errors include insufficient practice, lack of attention to problem details, and poor retention of mathematical concepts. Addressing these challenges requires a focus on strengthening conceptual understanding and providing diverse problem-solving opportunities for students.

Here is a discussion that illustrates the cognitive obstacles students face at the Process stage of APOS theory, along with how teachers can help them overcome these obstacles to improve their understanding of mathematical proofs:

Teacher: Now, how do you calculate the determinant of the simplified matrix?
Student 2: I tried using the determinant formula, which is ad−bc, but I am afraid I will make a mistake when simplifying.
Teacher: Okay, let us see together. What if we use the elements that we simplified earlier?
Student 3: Adi, we have a matrix, and now we calculate its determinant.
Student 1: The determinant is 2(ad−bc). I am a little hesitant to calculate it
Teacher: Good! Is anyone having trouble with this step? What happens if we use logarithms in a more complicated context?
Student 2: Do the properties of logarithms still apply, like log a(xn)=n log a(x)? I often get confused about when to apply them.
Teacher: Yes, they are important, and sometimes they can be confusing. The key is understanding when we can manipulate exponents or logarithms in that way. Remember, whenever we see an exponential or logarithmic form, we can try to exploit these properties.

Research on mathematical abstraction and connections reveals challenges students face in advanced topics like matrix algebra and logarithms. Students also have difficulty generalising concepts and connecting logarithms with other applications, such as converting to exponential form. These issues stem from various obstacles, including cognitive, genetic, psychological, and epistemological obstacles. To address these challenges, interventions like reinforcing prerequisite material, providing scaffolding, and using convergent questioning are recommended. Additionally, fostering connections between different representations, part-whole relationships, and mathematical procedures is crucial for problem-solving success. Improving instructional methods based on understanding students' mental constructions can help overcome mathematical abstraction and connection difficulties.

Here is a discussion that illustrates the cognitive obstacles students face at the Object stage of APOS theory, along with how teachers can help them overcome these obstacles to improve their understanding of mathematical proofs:

Teacher: *At this point, we want to look at logarithms and matrices not as separate elements but as an integrated object. What happens if we change the elements of this matrix to exponential form? Does the determinant remain the same?*

Student 1: Would we get the same result for the determinant? I do not think I understand the relationship between logarithms and exponentials.

Teacher: That is a good question. Let us try to understand that logarithms are the inverse of exponentials, so changing the form can affect the way we view the matrix. What if we try to visualise this matrix in a graphical context?

Student 2: I am still confused about how logarithms affect this matrix operation. Why do we need to know how these elements work together?"

Teacher: Logarithms give us a way to understand changes in a more structured form. By seeing these elements as part of a larger object, we can integrate their properties more easily. This will help us understand more concepts in the future."

The APOS theory provides a framework for understanding students' conceptualization of mathematical concepts. While APOS theory has been influential in mathematics education research, it is not universally accepted as a Kuhnian paradigm and may be better described as a Lakatosian research program.

Here is a discussion that illustrates the cognitive obstacles students face and how teachers help them overcome these obstacles, mainly focusing on the conceptual integration at the Schema stage:

Teacher: Look at the relationship between logarithms and matrices in a broader context. How can we apply these concepts to solve other problems, such as systems of exponential equations?

Student 3: Can we relate logarithms and matrices to solve systems of exponential equations? I am still not sure how these concepts are related."

Teacher: That is right! We can use the properties of logarithms and matrix operations to solve such systems. Imagine connecting these two concepts. What patterns do you notice when you compare the determinant results with different logarithm elements?

Student 2: I am beginning to see how these previously separate concepts can be related. However, I am still having trouble finding the right pattern."

Teacher: This is an important step in forming our mental schema. The more you connect these concepts, the easier it will be for you to recognise patterns and use them to solve other problems.

This discussion helps students address the cognitive obstacles they encounter in understanding logarithms and matrices. Obstacles such as confusion in applying the properties of logarithms, concerns in understanding matrix operations and difficulties in seeing relationships between mathematical concepts can be addressed stepwise from a more mechanical to a more integrated understanding. The APOS-based approach allows students to build more profound and flexible understandings, helping them overcome cognitive obstacles and preparing them for more complex mathematical challenges.

Group discussions provided further insight into the cognitive obstacles that participants faced. Based on the students' work, the APOS theoretical framework can effectively analyse cognitive obstacles in mathematical proof. The given problem requires solving a system of linear equations to determine the parameters a, b, and c in the quadratic equation $y=ax^2+bx+c$. This process demands a conceptual understanding of algebraic structures, procedural fluency in manipulating equations, and the ability to transition from an operational to an abstract comprehension of quadratic functions. Many students encounter difficulties at the action and process stages, where they focus on procedural execution without a deeper grasp of the underlying mathematical relationships. Challenges in transitioning to the object and schema stages often manifest in errors related to equation manipulation, misinterpretation of system constraints, and the inability to generalise results. Addressing these cognitive obstacles through targeted instructional strategies can enhance students' ability to construct mathematical proofs and develop a robust conceptual understanding of quadratic equations.

Figure 2 presents the written response of a participant solving a system of linear equations to determine the coefficients a, b, and c in the quadratic function $y=ax^2+bx+c$. The problem requires substituting given coordinate points into the general quadratic equation, forming *a* system of

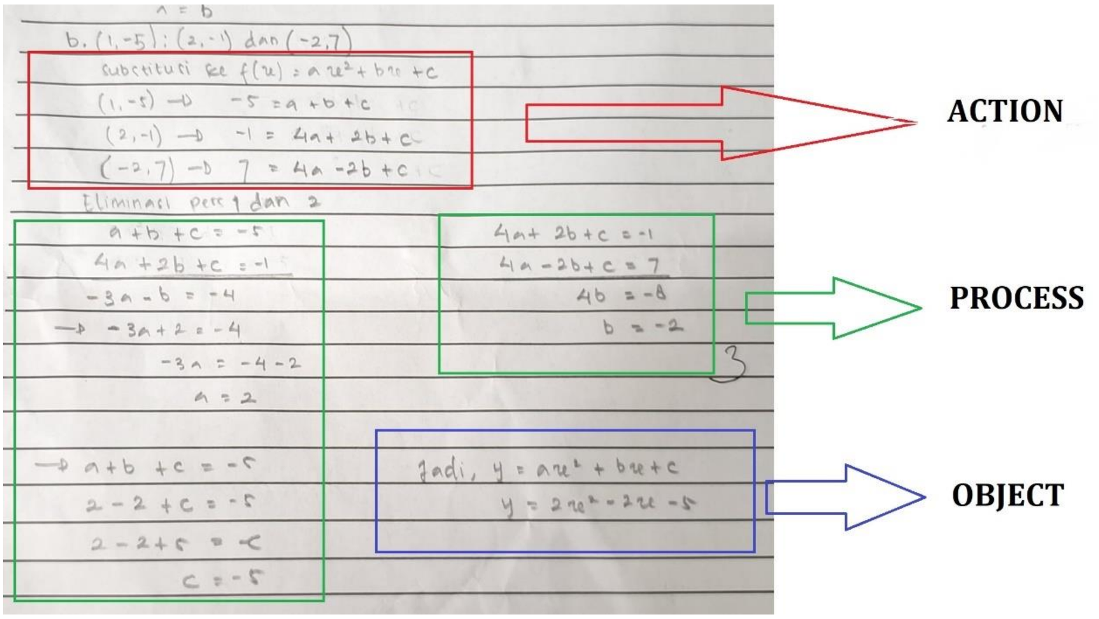

Figure 2. Participant's response highlighting cognitive obstacles

equations, and solving for the unknowns. Using the APOS framework, cognitive obstacles in the participant's problem-solving process can be analysed across different stages.

In the action stage, students demonstrate the initial ability to substitute point values (x, y) into a quadratic equation. Although this step is mechanical, some students tend to make errors in basic algebraic operations, such as incorrect addition or elimination of equations. A common obstacle at this stage is difficulty understanding the initial steps relevant to the problem, such as determining the relationship between the given points and the coefficients *a, b, and c*. In the process stage, the systematic elimination of variables indicates an emerging grasp of equation-solving techniques. However, minor computational errors or inconsistencies may highlight difficulties in comprehending relationships between equations.

Moving to the object stage, the participant arrives at specific values for *a, b,* and *c*, demonstrating an ability to conceptualise the quadratic function as a structured mathematical entity. However, any inconsistencies in their final solution could suggest gaps in recognising broader algebraic implications. At the schema stage, a fully developed understanding would require the participant to recognise alternative solution methods, generalise their approach to different quadratic problems, and validate the correctness of their results. This analysis underscores common cognitive obstacles in solving quadratic equations and highlights the need for instructional interventions that enhance both procedural fluency and conceptual comprehension.

The following is a conversation that highlights the cognitive obstacles that students face during the stages of solving a quadratic equation system problem, which illustrates cognitive obstacles at the Action Stage according to APOS theory:

Teacher: *We will solve the system of equations to find the values of aa, bb, and cc in the quadratic equation $y=ax^2+bx+c$. For example, we are given points such as (x_1,y_1), (x_2,y_2), and (x_3,y_3). Try to substitute the values of these points into the quadratic equation.*

Student 1: *Okay, I substitute the values of x1 and y1 into the equation. I get $y_1=ax_{12}+bx_1+c$. Then I continue for the other points.*

Teacher: *Good. However, pay attention to whether the substitution results are correct. Are you sure there are no addition or multiplication errors in basic algebra?*

Student 2: *I feel a little confused about the negative signs in the equation. Why are the results not as expected?*

Teacher: *Pay attention again to each step and the negative signs. At this stage, it is important to be careful with each algebraic operation you do.*

Research on students' difficulties with quadratic equations reveals common errors across educational contexts. These include mishandling signs, computational errors in introductory algebra, and challenges in applying equation-solving procedures. Students struggle with factorization, using the quadratic formula, and distinguishing between solving and simplifying equations. Performance patterns are similar across well-resourced and poorly-resourced schools, suggesting systemic issues in curriculum design. To address these challenges, studies recommend immediate feedback, whole-class discussions of errors, and targeted instructional strategies. Incorporating incorrect examples in teaching can improve equation-solving abilities, particularly for students with limited algebraic knowledge. Teachers are advised to pay more attention to integer operations and the cognitive shifts required in solving equations with variables on both sides.

Here is a conversation that highlights the cognitive obstacles students face at the Process stage of solving quadratic equation systems, illustrating the obstacles according to APOS theory at the Process stage and the need for reinforcing understanding:

> Teacher: Next, let us try the elimination process. From the system of equations *you have, try to eliminate one of the variables to reduce the number of equations. What do you get after the first step?"*
>
> *Student 3: I tried to eliminate c, but the result was not what I thought. There was something wrong with my steps.*
>
> *Teacher: Did you notice how the negative sign affected this elimination? Sometimes, mistakes happen because we are not consistent in handling these signs.*
>
> *Student 1: I struggle to subtract two equations with negative signs. I often get confused about whether I need to change the sign.*
>
> *Teacher: This is indeed a common mistake at this stage. When eliminating, be sure to double-check each step and sign and ensure that the logic of elimination remains consistent. Try to visualise these steps to make them clearer.*

Research suggests that students face challenges in mastering systems of linear equations (SLEs) and related algebraic concepts. Students may struggle to validate solutions at the object stage or understand that elimination results must satisfy all original equations. Some students achieve only an action-level understanding of unique solutions but struggle with non-unique solutions requiring process and object conceptions.

Interestingly, algebraic expertise involves retraining the visual system to perceive hierarchical structure in equations, with object-based attention playing a role in parsing expressions. Mathematical modelling tasks can help characterise students' learning routes and identify difficulties, particularly in validating results. These findings highlight the complexity of learning algebraic concepts and suggest potential strategies for improving instruction.

Here is a conversation highlighting the cognitive obstacles students face during the stages of solving quadratic equation systems problems, specifically focusing on the object stage of APOS theory and the process of forming a mathematical object:

> *Teacher: At this stage, we need to look at the elimination result as a more complete object. Does this elimination result satisfy all the equations you started with?*
>
> *Student 2: I feel confused because I only did the algebraic part. However, I am not sure if my elimination result is consistent with the whole equation.*
>
> *Teacher: This is a common problem. At this stage, you should start to see the system of equations and the elimination result as an integrated object, not just separate parts. Try to double-check each solution you find, and make sure that it satisfies all the original equations.*
>
> *Student 3: I sometimes get caught up in algebraic manipulations and ignore whether the result fits the whole equation. This makes me doubt my final solution.*

Research on students' understanding of quadratic equations reveals several challenges. Students often struggle to integrate their knowledge of systems of equations, algebraic operations, and parameter relationships within the quadratic context. The lack of flexibility in thinking is evident in students' inability to connect algebraic and geometric concepts, such as relating solutions to parabolas passing through specific points. Small algebraic errors frequently impact final results, reflecting weak mental schema structures. To address these issues, researchers suggest allocating more time for developing conceptual understanding and procedural fluency, incorporating figural pattern generalisation to stimulate visual-algebraic reasoning coordination, and explicitly considering mental constructions in the learning process.

The following conversation highlights the cognitive obstacles students face during the stages of solving quadratic equation systems problems, specifically focusing on the Schema stage of APOS theory and the integration of concepts at this stage:

Teacher: At this stage, we should be able to see how the relationship between the parameters aa, bb, and cc affects the form of the quadratic equation. Try to relate your results to its geometric form, which is the parabola that passes through these points.

Student 1: I find it difficult to relate the algebraic result to the form of the parabola. How do I know that the values of a, b, and c affect the form of the parabola?

Teacher: This is a very common challenge at this stage. Try to visualise the parabola using a graph. What happens if the value of a is greater or less? How does that affect the slope of the parabola?

Student 2: I feel confused because I only do the algebraic process without understanding how the results change the graphical form. I cannot yet connect the numbers to the graph that is formed.

Teacher: At this stage, you should be able to relate the algebraic result to the graphical form of the quadratic equation. This is important for developing a broader understanding because the results you get apply not only to the numbers but also to the geometric form.

This conversation illustrates how students encounter cognitive obstacles at each problem-solving stage according to the APOS theory. Students struggle to comprehend and correctly execute fundamental algebraic procedures at the action stage, often making errors in basic manipulations. Moving to the process stage, they begin to construct an understanding but continue to face challenges in algebraic manipulation, particularly in handling negative signs and maintaining procedural accuracy. At the object stage, difficulties arise in perceiving the system of equations as a structured mathematical entity rather than a sequence of isolated steps. In the schema stage, students cannot connect their algebraic solutions to geometric representations, specifically the parabolic nature of quadratic equations.

This analysis highlights the necessity of targeted learning interventions to facilitate students' progression through the APOS theory more effectively. For the Action stage, instructional strategies should emphasise a deep understanding of fundamental algebraic properties through structured exercises. During the process stage, students must be encouraged to reflect on their problem-solving steps and develop skills to verify the consistency of their results. To address challenges in the Object and Schema stages, incorporating visual representations—such as parabolic graphs—can enhance students' comprehension of the relationships between coefficients, equations, and their graphical interpretations. These findings further support the need for an APOS-based pedagogical approach that ensures seamless transitions between cognitive stages, ultimately mitigating mathematical proof and problem-solving obstacles.

DISCUSSION

The findings of this study provide critical insights into the cognitive obstacles pre-service mathematics teachers encounter in algebraic learning. These obstacles have significant implications for both instructional practice (Anwar et al., 2023; Nikou et al., 2024; Travé González et al., 2017) and educational theory (Al-Adwan et al., 2024; Allmon, 2011; Chabursky et al., 2024). This is also supported by Cristea et al. (2025) and Zakaria et al. (2025), who said identifying common cognitive obstacles emphasises the need for tailored instructional strategies that effectively address these challenges. Specifically, educators should prioritise conceptual understanding alongside procedural fluency (Kilp-Kabel & Mädamürk, 2024; Lee et al., 2024; Rivard, 2024), ensuring students progress beyond rote memorisation to deeply comprehend mathematical concepts' underlying structures. This study also suggests that structured peer collaboration can be an effective intervention for students at different proficiency levels by fostering deeper engagement with mathematical reasoning.

Cognitive obstacles at the early stages of learning indicate that students are still operating at a low procedural level (Susiswo et al., 2021). This may be attributed to the insufficient instructional emphasis on algorithmic mastery and limited exposure to real-world applications of logarithmic functions in mathematical proofs (Keller, 2012; Murre, 2023). For example, while students may recall the basic determinant formula, they often fail to apply it correctly due to misconceptions regarding logarithmic properties, such as log (a)+log (b)=log (ab). This difficulty points to an incomplete transition from the action stage to the process stage within the APOS framework, highlighting a critical gap in the development of conceptual understanding (Borji et al., 2020; Martínez-Planell & Trigueros, 2020). Addressing this gap requires instructional interventions to facilitate a smoother transition between these cognitive stages (Oktaç et al., 2022; Trigueros, Cabrera, et al., 2024). Guided problem-solving exercises that explicitly connect algorithmic procedures with conceptual reasoning can effectively bridge this gap (Abakah & Brijlall, 2024; Trigueros, Badillo, et al., 2024) . These findings underscore the necessity for an APOS-based pedagogical approach that enhances procedural fluency and strengthens students' ability to generalise mathematical principles across different contexts.

Students often struggle to perceive logarithms as mathematical objects that can be manipulated within broader structures, such as matrices. Logarithmic elements in the context of determinants should be understood as integrated units that interact systematically within matrix operations. However, students' inability to recognise these relationships indicates weak conceptual integration across different mathematical domains. At this stage, students should be able to connect multiple concepts—logarithms, matrices, and determinants—into a coherent cognitive framework. This finding suggests that instructional strategies must move beyond isolated procedural tasks and foster a more integrated understanding of these concepts.

Findings and conceptual understanding: APOS theory

The analysis of students' work in solving systems of linear equations to determine the parameters *a, b,* and *c* in the quadratic equation $y=ax^2+bx+c$ reveals distinct cognitive obstacles at each stage of the APOS framework. Research on pre-service mathematics teachers' understanding of complex mathematical concepts has identified various cognitive barriers within the APOS framework. Ndlovu and Brijlall (2016) found that many pre-service teachers remain at the Action and Process stages when learning determinant concepts, often demonstrating procedural knowledge without a deep conceptual understanding. Similar challenges are observed in logarithmic differentiation, where misconceptions persist among learners at the Action and early Process stages (Asghary et al., 2023). Additionally, difficulties in solving logarithmic equations, especially in proofs, highlight significant gaps in foundational knowledge of logarithmic concepts and rules (Okoye-Ogbalu & Nnadozie, 2024). These findings underscore the need for mathematics teacher education programs to emphasise both procedural fluency and conceptual understanding, as addressing misconceptions and enhancing competency will better prepare pre-service teachers for effective instruction.

Students face significant challenges when working with logarithms, often exhibiting errors rooted in both conceptual and procedural misunderstandings. Common mistakes include technical miscalculations, misapplication of algebraic rules, and difficulty applying logarithmic properties correctly (Bardini et al., 2004; Rafi & Retnawati, 2018). Rather than relying on rote memorisation and intuition, which often lead to incorrect reasoning, students should be encouraged to develop a solid understanding of logarithmic definitions (Dintarini, 2018; Tatira & Mukuka, 2024). Conceptual errors are the most prevalent among error types, followed by procedural and technical mistakes (Angraini et al., 2024; Yodiatmana & Kartini, 2022). These difficulties arise from weak foundational knowledge, leading to misconceptions such as treating "log" as a variable or failing to recognise structural relationships between different logarithmic expressions (Campo-Meneses et al., 2021; Yodiatmana & Kartini, 2022). While students perform adequately on routine calculations, they struggle with problems requiring higher-order thinking skills (Kania & Kusumah, 2025).

Findings from this study corroborate the patterns reported in prior research, particularly regarding students' dominance at the *action* stage in APOS theory, where reliance on mechanical procedures often overshadows conceptual understanding. Similar to the findings of Bardini et al. (2004) and Yodiatmana and Kartini (2022), our participants frequently misapplied logarithmic rules, revealing a lack of structural comprehension. Moreover, while previous studies identified isolated errors in procedural or technical steps, this study extends those insights by illustrating how such errors cluster within specific APOS stages, offering a more detailed mapping of cognitive obstacles in learning logarithms. Thus, consistent with Dintarini (2018) and Tatira and Mukuka (2024) our results emphasize the importance of strengthening conceptual foundations, especially during the transition from process to object stage. This deeper alignment between our findings and the literature reinforces the relevance of APOS theory as a lens to analyze and address the layered nature of algebraic difficulties among pre-service mathematics teachers.

At the Action stage, students primarily engage in the mechanical execution of algebraic procedures, such as substituting given coordinate points (x,y) into quadratic equations. This stage involves fundamental algebraic operations, yet students frequently make errors in basic manipulations, such as handling negative signs, addition, and subtraction. These errors indicate incomplete mastery of algebraic skills, which are essential before progressing to more complex problem-solving tasks. Consequently, the Action stage tends to emphasize mechanical execution without integrating the conceptual reasoning behind each step (Langi' et al., 2023; Trigueros, Badillo, et al., 2024). In contrast, the Process stage marks a shift in students' understanding—from performing individual operations to comprehending relationships between mathematical elements, such as those found in matrix theory. At this stage, students begin to recognize patterns in logarithmic simplifications and execute matrix multiplication with greater accuracy (Feriyanto & Putri, 2020; Kania et al., 2023). Despite this progress, misconceptions regarding logarithmic properties or matrix multiplication rules often persist, underscoring the need for a deeper, more nuanced understanding of these operations' interrelations. Moreover, during the Process stage, students begin to recognize relationships within systems of linear equations and apply methods such as elimination or substitution. A significant challenge at this stage is managing procedural complexity. Many students rely on memorized steps without fully comprehending the underlying logic, leading to errors when eliminating variables or processing signs. Additionally, students often fail to verify the consistency of intermediate results with the original equations, revealing a fragile conceptual grasp of the system. These findings suggest that instructional strategies should encourage students to critically engage with each step of their solution process, emphasizing the verification of results as key to strengthening their conceptual understanding.

At the Object stage, students are expected to view the system of equations and the elimination process as an integrated mathematical structure. However, obstacles at this stage include the lack of solution validation. While students may successfully determine the values of aaa, bbb, and ccc, they often neglect to check whether these values satisfy all original equations. This oversight suggests that students perceive problem-solving as a sequence of isolated steps, hindering their ability to generalize and apply strategies in different contexts. Reflective problem-solving tasks that encourage students to check their solutions against the entire system of equations are essential at this stage. Students must understand these concepts structurally (Abu-Hilal et al., 2013) and move beyond individual calculations (Padmanabhan et al., 2013). Inconsistencies in determinant calculations or misapplied logarithmic transformations may indicate gaps in their understanding of determinant formulas and logarithmic properties, suggesting the need for further conceptual development.

At the Schema stage, students should synthesize their understanding across multiple representations, linking algebraic solutions to their geometric interpretations (e.g., as a parabola). However, many students struggle to make this connection, demonstrating a limited ability to generalize their findings beyond the immediate problem. This indicates that current instructional strategies may not adequately emphasize the conceptual coherence between algebraic manipulations and their geometric significance. Integrating visual aids, such as graphs or geometric representations, could significantly enhance students' ability to make these conceptual links. At this stage, Oktaç et al. (2022) stated that students should develop the ability to generalize their approach to broader contexts, such as solving matrix equations involving logarithmic expressions in various applications. Learners should not only understand individual mathematical objects but also integrate them and apply their understanding to solve complex problems across different areas of mathematics.

These findings highlight the necessity for an APOS-based pedagogical approach that systematically addresses cognitive obstacles through structured learning interventions. Progressing from Action to Process requires reinforcing algebraic fluency with exercises that emphasize both procedural accuracy and conceptual understanding. Advancing from Process to Object requires reflective tasks that encourage students to critically analyze their solution steps, verify results, and understand the mathematical structures behind their work. Educators should incorporate visual representations (e.g., graphing quadratic functions) and contextual applications that help students internalize the interconnected relationships between algebraic expressions and their geometric interpretations, facilitating progression from Object to Schema.

Role of group discussion in overcoming cognitive obstacles

Group discussion plays a strategic role in addressing cognitive obstacles encountered by students, particularly in algebraic learning involving logarithms, matrices, and quadratic functions. Research on the APOS framework supports the idea that group discussions significantly enhance students' progression through each stage of mathematical understanding (Hlangwani & Dhlamini, 2024; Tuktamyshov & Gorskaya, 2024).

At the Action stage, students often rely on mechanical procedures without understanding the conceptual foundations of the mathematics involved. Group discussions help students identify and correct misconceptions (Saleemad et al., 2022; Wester, 2021), such as errors in applying logarithmic properties, by creating a space where they can articulate their reasoning and receive peer feedback. Such collaborative settings not only highlight individual mistakes but also encourage deeper engagement with the conceptual relationships underlying the procedures. As a result, students move beyond rote memorization, developing a more meaningful understanding of the material (Andrews et al., 2020; Schwarz et al., 2022). These discussions foster critical thinking and create opportunities for students to explore the rationale behind their actions, thus promoting the development of procedural fluency alongside conceptual awareness.

As students progress to the process stage, they begin to form connections between procedural steps and underlying theories. Group discussions offer a reflective space where students can critically evaluate their reasoning and refine their approaches. This collaborative validation process helps students recognize how logarithmic and matrix properties influence their solutions within broader mathematical contexts (Campo-Meneses et al., 2021; Kolar & Hodnik, 2021). By discussing the interactions between different mathematical operations, such as the properties of logarithms in the context of matrix transformations, students gain insight into how various mathematical elements integrate. This transition from procedural fluency to conceptual understanding is particularly enhanced by group interaction, which allows for the sharing of diverse perspectives on solving problems (Faridayanti et al., 2025; Lee et al., 2024).

At the object stage, students are expected to perceive mathematical concepts as integrated entities. Group discussions facilitate this by providing opportunities for students to explore the interaction between concepts (Ballard et al., 2023; Oppong et al., 2024) such as logarithms, matrices, and quadratic functions. For instance, students can analyze how logarithmic determinants behave within matrix structures, deepening their understanding of these interconnections and recognizing their role in more complex problem-solving scenarios (Junarti et al., 2022; Kania et al., 2024). By discussing how concepts interrelate, students develop a more holistic understanding of mathematical structures, enabling them to see the 'big picture' rather than isolated concepts.

Finally, at the schema stage, students must integrate their acquired knowledge into a comprehensive framework that connects logarithms, matrices, and quadratic functions across various applications. Group discussions expose students to multiple problem-solving approaches, encouraging them to explore alternative solutions and overcome conceptual obstacles (Moru & Mathunya, 2022). The collaborative nature of these discussions fosters higher-order cognitive development, enabling students to apply their knowledge in more abstract or applied contexts, such as solving real-world problems (Osman et al., 2016) or exploring advanced mathematical topics (Yuanita et al., 2023). By presenting and defending their approaches in a group setting, students can refine their thinking, challenge their assumptions, and extend their problem-solving strategies to new and unfamiliar contexts. This not only enhances their understanding of the material but also helps develop the cognitive flexibility needed for abstract mathematical reasoning (Ningrum et al., 2020; Richland & Simms, 2015).

In sum, group discussions offer a powerful tool for overcoming cognitive obstacles at each stage of the APOS framework. By fostering reflection, validation, and integration of mathematical concepts, these discussions help students progress from isolated procedural skills to a deep, interconnected understanding of mathematical ideas. Educators should therefore incorporate collaborative learning opportunities into their teaching practices, as these can significantly enhance students' ability to solve complex mathematical problems and apply their knowledge in diverse contexts.

CONCLUSION

This study underscores the significance of an APOS theory-based learning approach in overcoming cognitive obstacles in algebra, particularly with concepts such as logarithms, matrices, and quadratic functions. The study's findings reveal that most students and pre-service teachers remain at the Action stage, where they tend to rely on procedural steps without fully understanding the underlying concepts. Common cognitive obstacles include errors in applying logarithmic properties, difficulties in integrating logarithms into matrix operations, and an inability to view systems of equations as cohesive entities. Group discussions have proven effective in helping students navigate through these stages by providing a platform for reflection, collaboration, and validation of understanding. When combined with visual tools and exploration-based learning, this strategy enhances students' conceptual understanding and fosters critical thinking and problem-solving skills.

The implications of this study emphasise the need for a more structured and student-centred mathematics learning design, with a focus on facilitating gradual transitions through each stage of the APOS framework. Teachers should adopt group discussions, exploration-based activities, and problem-based learning methods to strengthen students' comprehension of complex mathematical concepts. The integration of graphical representations, simulations, and real-world application contexts can further support students in acquiring more holistic knowledge. Moreover, this study opens avenues for the development of APOS-based diagnostic tools, which can identify cognitive obstacles in real-time, enabling teachers to provide targeted, effective interventions. In conclusion, the results of this study contribute to the advancement of innovative mathematics education practices and support the creation of meaningful learning experiences for students.

Future research should explore the effectiveness of APOS-based interventions across different educational levels, including junior high school and vocational education settings, to assess the broader applicability of this approach. Longitudinal studies may also be conducted to examine how students' progression through the APOS stages evolves, particularly when supported by sustained, structured instructional designs. Additionally, mixed-method research that integrates qualitative insights with quantitative performance data could offer a more comprehensive understanding of how specific learning strategies affect students' conceptual transitions.

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BIBLIOGRAPHY

Abakah, F., & Brijlall, D. (2024). Finding an Effective Assessment Approach to Enhance the Teaching and Learning of Circle Geometry. *Africa Education Review*, *20*(3), 93–116.

https://doi.org/10.1080/18146627.2024.2402772

Abu-Hilal, M. M., Abdelfattah, F. A., Alshumrani, S. A., Abduljabbar, A. S., & Marsh, H. W. (2013). Construct validity of self-concept in TIMSS's student background questionnaire: A test of separation and conflation of cognitive and affective dimensions of self-concept among Saudi eighth graders. *European Journal of Psychology of Education*, *28*(4), 1201–1220. https://doi.org/10.1007/s10212-012-0162-1

Adigun, O. T., Quadri, O. G., Jacob, U. S., & Ngubane, S. A. (2025). An Exploration of E-Teaching Preparedness of Pre-Service Teachers of Learners with Special Needs. *Africa Education Review*, 1–25. https://doi.org/10.1080/18146627.2024.2402767

Agustyaningrum, N., Husna, A., Hanggara, Y., Abadi, A. M., & Mahmudi, A. (2020). Analysis of Mathematical Proof Ability in Abstract Algebra Course. *Universal Journal of Educational Research*, *8*(3), 823–834. https://doi.org/10.13189/ujer.2020.080313

Aksu, Z., KALAÇ, S., Özkaya, M., & KONYALIOĞLU, A. C. (2022). Pre-service Mathematics Teachers' Conceptual Knowledge Related To Basic Concepts And Operations. *Research on Education and Psychology*, *6*(2), 283–297. https://doi.org/10.54535/rep.1204295

Al-Adwan, A. S., Meet, R. K., Kala, D., Smedley, J., Urbaníková, M., & Al-Rahmi, W. M. (2024). Closing the divide: Exploring higher education teachers' perspectives on educational technology. *Information Development*. https://doi.org/10.1177/02666669241279181

Allmon, W. D. (2011). Why Don't People Think Evolution Is True? Implications for Teaching, In and Out of the Classroom. *Evolution: Education and Outreach*, *4*(4), 648–665. https://doi.org/10.1007/s12052-011-0371-0

Alsina, Á., Pincheira, N., & Delgado-Rebolledo, R. (2024). The professional practice of designing tasks: how do pre-service early childhood teachers promote mathematical processes in early algebra? *ZDM – Mathematics Education*, *56*(6), 1197–1210. https://doi.org/10.1007/s11858-024-01636-1

Andrews, D. A., Sekyere, E. O., & Bugarcic, A. (2020). Collaborative Active Learning Activities Promote Deep Learning in a Chemistry-Biochemistry Course. *Medical Science Educator*, *30*(2), 801–810. https://doi.org/10.1007/s40670-020-00952-x

Angraini, L. M., Kania, N., & Gürbüz, F. (2024). Students' Proficiency in Computational Thinking Through Constructivist Learning Theory. *International Journal of Mathematics and Mathematics Education*, 45–59. https://doi.org/10.56855/ijmme.v2i1.963

Anwar, L., Sa'dijah, C., Murtafiah, W., & Huljannah, M. (2023). Adversity quotient of Indonesian prospective mathematics teachers in solving geometry higher-order thinking skills problems. *Journal on Mathematics Education*, *15*(1), 79–98. https://doi.org/10.22342/jme.v15i1.pp79-98

Arnawa, I. M., Yanita, Y., Yerizon, Y., Ginting, B., & Nita, S. (2021). Does The Use of APOS Theory Promote Students' Achievement in Elementary Linear Algebra? *International Journal of Instruction*, *14*(3), 175–186. https://doi.org/10.29333/iji.2021.14310a

Asghary, N., Afkhami, R., & Medghalchi, A. (2023). Developing a Framework for Evaluating Student's Understanding at Figural Pattern Generalization. *PNA. Revista de Investigación En Didáctica de La Matemática*, *18*(1), 57–76. https://doi.org/10.30827/pna.v18i1.16566

Ballard, J., Gamage, S., Winfield, L., & Mooring, S. (2023). Cognitive discourse during a group quiz activity in a blended learning organic chemistry course. *Chemistry Teacher International*, *5*(3), 245–261. https://doi.org/10.1515/cti-2023-0007

Bardini, C., Pierce, R. U., & Stacey, K. (2004). Teaching linear functions in context with graphics calculators: Students' responses and the impact of the approach on their use of algebraic symbols. *International Journal of Science and Mathematics Education*, *2*(3), 353–376. https://doi.org/10.1007/s10763-004-8075-3

Bilondi, M. T, & Radmehr, F. (2023). Students' mathematical thinking of the tree concept: an integration of APOS with Tall's three worlds of mathematics. *Research in Mathematics Education*, 1–22. https://doi.org/10.1080/14794802.2023.2292260

Borji, V., Erfani, H., & Font, V. (2020). A combined application of APOS and OSA to explore undergraduate students' understanding of polar coordinates. *International Journal of Mathematical Education in Science and Technology*, *51*(3), 405–423. https://doi.org/10.1080/0020739X.2019.1578904

Campo-Meneses, K. G., Font, V., García-García, J., & Sánchez, A. (2021). Mathematical Connections Activated in High School Students' Practice Solving Tasks on the Exponential and Logarithmic Functions. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(9), em1998. https://doi.org/10.29333/ejmste/11126

Chabursky, S., Gutt, J., & Guglhör-Rudan, A. (2024). Ukrainian refugee adolescents in Germany – A qualitative study on schooling experiences and the fulfillment of basic psychological needs. *Children and Youth Services Review*, *160*, 107582. https://doi.org/10.1016/j.childyouth.2024.107582

Cinar, O. E., Rafferty, K., Cutting, D., & Wang, H. (2023). Using Augmented Reality to Enhance Learning and

Understanding of Abstract Programming Concepts. *2023 9th International Conference on Virtual Reality (ICVR)*, 498–504. https://doi.org/10.1109/ICVR57957.2023.10169459

Copur-Gencturk, Y., & Doleck, T. (2021). Linking teachers' solution strategies to their performance on fraction word problems. *Teaching and Teacher Education*, *101*, 103314. https://doi.org/10.1016/j.tate.2021.103314

Cristea, T. S., Heikkinen, S., Snijders, C., Saqr, M., Matzat, U., Conijn, R., & Kleingeld, A. (2025). Dynamics of self-regulated learning: The effectiveness of students' strategies across course periods. *Computers & Education*, *228*, 105233. https://doi.org/10.1016/j.compedu.2025.105233

Dewi, I. L. K., Zaenuri, Z., Dwijanto, D., & Mulyono, M. (2021). Identification of Mathematics Prospective Teachers' Conceptual Understanding in Determining Solutions of Linear Equation Systems. *European Journal of Educational Research*, *volume-10*-(volume-10-issue-3-july-2021), 1157–1170. https://doi.org/10.12973/eu-jer.10.3.1157

Dintarini, M. (2018). Understanding Logarithm: What are the Difficulties That Students Have? *Proceedings of the 5th International Conference on Community Development (AMCA 2018)*. https://doi.org/10.2991/amca-18.2018.65

Dubinsky, E., & Wilson, R. T. (2013). High school students' understanding of the function concept. *The Journal of Mathematical Behavior*, *32*(1), 83–101. https://doi.org/10.1016/j.jmathb.2012.12.001

Farida, N., Sesanti, N. R., & Ferdiani, R. D. (2019). Tingkat Pemahaman Konsep dan Kemampuan Mengajar Mahasiswa pada Mata Kuliah Kajian dan Pengembangan Matematika Sekolah 2. *MUST: Journal of Mathematics Education, Science and Technology*, *4*(2), 135. https://doi.org/10.30651/must.v4i2.2897

Faridayanti, I., Nasrullah, A., Hidayat, A., & Kania, N. (2025). The Impact of the Discovery Learning Model on Mathematical Communication Skills and Study Habits of Junior High School Students. *Indonesian Journal of Teaching and Learning (INTEL)*, *4*(1), 16–29. https://doi.org/10.56855/intel.v4i1.1330

Faustino, A. C., & Sales, E. R. de S. (2024). Pensamento algébrico nos anos iniciais: possibilidades com estudantes cegos e videntes. *Prometeica - Revista de Filosofía y Ciencias*, *31*, 347–358. https://doi.org/10.34024/prometeica.2024.31.16476

Feriyanto, F., & Putri, R. O. E. (2020). Developing Mathematics Module Based on Literacy and Higher Order Thinking Skills (HOTS) Questions to Train Critical Thinking Ability of High School Students in Mojokerto. *Journal of Physics: Conference Series*, *1594*(1). https://doi.org/10.1088/1742-6596/1594/1/012014

Fitriani, F., Siregar, Y. A., & Novitasari, W. (2021). Analisis kesulitan kemampuan komunikasi matematika mahasiswa menggunakan aplikasi google classroom pada matakuliah aljabar. *Journal of Didactic Mathematics*, *2*(1), 18–25. https://doi.org/10.34007/jdm.v2i1.596

Fitriawan, D., Ijuddin, R., & Sayu, S. (2022). ANALISIS MATERI SULIT DIPAHAMI DAN MISKONSEPSI MAHASISWA DALAM MATA KULIAH KALKULUS INTEGRAL. *Numeracy*, *9*(1), 27–38. https://doi.org/10.46244/numeracy.v9i1.1682

Hartono, S. (2025). Evolving trends in mathematical proof research in Indonesian mathematics education: A systematic review from design to data analysis. *Multidisciplinary Reviews*, *8*(12), 2025372. https://doi.org/10.31893/multirev.2025372

Hasan, B. (2020). KESULITAN SISWA DAN SCAFFOLDING DALAM MENYELESAIKAN MASALAH GEOMETRI RUANG. *Numeracy*, *7*(1), 49–64. https://doi.org/10.46244/numeracy.v7i1.998

Hlangwani, W., & Dhlamini, Z. B. (2024). Grade 12 learners' understanding of differentiation: tracking first principle and power rule using the ACE teaching cycle. *International Journal of Learning and Change*, *16*(6), 678–694. https://doi.org/10.1504/IJLC.2024.141954

Hyland, D., & O'Shea, A. (2022). How Well Do High-Achieving Undergraduate Students Understand School Algebra? *Canadian Journal of Science, Mathematics and Technology Education*, *22*(4), 818–834. https://doi.org/10.1007/s42330-022-00256-9

Istanto, I., Fakhruddin, F., Yanto, H., & Pramono, S. E. (2024). Altruistic leadership: A quantitative analysis of its effects on school culture, teacher integrity, and performance in Muhammadiyah middle schools in Indonesia. *International Journal of ADVANCED AND APPLIED SCIENCES*, *11*(10), 140–146. https://doi.org/10.21833/ijaas.2024.10.016

Jamil, N., Rosli, R., & Mahmud, M. S. (2025). Algebraic trends impact school mathematics education: A bibliometric review. *Multidisciplinary Reviews*, *8*(9), 2025261. https://doi.org/10.31893/multirev.2025261

Jiang, L., Sun, Y., & Zhao, H. (2024). Grounded theory approach to non-market strategy and organizational resilience: the case of Chinese private enterprise. *Kybernetes*, *53*(12), 5648–5670. https://doi.org/10.1108/K-03-2023-0386

Junarti, Zainudin, M., & Utami, A. D. (2022). The sequence of algebraic problem-solving paths: Evidence from structure sense of Indonesian student. *Journal on Mathematics Education*, *13*(3), 437–464. https://doi.org/10.22342/jme.v13i3.pp437-464

Jung, H., Newton, J., & Maher, E. M. (2024). Conceptualizing algebraic connections: Honoring voices from future mathematics teachers and the instructors who teach them. *International Electronic Journal of Mathematics Education*, *19*(4), em0797. https://doi.org/10.29333/iejme/15519

Kania, N., Fitriani, C., & Bonyah, E. (2023). Analysis of Students' Critical Thinking Skills Based on Prior Knowledge Mathematics. *International Journal of Contemporary Studies in Education (IJ-CSE)*, *2*(1), 49–58. https://doi.org/10.56855/ijcse.v2i1.248

Kania, N., & Kusumah, Y. S. (2025). THE MEASUREMENT OF HIGHER-ORDER THINKING SKILLS: A SYSTEMATIC LITERATURE REVIEW. *Malaysian Journal of Learning and Instruction*, *22*(1), 97–116. https://doi.org/10.32890/mjli2025.22.1.6

Kania, N., Kusumah, Y. S., Dahlan, J. A., Nurlaelah, E., Gürbüz, F., & Bonyah, E. (2024). Constructing and providing content validity evidence through the Aiken's V index based on the experts' judgments of the instrument to measure mathematical problem-solving skills. *REID (Research and Evaluation in Education)*, *10*(1), 64–79. https://doi.org/10.21831/reid.v10i1.71032

Keller, N. (2012). A tight quantitative version of Arrow's impossibility theorem. *Journal of the European Mathematical Society*, *14*(5), 1331–1355. https://doi.org/10.4171/jems/334

Kilp-Kabel, T., & Mädamürk, K. (2024). The developmental trajectories of math skills and its relation to math interest in Grades three and five. *European Journal of Psychology of Education*, *39*(3), 2681–2699. https://doi.org/10.1007/s10212-024-00807-8

Kolar, V. M., & Hodnik, T. (2021). Mathematical literacy from the perspective of solving contextual problems. *European Journal of Educational Research*, *10*(1), 467–483. https://doi.org/10.12973/EU-JER.10.1.467

Langi', E. L., Juniati, D., & Abadi. (2023). Students as Prospective Teachers' Understanding of Integral Based on the APOS Theory in Terms of Gender Difference. *Journal of Higher Education Theory and Practice*, *23*(4). https://doi.org/10.33423/jhetp.v23i4.5897

Lee, Y., Mackenzie, M. J., & Lach, L. (2024). Toward a developmental transactional model of educational upward mobility. *Review of Education*, *12*(3). https://doi.org/10.1002/rev3.70007

López-Martín, M. del M., Aguayo-Arriagada, C. G., & García López, M. del M. (2022). Preservice Elementary Teachers' Mathematical Knowledge on Fractions as Operator in Word Problems. *Mathematics*, *10*(3), 423. https://doi.org/10.3390/math10030423

Martín Molina, J. C., Maz Machado, A., Madrid, M. J., & Rodríguez Baiget, M. J. (2024). Comprehension of linear systems with two unknowns in secondary education. *International Journal of Evaluation and Research in Education (IJERE)*, *13*(6), 3914. https://doi.org/10.11591/ijere.v13i6.28206

Martínez-Planell, R., & Trigueros, M. (2020). Students' understanding of Riemann sums for integrals of functions of two variables. *The Journal of Mathematical Behavior*, *59*, 100791. https://doi.org/10.1016/j.jmathb.2020.100791

Mera, C., Delgado, C., Aragón, E., Menacho, I., Canto, M. D. C., & Navarro, J. I. (2022). Contributions of the psychology of mathematical cognition in early childhood education using apps. *Frontiers in Psychology*, *13*. https://doi.org/10.3389/fpsyg.2022.913970

Moru, E. K. (2020). An APOS Analysis of University Students' Understanding of Derivatives: A Lesotho Case Study. *African Journal of Research in Mathematics, Science and Technology Education*, *24*(2), 279–292. https://doi.org/10.1080/18117295.2020.1821500

Moru, E. K., & Mathunya, M. (2022). A constructivist analysis of Grade 8 learners' errors and misconceptions in simplifying mathematical algebraic expressions. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *7*(3), 130–144. https://doi.org/10.23917/jramathedu.v7i3.16784

Murre, J. M. J. (2023). How averaging individual curves transforms their shape: Mathematical analyses with application to learning and forgetting curves. *Journal of Mathematical Psychology*, *117*, 102816. https://doi.org/10.1016/j.jmp.2023.102816

Mustafa, G., & Derya, C. (2016). A research on future mathematics teachers instructional explanations: The sample of Algebra. *Educational Research and Reviews*, *11*(16), 1500–1508. https://doi.org/10.5897/ERR2016.2823

Mutambara, L. H. N., & Bansilal, S. (2022). A case study of in-service teachers' errors and misconceptions in linear combinations. *International Journal of Mathematical Education in Science and Technology*, *53*(11), 2900–2918. https://doi.org/10.1080/0020739X.2021.1913656

Mutambara, L. H. N., & Tsakeni, M. (2022). Cognitive obstacles in the learning of complex number concepts: A case study of in-service undergraduate physics student-teachers in Zimbabwe. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(10), em2158. https://doi.org/10.29333/ejmste/12418

Mwakililo, P. S., Mauki, C. M., & Lyakurwa, S. E. (2025). Coping with dual roles: a case of adolescent student mothers in Tanzania's secondary schools. *Asian Education and Development Studies*. https://doi.org/10.1108/AEDS-07-2024-0158

Ndiung, S., & Menggo, S. (2024). Project-Based Learning in Fostering Creative Thinking and Mathematical

Problem-Solving Skills: Evidence from Primary Education in Indonesia. *International Journal of Learning, Teaching and Educational Research*, *23*(8), 289–308. https://doi.org/10.26803/ijlter.23.8.15

Ndlovu, Z., & Brijlall, D. (2015). Pre-service Teachers' Mental Constructions of Concepts in Matrix Algebra. *African Journal of Research in Mathematics, Science and Technology Education*, *19*(2), 156–171. https://doi.org/10.1080/10288457.2015.1028717

Ndlovu, Z., & Brijlall, D. (2016). Pre-service Mathematics Teachers' Mental Constructions of the Determinant Concept. *International Journal of Educational Sciences*, *14*(1–2), 145–156. https://doi.org/10.1080/09751122.2016.11890488

Nga, N. T., Dung, T. M., Trung, L. T. B. T., Nguyen, T.-T., Tong, D. H., Van, T. Q., & Uyen, B. P. (2023). The Effectiveness of Teaching Derivatives in Vietnamese High Schools Using APOS Theory and ACE Learning Cycle. *European Journal of Educational Research*, *volume-12*-(volume-12-issue-1-january-2023), 507–523. https://doi.org/10.12973/eu-jer.12.1.507

Nhat, N. C., Oanh, T. T. K., & Hang, P. T. T. (2024). The Effect of Stem Education on Academic Performance: A Meta-Analysis Study. *International Journal of Learning, Teaching and Educational Research*, *23*(11), 180–195. https://doi.org/10.26803/ijlter.23.11.9

Nikou, S. A., Perifanou, M., & Economides, A. A. (2024). Exploring Teachers' Competences to Integrate Augmented Reality in Education: Results from an International Study. *TechTrends*, *68*(6), 1208–1221. https://doi.org/10.1007/s11528-024-01014-4

Ningrum, H. U., Mulyono, M., & Isnarto, I. (2020). Mathematical Connection Ability Based on Self-Efficacy in IDEAL Problem Solving Model Assisted by ICT. *Unnes Journal of* …. https://journal.unnes.ac.id/sju/index.php/ujmer/article/view/32797

Okoye-Ogbalu, I., & Nnadozie, V. (2024). Subject competency and teacher knowledge: An exploration of second-year pre-service mathematics teachers' difficulties in solving logarithmic problems using basic rules for logarithm. *Journal of Mathematics and Science Teacher*, *4*(1), em054. https://doi.org/10.29333/mathsciteacher/13887

Oktaç, A., Vázquez Padilla, R., Ramírez Sandoval, O., & Villabona Millán, D. (2022). Transitional points in constructing the preimage concept in linear algebra. *International Journal of Mathematical Education in Science and Technology*, *53*(5), 1170–1189. https://doi.org/10.1080/0020739X.2021.1968523

Ontiveros, Y. O., González, J. R., & Palencia, J. L. D. (2025). Integrating creative thinking and cultural insights through the anthropological theory of didactics in university algebra. *Journal of Education and Learning (EduLearn)*, *19*(2), 742–750. https://doi.org/10.11591/edulearn.v19i2.21461

Oppong, R. A., Adjei, E. A., Amoah, E. K., Okuleteh, E., & Larri, B. (2024). Analyses of Pre-Service Teachers' Errors in Solving Algebraic Tasks in Ghana. *International Journal of Mathematics and Mathematics Education*, 155–174. https://doi.org/10.56855/ijmme.v2i3.1134

Osman, S., Abu, M., Mohammad, S., & Mokhtar, M. (2016). Identifying Pertinent Elements of Critical Thinking and Mathematical Thinking Used in Civil Engineering Practice in Relation to Engineering Education. *The Qualitative Report*. https://doi.org/10.46743/2160-3715/2016.2203

Padernal, R. E., & Tupas, S. V. (2024). First-year students' mathematical skills: assessing and developing an innovative strategy for teaching mathematics. *International Journal of Evaluation and Research in Education (IJERE)*, *13*(5), 2890. https://doi.org/10.11591/ijere.v13i5.29182

Padmanabhan, G., De Saram, D. D., Schanandore, T. C., Schanandore, J., & Pieri, R. V. (2013). A surveying course as summer experience for. *ASEE Annual Conference and Exposition, Conference Proceedings*. https://www.scopus.com/inward/record.uri?eid=2-s2.0-85029034669&partnerID=40&md5=0362f492dc25b2a09f3ba38fec5a391a

Pape, S., Gunpinar, Y., JohnBull, R., & Bell, C. (2022). Advancing Algebra Achievement Using Affordances of Classroom Connectivity Technology: The Case for Noticing through Discussion. *Mathematics*, *10*(15), 2641. https://doi.org/10.3390/math10152641

Pasigon, C. P. (2024). Mathematical Proficiency, Scientific Reasoning, Metacognitive Skills, and Performance of Learners in Physics: A Mathematical Model. *International Journal of Learning, Teaching and Educational Research*, *23*(4), 252–278. https://doi.org/10.26803/ijlter.23.4.14

Pham, H. T., & Nguyen, N. T. V. (2023). Pedagogical student assessment tools for learning outcome assessment skills. *Cypriot Journal of Educational Sciences*, *18*(1), 158–174. https://doi.org/10.18844/cjes.v18i1.8514

Pinter, L., & Siddiqui, M. F. H. (2024). Enhancing Calculus Learning through Interactive VR and AR Technologies: A Study on Immersive Educational Tools. *Multimodal Technologies and Interaction*, *8*(3), 19. https://doi.org/10.3390/mti8030019

Rafi, I., & Retnawati, H. (2018). What are the common errors made by students in solving logarithm problems? *Journal of Physics: Conference Series*, *1097*, 012157. https://doi.org/10.1088/1742-6596/1097/1/012157

Richland, L. E., & Simms, N. (2015). Analogy, higher order thinking, and education. In *Wiley Interdisciplinary Reviews: Cognitive Science* (Vol. 6, Issue 2, pp. 177–192). Wiley. https://doi.org/10.1002/wcs.1336

Rivard, S. (2024). Unpacking the process of conceptual leaping in the conduct of literature reviews. *The Journal of Strategic Information Systems*, *33*(1), 101822. https://doi.org/10.1016/j.jsis.2024.101822
Romadhon, N. I., Amir, M. F., & Wardana, M. D. K. (2024). Assessing students' mathematical reasoning in problem-based learning: a gender perspective. *International Journal of Evaluation and Research in Education (IJERE)*, *13*(6), 3763. https://doi.org/10.11591/ijere.v13i6.29580
Roulstone, A., Morsanyi, K., & Bahnmueller, J. (2024). Performance on curriculum-based mathematics assessments in developmental dyscalculia: the effect of content domain and question format. *Psychological Research*, *88*(8), 2444–2454. https://doi.org/10.1007/s00426-024-02015-x
Saleemad, K., Noklang, S., & Dudsdeemaytha, J. (2022). Leadership Characteristics for Muslim School Leaders in a Multicultural Context in Thailand. *International Journal of Instruction*, *15*(2), 293–306. https://doi.org/10.29333/iji.2022.15217a
Sari, I. P., Suryadi, D., Herman, T., Dahlan, J. A., & Supriyadi, E. (2024). Learning obstacles on fractions: A scoping review. *Infinity Journal*, *13*(2), 377–392. https://doi.org/10.22460/infinity.v13i2.p377-392
Schillinger, T. (2021). Self-efficacy of Kindergarten Teachers' Mathematical Instruction. *Early Childhood Education Journal*, *49*(4), 623–632. https://doi.org/10.1007/s10643-020-01101-0
Schwarz, C. V., Ke, L., Salgado, M., & Manz, E. (2022). Beyond assessing knowledge about models and modeling: Moving toward expansive, meaningful, and equitable modeling practice. *Journal of Research in Science Teaching*, *59*(6), 1086–1096. https://doi.org/10.1002/tea.21770
Siswono, T. Y. E., Hartono, S., Wachidul Kohar, A., Karim, K., & Lastiningsih, N. (2019). How do Prospective Teachers Manage Students' Learning of Mathematics? *TEM Journal*, 677–685. https://doi.org/10.18421/TEM82-49
Soneira, C., Bansilal, S., & Govender, R. (2021). Insights into the reversal error from a study with South African and Spanish prospective primary teachers. *Pythagoras*, *42*(1). https://doi.org/10.4102/pythagoras.v42i1.613
Susiswo, S., Retno Murniasih, T., Sa'dijah, C., Muksar, M., & Murtafiah, W. (2021). The Development of an Instrument on Negative Fractions to Measure the Cognitive Obstacle Based on Mental Mechanism Stages. *TEM Journal*, 1357–1362. https://doi.org/10.18421/TEM103-44
Suwardika, G., Sopandi, A. T., Indrawan, I. P. O., & Masakazu, K. (2024). A flipped classroom with whiteboard animation and modules to enhance students' self-regulation, critical thinking and communication skills: a conceptual framework and its implementation. *Asian Association of Open Universities Journal*, *19*(2), 135–152. https://doi.org/10.1108/AAOUJ-10-2023-0115
Tatira, B. (2021). Mathematics Education Students' Understanding of Binomial Series Expansion Based on the APOS Theory. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(12), em2035. https://doi.org/10.29333/ejmste/11287
Tatira, B., & Mukuka, A. (2024). Unpacking pre-service teachers' conceptualization of logarithmic differentiation through the APOS theory. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(12), em2541. https://doi.org/10.29333/ejmste/15655
Thomaidis, Y., & Tzanakis, C. (2022). Historical knowledge and mathematics education: a recent debate and a case study on the different readings of history and its didactical transposition. *ZDM – Mathematics Education*, *54*(7), 1449–1461. https://doi.org/10.1007/s11858-022-01370-6
Travé González, G., Pozuelos Estrada, F. J., & Trave Gonzalez, G. (2017). How teachers design and implement intstructional materials to improve classroom practice. *Intangible Capital*, *13*(5), 971. https://doi.org/10.3926/ic.1062
Trigueros, M., Badillo, E., Sánchez-Matamoros, G., & Hernández-Rebollar, L. A. (2024). Contributions to the characterization of the Schema using APOS theory: Graphing with derivative. *ZDM – Mathematics Education*, *56*(6), 1093–1108. https://doi.org/10.1007/s11858-024-01615-6
Trigueros, M., Cabrera, A. C., & Aguilar, M. S. (2024). Mental constructions for the learning of the concept of vector space. *ZDM – Mathematics Education*, *56*(7), 1417–1431. https://doi.org/10.1007/s11858-024-01640-5
Tuktamyshov, N. K., & Gorskaya, T. Y. (2024). APOS Theory in Learning Mathematics (Using Trigonometry as an Example). *Integration of Education*, *28*(1), 111–124. https://doi.org/10.15507/1991-9468.114.028.202401.111-124
Umiralkhanov, A., Abylkassymova, A., Tuyakov, Y., Zhadrayeva, L., & Ardabayeva, A. (2024). Continuity in Teaching the Course Algebra and the Beginning of Analysis: Adaptation of Students to New Conditions. *Qubahan Academic Journal*, *4*(3), 796–808. https://doi.org/10.48161/qaj.v4n3a1050
Usmayati, U., & Gürbüz, F. (2024). *Empowering Students with Discovery Learning in Circle Geometry for Better Problem-Solving Uum*. *01*(01), 11–20.
Volikova, M. M., Armash, T. S., Yechkalo, Y. V, & Zaselskiy, V. I. (2019). Practical use of cloud services for organization of future specialists professional training. In K. A.E. & S. V.N. (Eds.), *CEUR Workshop*

Proceedings (Vol. 2433, pp. 486–498). CEUR-WS. https://www.scopus.com/inward/record.uri?eid=2-s2.0-85072756023&partnerID=40&md5=43a0f48d8fd8a5e7ad478f8832d6ad22

Wester, J. S. (2021). Students' Possibilities to Learn From Group Discussions Integrated in Whole-class Teaching in Mathematics. *Scandinavian Journal of Educational Research*, *65*(6), 1020–1036. https://doi.org/10.1080/00313831.2020.1788148

Wilkins, J. L. M., & Norton, A. (2018). Learning progression toward a measurement concept of fractions. *International Journal of STEM Education*, *5*(1), 27. https://doi.org/10.1186/s40594-018-0119-2

Yerizon, D., Arnawa, I. M., Yanita, D., Ginting, B., & Nita, S. (2019). Students' Errors in Learning Elementary Group Theory: A Case Study of Mathematics Students at Andalas University. *Universal Journal of Educational Research*, *7*(12), 2693–2698. https://doi.org/10.13189/ujer.2019.071216

Yodiatmana, Y., & Kartini, K. (2022). Analysis of Student Errors in Solving Basic Logarithmic Problems Using Kastolan Error Analysis. *Jurnal Gantang*, *7*(2), 129–136. https://doi.org/10.31629/jg.v7i2.4689

Yuanita, P., Maimunah, M., Kartini, K., & Arif, N. (2023). Analysis of SOLO taxonomy-based test instruments in approaching the Merdeka curriculum. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 12–29. https://doi.org/10.23917/jramathedu.v8i1.1270

Zakaria, M. I., Hanri, C., Noer, S. H., Triana, M., & Widyastuti, W. (2025). 4C skills teaching activities for mathematics teachers: application of modified nominal group technique. *Journal of Education and Learning (EduLearn)*, *19*(2), 626–633. https://doi.org/10.11591/edulearn.v19i2.22337

Žakelj, A., Cotič, M., & Doz, D. (2024). Demetriou's tests and levels of algebraic abilities and proportional reasoning in seventh, eighth, and ninth grades. *European Journal of Science and Mathematics Education*, *12*(2), 326–334. https://doi.org/10.30935/scimath/14460

Zayyadi, M., Nusantara, T., Hidayanto, E., Sulandra, I. M., & As'ari, A. R. (2019). Exploring prospective student teacher's question on mathematics teaching practice. *Journal of Technology and Science Education*, *9*(2), 228. https://doi.org/10.3926/jotse.465