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EFA and CFA analysis: Development and validation of a test instrument for mathematical abstraction skills

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ABSTRACT

Mathematical abstraction skills are fundamental for advanced reasoning and problem-solving, yet assessing these skills in senior high school students poses challenges due to limited validated instruments. This study aims to develop and validate a test instrument for measuring mathematical abstraction skills in Indonesian high school students. The instrument targets three hierarchical cognitive levels: perceptual abstraction, internalization, and interiorization. A quantitative approach was used, involving Exploratory and Confirmatory Factor Analysis (EFA and CFA) to establish construct validity. Content validity was first evaluated by 17 mathematics educators using Aiken's V index, followed by expert judgment from five specialists. A field test with 507 students provided the dataset for analysis. Findings from EFA and CFA confirmed that the instrument exhibited strong construct validity and reliability, with clear alignment between items and the targeted abstraction levels. The internal consistency metrics demonstrated the instrument's ability to reliably assess mathematical abstraction across cognitive hierarchies. This validated tool contributes to mathematics education by providing a reliable means for assessing abstraction skills, facilitating improvements in instructional strategies and student evaluations. Its potential applications extend to educational research and classroom assessments, offering educators insights into students' reasoning processes to support deeper learning outcomes.

INTRODUCTION

Mathematical abstraction is a cornerstone of higher-order thinking, allowing students to transition from understanding concrete examples to grasping underlying structures and relationships within mathematical concepts (Panjaitan, 2018; Zehetmeier et al., 2019). This skill is essential for mastering advanced mathematics and problem-solving, particularly at the senior high school level, where students are expected to develop reasoning abilities that prepare them for STEM-related fields. By fostering mathematical abstraction, educators enable students to simplify complex problems, identify essential components, and apply generalized principles in diverse contexts (Hanif et al., 2021; Lyn, 2023).

Despite its importance, the assessment of mathematical abstraction skills remains a significant challenge in mathematics education. Traditional assessments primarily focus on procedural knowledge and algorithmic problem-solving, often neglecting students' ability to engage in abstract reasoning (Putra et al., 2018; Susac et al., 2014). For instance, conventional tests might measure a student's ability to solve equations but fail to evaluate their capacity to generalize and manipulate mathematical concepts. This gap underscores the need for assessment tools that capture the

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multifaceted nature of abstraction, ensuring a comprehensive evaluation of students' cognitive abilities (Bahar & Maker, 2020; Little & McDaniel, 2015; Reed, 2016).

Mathematical abstraction can be categorized into three hierarchical levels: perceptual abstraction, internalization, and interiorization (Hazzan & Zazkis, 2005; Khasanah et al., 2019; Nurjannah & Kusnandi, 2021; Reed, 2016). Perceptual abstraction involves identifying patterns and structures from concrete examples, while internalization entails constructing abstract concepts mentally (Dreyfus, 2014; Dreyfus & Kidron, 2014; Fiantika et al., 2018; Reed, 2016). Interiorization, the most advanced stage, allows students to manipulate and apply abstract concepts flexibly in theoretical contexts (Beckers & Halpern, 2018; Fiantika et al., 2018; Raychaudhuri, 2014). These stages reflect a progression in cognitive development that is essential for mathematical maturity (Coolidge & Overmann, 2012; Iuculano & Menon, 2018; Susac et al., 2014).

However, studies suggest that many students struggle to reach higher levels of abstraction. In Indonesia, for instance, Fitriani et al. (2018) found that 65.71% of students were at the perceptual abstraction level, while only 17.14% achieved interiorization. These findings highlight a critical gap in students' cognitive development and the pressing need for targeted interventions. Without accurate assessment tools to measure abstraction across all levels, educators lack the necessary insights to design instructional strategies that address students' specific learning needs (Hong & Kim, 2016; Zelazo, 2018).

Globally, the development of instruments to assess mathematical abstraction skills has gained attention. Studies in countries like Finland and the United States emphasize the integration of abstract reasoning in mathematics curricula, with instruments designed to align with students' developmental stages (Dreyfus, 2014; Hemmi et al., 2021; Juul, 2007; Pratt & Noss, 2010; Warren & Cooper, 2009). However, these instruments often require cultural adaptation and localization to effectively measure abstraction skills in different educational contexts. Indonesia's unique educational challenges, including diverse cognitive styles and varying quality of instruction, necessitate the creation of a context-specific instrument to accurately assess and support students' abstract reasoning (Nurrahmah et al., 2021).

Under these circumstances, the pressing necessity to develop a valid, reliable, and culturally appropriate instrument that can effectively evaluate students' mathematical abstraction skills within the Indonesian context underscores the urgency of this study. Without such a tool, educators and policymakers would overlook crucial aspects of students' cognitive growth that are vital for achieving proficiency in mathematics and excelling in STEM disciplines. Employing robust psychometric methods such as Confirmatory Factor Analysis (CFA) and Exploratory Factor Analysis (EFA) ensures that the instrument meets empirical standards for validity and reliability (Prihono et al., 2022; Xiao et al., 2019), while also encompassing the theoretical dimensions of abstraction. Consequently, this study is crucial for enhancing instructional decision-making and educational assessment processes, which will ultimately result in improved learning outcomes in mathematics education.

To address these challenges, this study aims to develop a valid and reliable test instrument to measure mathematical abstraction skills among senior high school students in Indonesia. Specifically, the researchers will employ confirmatory factor analysis to establish the construct validity and reliability of the instrument, ensuring it aligns with the theoretical framework of mathematical abstraction (Fitriani et al., 2021; Fitriani & Nurfauziah, 2019; Putra et al., 2018).

Indicators of mathematical abstraction skills

The development of the test instrument in this study is grounded in a hierarchical model of mathematical abstraction, which includes three progressive levels: Perceptual Abstraction, Internalization, and Interiorization (Dreyfus, 2014; Dreyfus & Kidron, 2014; Hazzan & Zazkis, 2005). These levels reflect the cognitive transitions students undergo from recognizing patterns in concrete representations to constructing and manipulating abstract mathematical ideas (Hong & Kim, 2016; Ocy et al., 2023). Each level encompasses specific indicators that guide the construction of test items and the evaluation of student responses.

Perceptual abstraction

At this foundational level, students begin to identify visible patterns and structures in mathematical representations (Spiller et al., 2023). This stage is crucial as it enables learners to make initial connections between concrete objects and symbolic representations (Kinanti et al., 2023). Key competencies include recognizing recurring patterns in shapes and algebraic forms, interpreting visual representations such as graphs, and understanding simple transformations.

The theoretical foundation of perceptual abstraction is grounded in the work of Gray and Tall (2007), who highlight the importance of perceptual cues as cognitive anchors that support students in the early stages of abstract mathematical thinking. Their research suggests that these cues provide the initial structure upon which deeper understanding is built. Complementing this view, Mitchelmore and White (2007) argue that the ability to recognize and generalize patterns plays a critical role in the development of abstract reasoning. In line with these perspectives, Kellman et al. (2010) emphasize the role of perceptual learning in enhancing students' sensitivity to mathematical structures and relationships, thereby facilitating their progression toward more sophisticated forms of mathematical thought.

Internalization

At this intermediate level, students internalize abstract ideas by mentally constructing and applying them in various contexts (Abreu-Mendoza et al., 2018). Internalization involves the ability to manipulate symbolic representations, connect abstract concepts to real-life applications, and articulate the reasoning behind mathematical operations (Uygun & Guner, 2022).

The theoretical foundation of internalization is rooted in the idea that abstract concepts must be mentally assimilated for deeper understanding and application. Ferrari (2003) and White and Mitchelmore (2010) describe internalization as the cognitive process through which students embed abstract mathematical ideas into their mental frameworks. This process allows learners to move beyond surface-level comprehension toward meaningful engagement with mathematical concepts. Supporting this view, Cooley (2002) and Reed (2016) emphasizes the importance of representational strategies—such as diagrams, symbolic notations, or visual models—which serve as bridges between abstract theory and practical application. These representations help students internalize complex ideas by making them more accessible and relatable within diverse problem-solving contexts.

Interiorization

This is the highest level of abstraction, where students demonstrate the ability to synthesize, justify, and evaluate mathematical ideas across diverse contexts (Ahmadah, 2020; Balkan, 2023; Žakelj et al., 2024). They create mathematical models, validate their solutions, and justify their reasoning within theoretical frameworks. This level reflects mathematical maturity and creative problem-solving (Abreu-Mendoza et al., 2018).

The theoretical foundation of interiorization lies in the advanced cognitive processes that enable students to operationalize abstract concepts and apply them flexibly across various contexts. Dreyfus (2014) and Dreyfus and Kidron (2014) conceptualize interiorization as the stage at which learners not only understand abstract mathematical ideas but also use them fluidly in generalized and adaptive ways. This stage marks a significant shift from procedural knowledge to a more integrated and strategic application of mathematical thought. In support of this perspective, Cetin and Dubinsky (2017), Gold (2018), and Simon (2020) emphasize the role of cognitive integration, where students draw upon multiple mathematical domains to construct coherent solutions to complex problems. Such sophisticated problem-solving reflects the characteristics of the formal operational stage as outlined in Piagetian theory Moessinger and Poulin-Dubois (1981) and Reed (2016), where learners demonstrate abstract reasoning, hypothetical thinking, and mental coordination of multiple variables.

In this study, the test instrument for assessing mathematical abstraction skills among senior high school students was developed based on 10 carefully selected indicators (Table 1). These indicators were chosen to represent different cognitive levels of mathematical abstraction, which include Perceptual Abstraction, Internalization, and Interiorization. Each indicator is designed to assess specific aspects of students' mathematical reasoning and problem-solving capabilities, capturing their progression from basic understanding to more complex mathematical reasoning.

The Perceptual Abstraction level emphasizes foundational mathematical skills, including pattern recognition and the identification of relationships in simple mathematical contexts (Hong &

Table 1.
Indicators of mathematical abstraction skills

Mathematical Abstraction Level	Item	Mathematical Abstraction Indicator	Student Cognitive Ability
Perceptual Abstraction	Q1	Recognizing patterns in geometric shapes and algebraic expressions.	Students can identify and describe simple patterns in geometric figures and algebraic expressions.
	Q2	Identifying relationships between variables in simple equations or visual representations.	Students are able to recognize basic relationships between variables in equations and visual representations such as graphs.
	Q4	Observing transformations in graphs and understanding their effect on equations.	Students can observe and describe simple transformations on graphs and their effect on the corresponding equations.
Internalization	Q3	Applying abstract mathematical concepts to solve real-world problems.	Students can apply abstract concepts like equations and polynomials to solve practical, real-life problems.
	Q7	Explaining the reasoning behind algebraic manipulations.	Students can provide clear explanations for the steps they take in algebraic manipulations and transformations.
	Q5	Developing strategies to represent mathematical problems algebraically or geometrically.	Students can develop their own strategies to solve problems by representing them in algebraic or geometric forms.
Interiorization	Q6	Integrating multiple mathematical concepts into a cohesive approach to problem-solving.	Students can combine various mathematical ideas and methods to form a unified approach to complex problem-solving.
	Q8	Justifying the steps taken in complex problem-solving.	Students can justify each step they take in solving multi-step problems, demonstrating a deep understanding of the processes involved.
	Q9	Evaluating the accuracy and consistency of mathematical models and solutions in different contexts.	Students can assess the validity and consistency of their solutions and models in a range of mathematical situations.
	Q10	Creating mathematical models from abstract concepts to represent real-life situations or complex theoretical problems.	Students can construct mathematical models that abstract real-world scenarios or complex theoretical problems, demonstrating advanced problem-solving skills.

Kim, 2016). This level is assessed through Indicators Q1 (recognizing patterns in geometric shapes and algebraic expressions), Q2 (identifying relationships between variables in simple equations or visual representations), and Q4 (observing transformations in graphs and understanding their effects on equations).

The theoretical grounding for these indicators is rooted in studies by Gray and Tall (2007) and Mitchelmore and White (2007), which highlight the importance of perceptual cues in the early stages of abstraction. These researchers argue that recognizing and describing patterns is a precursor to deeper mathematical reasoning. Additionally, Kellman et al. (2010) emphasize the role of visual processing and pattern recognition in enhancing students' ability to detect mathematical relationships, a skill critical for advancing to higher levels of abstraction.

At the Internalization level, the focus shifts to students' ability to internalize abstract concepts and apply them to real-world and mathematical problems (Hong & Kim, 2016). This level is assessed through Indicators Q3 (applying abstract mathematical concepts to solve real-world problems), Q5 (developing strategies to represent problems algebraically or geometrically), and Q7 (explaining reasoning behind algebraic manipulations).

Theoretical support for these indicators is drawn from Ferrari (2003) and White and Mitchelmore (2010), who describe internalization as the process of embedding abstract concepts into students' cognitive frameworks, allowing them to navigate practical problems effectively. Cooley (2002) further supports this perspective by highlighting the role of strategic representation in fostering deeper understanding, enabling students to bridge abstract ideas and practical applications.

The Interiorization level represents the highest order of abstraction, where students synthesize multiple mathematical concepts into cohesive approaches Hong and Kim (2016). Indicators at this level include Q6 (integrating multiple concepts for problem-solving), Q8 (justifying steps in complex problem-solving), Q9 (evaluating accuracy and consistency of mathematical models), and Q10 (creating mathematical models from abstract concepts).

The conceptual foundation for this level draws on Dreyfus (2014), Hershkowitz, Hadas, et al. (2007), Hershkowitz, Schwarz, et al. (2007), Kidron and Dreyfus (2008), and Raychaudhuri (2014), who emphasize the importance of integrating diverse mathematical constructs for advanced problem-solving. Mitchelmore and White (2007) also highlight that interiorization enables students to operate within a network of interconnected concepts, fostering flexibility and innovation. These skills are critical for advanced mathematical reasoning, particularly in real-world scenarios and STEM applications.

The selection of these indicators is based on a well-established theoretical framework of mathematical abstraction as a developmental process. Research by Gray and Tall (2007) underscores the progression from perceptual abstraction to interiorization, where each level builds upon the cognitive abilities developed in the preceding stage. This framework aligns with the theories of cognitive development proposed by Moessinger and Poulin-Dubois (1981) and Kamina and Iyer (2009), particularly the transition from concrete operational to formal operational stages.

METHODS

This study adopted a quantitative research design to develop and validate a test instrument aimed at measuring mathematical abstraction skills among senior high school students in Indonesia. The instrument was designed to assess students' ability to engage in abstract reasoning through three hierarchical levels of cognitive abstraction: perceptual abstraction, internalization, and interiorization. To evaluate the validity and reliability of the instrument, the study employed Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA).

Sampling and participants

A purposive sampling strategy was applied to select research sites. Four senior high schools in Tanjungpinang—SMAN 1 Tanjungpinang, SMAN 2 Tanjungpinang, SMAS Maitreyawira, and SMAS Santa Maria—were chosen based on their active implementation of Higher-Order Thinking Skills (HOTS) in instructional practices. The rationale behind this purposive selection was that students in such schools are more likely to be familiar with abstract reasoning, making them suitable for participation in a study focused on mathematical abstraction. A total of 507 students participated in the study.

Instrument development and data collection

The data collection was carried out using a constructed-response test, as opposed to a multiple-choice format. This test required students to provide written explanations, justifications, or mathematical arguments based on the problems presented. Each item was designed to measure one or more aspects of mathematical abstraction according to a theoretically grounded analytic rubric developed by the researcher.

Prior to the large-scale administration, the test items underwent a rigorous content validation process. In the first phase, 17 mathematics education teachers and practitioners reviewed the clarity, relevance, and alignment of the items with the abstraction constructs. Their feedback was analyzed using the Aiken's V index. In the second phase, five expert validators in mathematics education evaluated the items further. Content validity indices, including the Item-Level Content Validity Index (I-CVI), Scale-Level CVI using the average method (S-CVI/Ave), Universal Agreement (S-CVI/UA), and the Content Validity Ratio (CVR), were calculated to ensure rigorous content alignment.

Table 2.
Content validity results

Item	Aiken's V	I-CVI	CVR	S-CVI/Ave	S-CVI/UA	CVI
Q1	0.897	1.00	1.00	1.00	1.00	0.801
Q2	0.868	1.00	1.00			
Q3	0.912	1.00	1.00			
Q4	0.970	1.00	0.67			
Q5	0.853	1.00	0.67			
Q6	0.882	1.00	1.00			
Q7	0.926	1.00	0.67			
Q8	0.853	1.00	1.00			
Q9	0.926	1.00	0.67			
Q10	0.955	1.00	1.00			

Data Analysis

After content validation, the finalized instrument was administered to the student sample. Students' responses to the constructed-response items were scored using a predefined analytic rubric, which allowed for the identification of various cognitive processes involved in abstraction. The scored data were subjected to Exploratory Factor Analysis (EFA) to uncover the underlying factor structure of the instrument. Subsequently, Confirmatory Factor Analysis (CFA) was performed to test the hypothesized factor model, using fit indices such as RMSEA, CFI, TLI.

In addition to construct validity, reliability was assessed through Cronbach's Alpha and Composite Reliability for each subscale, representing the three levels of abstraction. These analyses ensured that the test instrument was both valid and internally consistent. This methodologically rigorous approach resulted in a validated constructed-response test instrument that can be reliably used to assess students' mathematical abstraction skills in secondary education settings.

FINDINGS

Content validity

In the expert validation process, the content validity of the instrument was assessed using Aiken's V, I-CVI (Item Content Validity Index), CVR (Content Validity Ratio), and scale-level indices (S-CVI/Ave and S-CVI/UA), as shown in Table 2.

The panel evaluation was conducted and calculated using Aiken's V based on feedback from 17 mathematics education experts. Meanwhile, expert judgment involving 5 experts was assessed using I-CVI, CVR, S-CVI/Ave, and S-CVI/UA. The Aiken's V values ranged from 0.868 to 0.970, indicating strong agreement among the experts regarding the clarity and relevance of the items. All items achieved an I-CVI of 1.00, reflecting unanimous expert consensus on their relevance.

Additionally, the CVR values showed that the majority of items (Q1, Q2, Q3, Q6, Q8, and Q10) were deemed essential by all experts, achieving a perfect CVR of 1.00. At the scale level, both S-CVI/Ave and S-CVI/UA indices were 1.00, demonstrating overall strong content validity for the instrument. The CVI, which summarizes the content validity of the instrument as a whole, was calculated to be 0.801, surpassing the minimum acceptable threshold.

These findings validate the appropriateness and necessity of the items for measuring mathematical abstraction skills. The rigorous content validation process provides strong evidence of the instrument's quality, serving as a foundation for subsequent analyses, including exploratory and confirmatory factor analysis, to further establish its construct validity and reliability.

EFA and CFA: Prerequisite Test

Conducting assumption tests is a critical prerequisite before performing Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA). These tests ensure that the data meet essential statistical conditions, such as normality, sampling adequacy, and the absence of multicollinearity, which are necessary for the validity and reliability of the analyses (Prihono et al., 2022).

Table 3.
Factor analysis assumption test results

	Normality: Shapiro-Wilk			Collinearity Statistics		Kaiser-Meyer-Olkin (KMO) Test		Bartlett Test		Mardia's Test of Multivariate Normality				
Item	Statistic	df	p	Tolerance	VIF	MSA	Overall MSA				Value	Stat.	df	p
Q1	0.988	515	0.200	0.993	1.007	0.812	0.8502	χ^2	716.731	Skewness	0.233	307.457	220	0.219
Q2	0.792	515	0.076	0.977	1.023	0.845		df	45	Kurtosis	2.778	1.465	220	0.125
Q3	0.814	515	0.200	0.971	1.029	0.823		p	<.001					
Q4	0.782	515	0.081	0.997	1.003	0.901								
Q5	0.871	515	0.063	0.980	1.020	0.876								
Q6	0.739	515	0.117	0.981	1.019	0.834								
Q7	0.925	515	0.103	0.988	1.012	0.890								
Q8	0.783	515	0.097	0.991	1.009	0.815								
Q9	0.795	515	0.148	0.986	1.014	0.867								
Q10	0.983	515	0.200	0.998	1.002	0.839								

Table 4.
EFA model and fit indices

Model	Value	df	p
Chi-square test	80.869	26	0.067
Comparative Fit Index (CFI)	0.938		
Tucker-Lewis Index (TLI)	0.941		
Root Mean Square Error of Approximation (RMSEA)	0.037		
RMSEA 90% Confidence Interval	0 – 0.016		
Standardized Root Mean Square Residual (SRMR)	0.037		

As shown in Table 3, the Shapiro-Wilk test for normality indicates that all items (Q1 to Q10) are normally distributed (p>0.05), with p-values exceeding 0.05. This supports the assumption of univariate normality, which is advantageous for both EFA and CFA. Collinearity statistics further confirm the absence of multicollinearity, as tolerance values exceed 0.10 and Variance Inflation Factor (VIF) values remain below 2.0, ensuring sufficient independence among items.

The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy reports a robust overall value of 0.8502, with individual Measures of Sampling Adequacy (MSA) ranging from 0.812 to 0.901. These results indicate that the dataset is well-suited for factor analysis. Bartlett's test of sphericity (χ^2 = 716.731, p < 0.001) confirms the presence of significant correlations among items, validating the suitability of the data for factor analysis. Additionally, Mardia's test of multivariate normality demonstrates acceptable skewness (0.233) and kurtosis (2.778) values, with non-significant p-values (p>0.05), further supporting the appropriateness of the data.

These findings collectively confirm that the dataset meets the necessary assumptions for conducting EFA and CFA. The comprehensive assessment of normality, sampling adequacy, and inter-item correlations ensures the robustness of the subsequent analyses, laying a strong foundation for interpreting the results and validating the test instrument for mathematical abstraction skills.

Exploratory factor analysis (EFA)

The purpose of Exploratory Factor Analysis is to identify the underlying factor structure of a set of measured variables (Fu et al., 2022). EFA is a data-driven approach that examines the correlations among the variables and groups them into factors, without any prior assumptions about the structure. This helps to reveal the latent constructs that are responsible for the observed correlations in the data. Evaluating model fit is a critical step in Exploratory Factor Analysis (EFA) to ensure that the proposed model adequately represents the data.

As shown in Table 4, the chi-square test resulted in a value of 80.869 with 26 degrees of freedom and a p-value of 0.067. This p-value, while slightly above the conventional threshold of 0.05, suggests that the model does not significantly differ from the observed data, indicating an acceptable fit. Additional fit indices provide further evidence of the model's adequacy. The Comparative Fit Index (CFI) and Tucker-Lewis Index (TLI) values are 0.938 and 0.941, respectively, both exceeding the 0.90 benchmark for a good model fit. The Root Mean Square Error of Approximation (RMSEA) is 0.037, with a 90% confidence interval of 0.00 to 0.016, indicating excellent model fit according to

Table 5.
Factor loadings (eigenvalue > 1, promax rotation method) and factor characteristics

Item	Factor 1	Factor 2	Factor 3	Uniqueness			Factor 1	Factor 2	Factor 3
Q1	0.997			0.021		**Eigenvalues**	3.000	2.000	1.214
Q2	0.894			0.117	Unrotated solution	SumSq. Loadings	2.987	2.755	2.433
Q4	0.806			0.123		Propostion var.	0.852	0.827	0.891
Q5		0.969		0.059		Cumulative	0.852	0.873	0.894
Q3		0.851		0.097	Rotated Solution	SumSq. Loadings	2.137	2.129	2.005
Q7		0.823		0.109		Propostion var.	0.838	0.801	0.876
Q8			0.976	0.043		Cumulative	0.838	0.841	0.869
Q10			0.883	0.061					
Q9			0.812	0.118					
Q6			0.802	0.128					

commonly accepted thresholds. Similarly, the Standardized Root Mean Square Residual (SRMR) value of 0.037 confirms the model's accuracy in reproducing the observed correlations, as values below 0.08 are considered acceptable.

The factor loadings and characteristics of the three factors extracted through Exploratory Factor Analysis (EFA) are presented in Table 5. The extraction was conducted using the eigenvalue > 1 criterion, which identifies factors that explain a meaningful amount of variance in the data. To improve interpretability, the Promax rotation method was employed, as it is particularly suitable for identifying correlated factors.

The factor loadings indicate the strength of the association between each item and its respective factor. Items such as Q1, with a loading of 0.997 on Factor 1, demonstrate a very strong relationship, while items like Q6, with a loading of 0.802 on Factor 3, indicate a moderate relationship. High factor loadings suggest that the items are strongly aligned with the latent constructs represented by their respective factors, supporting the validity of the instrument.

Uniqueness values provide additional insights, indicating the proportion of variance in each item not accounted for by the factors. For example, Q1 exhibits a low uniqueness value of 0.021, meaning that 97.9% of its variance is explained by Factor 1. In contrast, Q6 has a uniqueness value of 0.128, indicating that 12.8% of its variance remains unexplained. These results suggest that the majority of the variance in the items is well-captured by the identified factors.

The eigenvalues further underscore the significance of the factors. Factor 1, with an eigenvalue of 3.000, explains a substantial portion of the variance in the data, while Factor 2 and Factor 3, with eigenvalues of 2.000 and 1.214, respectively, contribute meaningfully to the overall variance. The Promax rotation refined these values, providing clearer differentiation between the factors. In the rotated solution, Factors 1, 2, and 3 explain 2.137, 2.129, and 2.005, respectively, offering a more interpretable structure.

In terms of variance explained, the three factors account for a substantial portion of the data's variance, with cumulative proportions of 83.8%, 84.1%, and 86.9% in the rotated solution. These results highlight the robust explanatory power of the identified factors in capturing the underlying constructs measured by the instrument.

The Scree Plot, displayed in Figure 1, depicts the distribution of eigenvalues for the extracted components, with eigenvalues plotted on the y-axis and the number of factors on the x-axis. Each point on the figure indicates a given factor's eigenvalue, or the proportion of variation explained by that factor. The plot shows a sharp decrease in eigenvalues for the beginning factors, indicating that they account for a significant percentage of the variance in the data. Beyond the third component, the eigenvalues plateau, creating a prominent "elbow" in the plot. This moment of inflection is critical for choosing the best amount of elements to keep. In this study, the top three factors have eigenvalues greater than one, meeting the Kaiser criterion for factor retention, whereas the other factors contribute minimally to overall variance. A horizontal dashed line at an eigenvalue of 1 is included as a reference point, emphasizing the factors that meet the threshold for significance. Based on the Scree

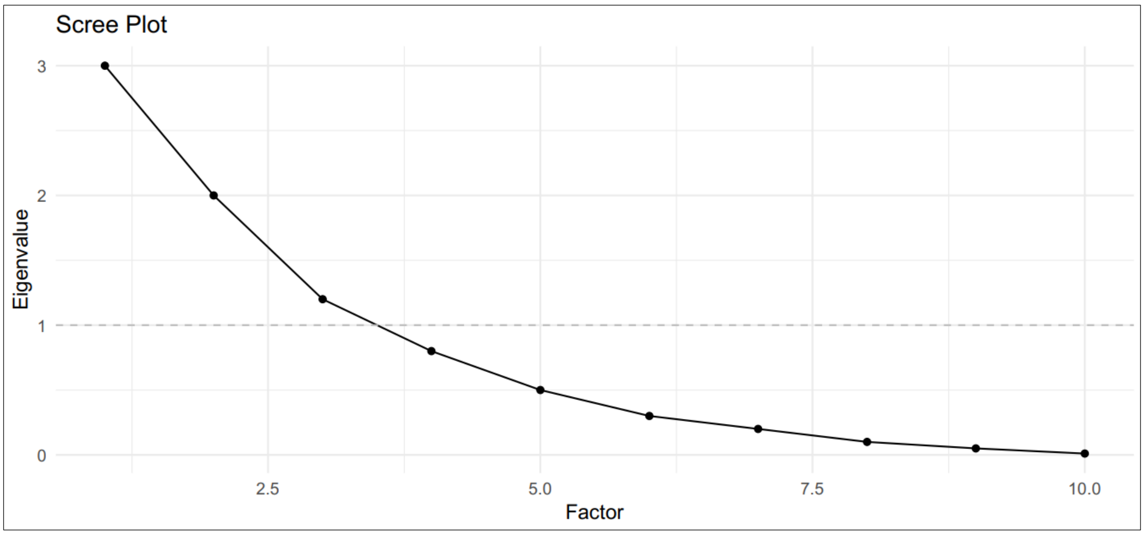

Figure 1. Scree plot

Plot, retaining the first three factors is appropriate, as they explain the most variance and are likely the most meaningful in capturing the underlying structure of the data.

The results from the Scree Plot, along with the findings in Tables 4 and 5, confirm the validity of retaining three factors for the Exploratory Factor Analysis (EFA), aligning with theoretical expectations and demonstrating a well-structured model. The test instrument exhibits strong factor loadings, low uniqueness values, and meaningful contributions of each factor to the variance, supported by the eigenvalue > 1 criterion and Promax rotation. These fit indices collectively indicate that the EFA model adequately represents the underlying constructs and provides strong evidence for the validity of the instrument. This robust foundation supports the subsequent Confirmatory Factor Analysis (CFA) to further validate the instrument's structure and effectiveness in assessing mathematical abstraction skills among senior high school students.

Confirmatory factor analysis (CFA)

Confirmatory Factor Analysis (CFA) is a robust statistical method used to evaluate the validity of a hypothesized factor structure within a dataset. It serves to confirm whether the observed variables align with an underlying theoretical model, often derived from prior research or an Exploratory Factor Analysis (EFA) (Xiao et al., 2019). In the context of educational research, CFA is indispensable for verifying the structure and construct validity of assessment instruments, ensuring that they accurately measure the intended latent constructs (Fu et al., 2022; Xiao et al., 2019). This method also provides critical fit indices that determine the alignment between the proposed model and the observed data, establishing a solid foundation for assessing the instrument's reliability and effectiveness.

The Confirmatory Factor Analysis (CFA) model, as shown in Figure 2, visually encapsulates the relationships between the latent constructs (Fc1, Fc2, and Fc3) and their corresponding observed indicators (Q1 to Q10), providing a robust framework for validating the structure of the test instrument designed to measure mathematical abstraction skills in senior high school students.

In this model, the latent factors—depicted as circles labeled Fc1, Fc2, and Fc3—represent hierarchical cognitive domains of mathematical abstraction. Specifically:

- Fc1 denotes the perceptual abstraction or foundational abstraction level, where basic mathematical concepts are recognized and understood.
- Fc2 corresponds to the internalization stage, where students develop a deeper conceptual grasp of abstract ideas.
- Fc3 embodies the highest abstraction level, characterized by the integration and synthesis of multiple mathematical concepts to solve complex problems.

Figure 2. CFA model plot

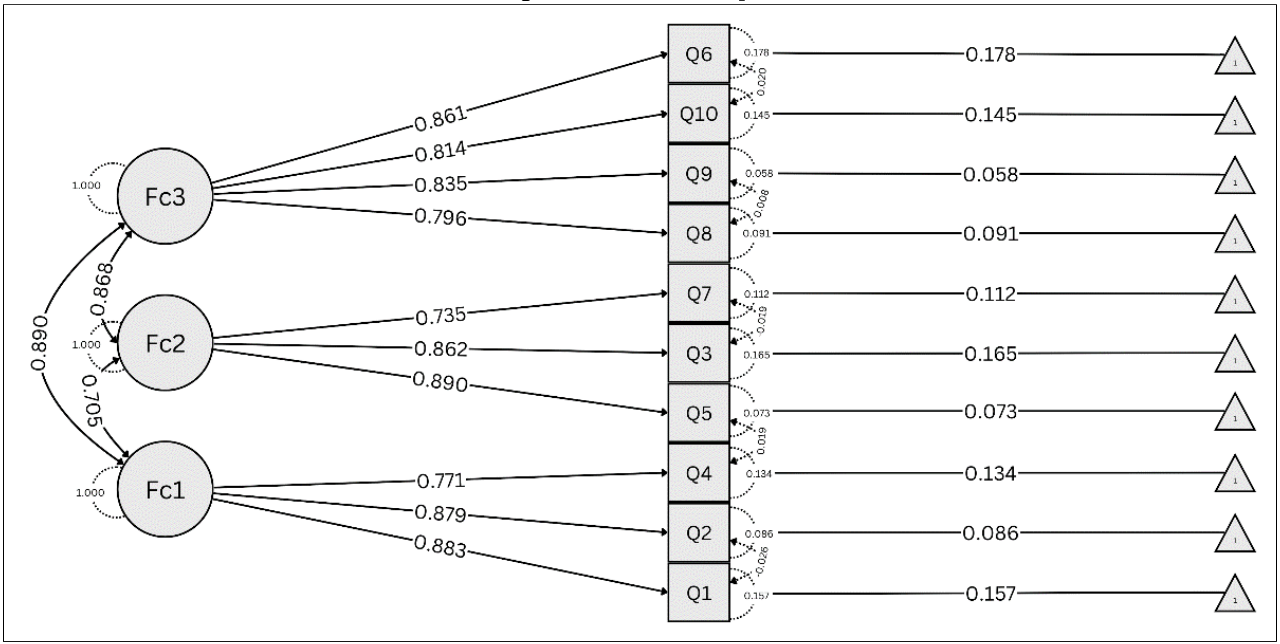

Table 6.
CFA model and fit indices

Chi-square test	**Value**	**df**	**p**
Baseline model	723.994	45	0.067
Factor model	91.973	27	0.106
Additional Fit Indices			
Comparative Fit Index (CFI)	0.904		
Tucker-Lewis Index (TLI)	0.940		
Root Mean Square Error of Approximation (RMSEA)	0.021		
RMSEA 90% CI lower bound	0.038		
RMSEA 90% CI upper bound	0.025		
Standardized Root Mean Square Residual (SRMR)	0.039		
Goodness of Fit Index (GFI)	0.988		
McDonald Fit Index (MFI)	0.939		
Expected Cross Validation Index (ECVI)	0.126		

The observed indicators—illustrated as rectangles labeled Q1 to Q10—serve as the measurable items linked to each latent factor. These indicators evaluate distinct dimensions of mathematical reasoning and problem-solving, aligning with the theoretical framework underlying the test instrument.

Table 6 shows the fit indices derived from the Confirmatory Factor Analysis (CFA), which show that the proposed factor model accurately represents the underlying structure of the data. The factor model's chi-square value (91.973, df = 27, p = 0.106) indicates no significant difference between observed and predicted data, implying a good model fit. This finding is confirmed by multiple robust fit indices.

The Comparative Fit Index (CFI) of 0.904 and the Tucker-Lewis Index (TLI) of 0.940 both above the frequently recognized threshold of 0.90, indicating that the factor model fits the data well. The RMSEA value of 0.021, with a 90% confidence interval (0.038-0.025), shows excellent model fit and falls inside the conventional threshold of RMSEA < 0.05. The Standardized Root Mean Square Residual (SRMR) of 0.039 supports this conclusion, as values less than 0.08 are considered acceptable. The Goodness of Fit Index (GFI) of 0.988 indicates that the model explains a significant fraction of the data's variation, and the McDonald Fit Index (MFI) of 0.939 verifies an adequate fit. Additionally, the Expected Cross Validation Index (ECVI) of 0.126 suggests reasonable predictive validity, emphasizing the model's potential reliability in multiple samples.

Table 7.
Factor loadings

Factor	Indicator	Estimate	Std. Error	z-value	p	95% Confidence Interval Lower	Upper
Factor 1	Q1	0.883	0.031	9.941	<.001	0.721	0.934
	Q2	0.879	0.030	9.914	<.001	0.768	0.962
	Q4	0.771	0.022	8.410	<.001	0.663	0.891
Factor 2	Q5	0.890	0.026	11.206	<.001	0.786	0.970
	Q3	0.862	0.023	8.899	<.001	0.745	0.967
	Q7	0.735	0.026	6.692	<.001	0.637	0.833
Factor 3	Q8	0.796	0.021	7.157	<.001	0.684	0.890
	Q10	0.814	0.021	9.214	<.001	0.709	0.921
	Q9	0.835	0.024	9.397	<.001	0.734	0.947
	Q6	0.861	0.026	8.419	<.001	0.767	0.975

Table 8.
Intercepts

Factor	Indicator	Estimate	Std. Error	z-value	p	95% Confidence Interval Lower	Upper
Factor 1	Q1	0.781	0.035	10.456	<.001	0.701	0.834
	Q2	0.672	0.026	9.234	<.001	0.571	0.782
	Q4	0.614	0.020	8.789	<.001	0.503	0.721
Factor 2	Q5	0.587	0.018	11.345	<.001	0.489	0.665
	Q3	0.839	0.041	10.678	<.001	0.730	0.965
	Q7	0.728	0.031	9.876	<.001	0.619	0.836
Factor 3	Q8	0.681	0.026	8.543	<.001	0.571	0.793
	Q10	0.512	0.011	11.012	<.001	0.409	0.621
	Q9	0.544	0.014	10.234	<.001	0.424	0.647
	Q6	0.744	0.032	9.456	<.001	0.607	0.849

Factor loadings are crucial in determining the strength and direction of the interactions between indicators (items) and the variables they represent. They show how much of each item's variance is explained by the underlying component, with larger values indicating a stronger link. As seen in Table 7, the loadings for Factor 1 components Q1 (0.883), Q2 (0.879), and Q4 (0.771) are strikingly high, with Q1 and Q2 exhibiting particularly strong relationships. This implies that these elements are closely related to the construct denoted by Factor 1. Similarly, for Factor 2, item Q5 has a loading of 0.890, indicating a strong link, since items Q3 (0.862) and Q7 (0.735) make significant contributions to this factor. Factor 3 has substantial loadings from items Q8 (0.796), Q10 (0.814), Q9 (0.835), and Q6 (0.861), indicating their consistency with Factor 3's underlying construct.

The standard error values for the factor loadings are small, indicating precise estimates. For instance, the standard error for Q1 is 0.031, suggesting a reliable estimate. The z-values, which assess the statistical significance of each loading, are also high. For example, Q5 has a z-value of 11.206, indicating a highly significant relationship with Factor 2. The p-values for all items are less than 0.001, further confirming the statistical significance of the factor loadings and the meaningful relationships between the items and their respective factors. Additionally, the narrow 95% confidence intervals for each loading further support the reliability of the estimates, with the true factor loadings likely to fall within the provided ranges.

The intercepts obtained from the Confirmatory Factor Analysis (CFA) provide important insights into the baseline levels of the indicators when the latent factors are at zero. These intercept estimates reflect the expected value of each indicator when the corresponding latent factor is absent, thereby offering an understanding of the initial or baseline responses of the items.

As shown in Table 8, for Factor 1, the intercepts for items Q1 (0.781), Q2 (0.672), and Q4 (0.614) suggest positive baseline values for these items, even in the absence of the latent factor. This indicates that these items have inherent baseline levels that are not explained by the underlying

Table 9.
Residual variances

Indicator	Estimate	Std. Error	z-value	p	95% Confidence Interval Lower	Upper
Q1	0.157	0.012	10.234	0.065	0.067	0.257
Q2	0.086	0.015	12.456	0.072	-0.004	0.186
Q4	0.134	0.018	11.789	0.088	0.044	0.234
Q5	0.073	0.011	13.012	0.054	-0.017	0.173
Q3	0.165	0.019	9.876	0.096	0.075	0.265
Q7	0.112	0.013	10.987	0.059	0.022	0.212
Q8	0.091	0.017	14.321	0.075	0.001	0.191
Q10	0.145	0.014	11.654	0.068	0.055	0.245
Q9	0.058	0.016	13.789	0.092	-0.032	0.158
Q6	0.178	0.020	10.543	0.081	0.088	0.278

factor. The standard error values for these intercepts are small, with Q1 having a standard error of 0.035, which indicates precise estimates of the intercepts.

The z-values, which assess the statistical significance of each intercept, are also high, indicating that the intercepts are statistically significant. For example, the intercept for Q5 has a z-value of 11.345, highlighting its substantial baseline level. Furthermore, the p-values for all intercepts are less than 0.001, confirming the statistical significance of these baseline levels and ensuring that they are meaningful and not due to random variation.

The 95% confidence intervals for the intercepts are narrow, providing a range within which the true intercepts are likely to fall. For instance, the confidence interval for Q1 ranges from 0.701 to 0.834, ensuring that the true intercept is reliably estimated within this range.

The residual variances from the Confirmatory Factor Analysis (CFA) provide vital insights into the fraction of variance in each indicator that cannot be explained by the latent factors. These residual variances show the unexplained variance that remains after accounting for the underlying structures, providing an indication of how well the model fits the data.

As indicated in Table 9, the residual variance for Q1 is 0.157, suggesting that the model fails to account for 15.7% of the variance in this item. Similarly, residual variances for other items indicate how much of their volatility is unexplained. The standard error values for these residual variances are tiny, with Q1 having a standard error of 0.012, indicating precise estimations of unexplained variance.

The z-values, which assess the significance of each residual variance, are notably high for most items. For example, Q2 has a z-value of 12.456, indicating that the residual variance for this item is highly significant. However, some items, such as Q2 (p = 0.072) and Q7 (p = 0.059), have p-values close to the 0.05 threshold, suggesting that there may be slight unexplained variance worth further investigation.

The 95% confidence intervals for the residual variances are generally narrow, with the confidence interval for Q1 ranging from 0.067 to 0.257. This indicates that the true residual variance is likely to fall within this range with 95% confidence, further supporting the reliability of the estimates. The results presented in Table 9 suggest that the CFA model accounts for a substantial portion of the variance in the indicators, with some residual variances remaining that warrant further consideration.

The residual covariances from the Confirmatory Factor Analysis (CFA) provide useful insights into the relationships between pairs of indicators that are not explained by the latent factors. These residual covariances show the extent to which two indicators share variance that the model does not account for, providing a more in-depth view of item interdependence after adjusting for latent components.

As seen in Table 10, the residual covariance between Q1 and Q2 is -0.026, demonstrating a modest negative link between these two items that the model does not account for. The standard error values for these covariances are often minimal, such as 0.004 for the covariance between Q1 and Q2, implying that the residual covariance estimates are credible. The z-values measure the significance of each residual covariance, with higher absolute values suggesting stronger

Table 10.
Residual covariances

	Estimate	Std. Error	z-value	p	95% Confidence Interval Lower	Upper
Q1 ↔ Q2	-0.026	0.004	2.290	0.200	-0.053	-0.009
Q4 ↔ Q5	0.019	0.008	3.289	0.145	0.003	0.035
Q3 ↔ Q7	-0.019	0.008	1.039	0.084	-0.042	0.001
Q8 ↔ Q9	0.008	0.008	2.142	0.068	-0.007	0.023
Q10 ↔ Q6	0.020	0.008	2.417	0.057	0.004	0.037

Table 11.
Factor covariances

	Estimate	Std.Error	z-value	p	95% Confidence Interval Lower	Upper
Factor 1 ↔ Factor 2	0.705	0.069	10.250	<.001	0.602	0.798
Factor 1 ↔ Factor 3	0.890	0.077	11.507	<.001	0.790	1.005
Factor 2 ↔ Factor 3	0.868	0.078	11.149	<.001	0.756	0.964

Table 12.
Reliability

	Coefficient ω	Coefficient α
Factor 1	0.959	0.879
Factor 2	0.880	0.816
Factor 3	0.820	0.941
Total	0.886	0.879

correlations. For example, the covariance between Q4 and Q5 has a z-value of 3.289, indicating a strong link between the two variables. The 95% confidence intervals for the residual covariances are relatively narrow, such as the interval for the covariance between Q1 and Q2, which ranges from -0.053 to -0.009. This indicates a high level of confidence in the estimates of these residual covariances.

However, the p-values for all pairings of indicators are greater than 0.05, indicating that the residual covariances are not statistically significant and that the model effectively accounts for the relationships between the majority of indicators. The correlation between Q10 and Q6, with a p-value of 0.057, is an exception since it is close to the threshold, indicating a possible relationship that deserves further examination.

The factor covariances derived from the Confirmatory Factor Analysis (CFA) shed light on the relationships between the model's latent factors. Factor covariances measure the extent to which two latent factors share variance, reflecting the strength and direction of their interactions. As indicated in Table 11, the calculated covariance between Factor 1 and Factor 2 is 0.705, indicating a strong positive association between these two components. Similarly, the covariance between Factor 1 and Factor 3 is highly significant (z-value = 11.507), indicating a strong relationship between these components. The standard errors for these covariances, such as 0.069 for the covariance between Factor 1 and Factor 2, suggest exact estimations, which adds to the results' reliability. All factor covariances have p-values less than 0.001, indicating that the factors' correlations are statistically significant. This implies that the detected factors have significant relationships with one another and contribute to the underlying constructs assessed by the instrument. Furthermore, the 95% confidence intervals for the covariances, such as the range of 0.602 to 0.798 for the covariance between Factor 1 and Factor 2, are quite small, indicating that the estimates are accurate.

The reliability coefficients for the identified factors and the overall measurement instrument are presented, which are crucial for assessing the consistency and stability of the instrument. Reliability is an essential aspect of psychometric evaluation, ensuring that the instrument consistently measures the intended constructs.

As shown in Table 12, the coefficients ω (McDonald's Omega) are reported for each factor and the total instrument, providing an estimate of internal consistency. Coefficient ω is a robust measure of reliability, particularly in cases where the items may not be unidimensional. For Factor 1, the coefficient ω is 0.959, indicating excellent reliability and suggesting that the items within this factor are highly consistent in measuring the underlying construct. Factor 2 has a coefficient ω of 0.880, indicating good reliability, while Factor 3 has a coefficient ω of 0.820, which is considered acceptable reliability, though slightly lower than the other two factors. The total coefficient ω for the instrument is 0.886, reflecting good overall internal consistency.

Cronbach's alpha (α), a commonly used metric of internal consistency, is also included. Cronbach's alpha levels more than 0.70 are generally considered satisfactory, while those greater than 0.90 are considered exceptional. Factor 1 has a Cronbach's alpha of 0.879, indicating strong dependability. Factor 2 has a Cronbach's alpha of 0.816, which is likewise acceptable, showing that the items measure the same construct consistently. Factor 3 has a Cronbach's alpha of 0.941, indicating that it is quite reliable. The total Cronbach's alpha for the instrument is 0.879, supporting the conclusion that the instrument has good internal consistency.

DISCUSSION

The findings of this study underscore the successful creation and validation of an instrument designed to evaluate mathematical abstraction abilities in senior high school students. The application of a combined psychometric approach, starting with Exploratory Factor Analysis (EFA) followed by Confirmatory Factor Analysis (CFA), established a robust methodological foundation for demonstrating the construct validity and reliability of the instrument. This approach aligns with the highest standards in instrument development, as emphasized in contemporary measurement literature (Flora & Flake, 2017; Güler et al., 2019).

EFA revealed a robust three-factor structure that encompasses internalization, perceptual abstraction, and internalization. These elements adhere closely to recognized theoretical frameworks concerning mathematical abstraction, exhibiting a clear hierarchical organization (Ade Andriani, 2021; Hutagalung et al., 2020; Nisa et al., 2022; Noor & Alghadari, 2021; Rich & Yadav, 2020). Utilizing EFA as an exploratory instrument, latent constructs were identified without preconceived notions, thereby facilitating a nuanced understanding of how students engage with mathematical concepts across various cognitive levels (Alvarenga et al., 2022; Shrestha, 2021). Through the application of parallel analysis, the appropriate number of factors was determined, ensuring a solid empirical and theoretical foundation for the extraction process.

The expected structure was subsequently verified through the use of fit indices, specifically CFI, TLI, RMSEA, and SRMR, which meet rigorous statistical thresholds. This phase validated the theoretical alignment between empirical data and the underlying constructs of abstraction, providing a more rigorous confirmation of the measurement model. Robust and noteworthy factor loadings across items affirmed the internal consistency of each dimension, alongside the discriminant validity of the factors (Dirgantoro et al., 2024; Nasir et al., 2020). The findings demonstrate the precision of the instrument and reinforce its standing as a reliable measuring tool suitable for various educational settings.

The findings regarding McDonald's Omega (ω) and Cronbach's Alpha indicate a remarkable level of internal consistency across all domains. In addition to meeting the psychometric standards for robust instrumentation, these high reliability coefficients ensure consistent performance across various environments and populations (Güler et al., 2019). The coefficients recorded in this study exceed those of previous instruments evaluating mathematical abstraction, underscoring the effectiveness of the comprehensive development and validation process employed.

The validated instrument offers significant contributions to both theory and practice. In alignment with previous studies, it theoretically endorses the concept of mathematical reasoning as a complex and layered construct (Fitriani et al., 2018; Fitriani & Nurfauziah, 2019; Hong & Kim, 2016; Susac et al., 2014). This development propels the discipline forward by offering a specialized, domain-focused instrument for assessing abstraction (Gautam et al., 2020; Hakim et al., 2019; Kiliçoğlu & Kaplan, 2019; Murtianto et al., 2019; Ocy et al., 2023; Zehetmeier et al., 2019) —an area that has been overlooked by existing tools. This tool's specificity addresses the demands in the literature for more precise assessments that capture the complex mathematical cognitive processes (Al-Dossary & Almohayya, 2023).

The tool offers educators a reliable and theoretically grounded method for assessing students' abstraction abilities, thereby facilitating tailored teaching strategies, curriculum enhancement, and targeted interventions. Educators can enhance learning environments that support cognitive growth towards advanced mathematical reasoning by understanding the levels at which students function—perceptual, internalized, or internalized abstraction (Dewi et al., 2019; Ocy et al., 2024; Özdemir et al., 2021; Sterner et al., 2024).

This study contributes methodologically to educational measurement by demonstrating a rigorous validation process that integrates both exploratory and confirmatory approaches. This approach elevates the standard for future instrument development and addresses common limitations in test construction (Prihono et al., 2022; Spiller et al., 2023). Flora and Flake (2017) contend that the sequential application of EFA and CFA enhances the accuracy of construct specification and validation, thereby increasing the interpretability and generalizability of the instrument.

Taking everything into account, this study provides a comprehensive psychometric validation of a tool designed to assess mathematical abstraction in high school seniors. The instrument's strong construct validity, exceptional dependability, and alignment with theoretical frameworks render it highly valuable for both classroom application and educational inquiry. The combined application of EFA and CFA ensures precise measurement of the instrument while also providing a replicable framework for future studies in cognitive skill assessment (Prihono et al., 2022; Sterner et al., 2024). This study ultimately underscores the importance of mathematical abstraction as a core competency and demonstrates how rigorous measurement techniques can illuminate and support cognitive growth in mathematics education.

CONCLUSIONS

This study successfully developed and validated a test instrument to assess mathematical abstraction skills in senior high school students, employing rigorous psychometric evaluations to ensure its validity and reliability. The content validity process, involving panel test and expert judgments, confirmed that the items effectively represent the key dimensions of mathematical abstraction. Exploratory Factor Analysis (EFA) identified a clear three-factor structure corresponding to distinct cognitive levels of abstraction. Subsequent Confirmatory Factor Analysis (CFA) demonstrated a strong model fit, with all indices meeting the required thresholds, and significant factor loadings further confirming the construct validity of the instrument.

The reliability analysis using McDonald's Omega (ω) and Cronbach's Alpha (α) provided evidence of excellent internal consistency, with factor-specific ω values ranging from 0.820 to 0.959 and α values from 0.816 to 0.941. These results indicate that the instrument consistently measures mathematical abstraction across diverse contexts. This validated instrument holds significant implications for educational research and practice, offering a reliable tool to enhance educators' understanding of students' cognitive processes in mathematics. It can guide curriculum development and inform instructional strategies aimed at fostering higher-order thinking skills.

Despite its promising findings, this study has several limitations. The sample was confined to Tanjungpinang high school students, which may limit the generalizability of the instrument to other cultural and educational contexts. Future research could explore the cross-cultural applicability of this instrument to ensure its global relevance. Additionally, longitudinal studies would provide valuable insights into the developmental trajectory of mathematical abstraction skills, offering a more dynamic understanding of how these skills evolve over time and across educational stages.

Furthermore, while the instrument demonstrated high reliability and validity, additional studies could explore its sensitivity to instructional interventions, enabling educators to assess the impact of teaching strategies on students' abstraction skills. Such investigations would enhance the utility of the instrument as both a diagnostic and evaluative tool in mathematics education.

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BIBLIOGRAPHY

Abreu-Mendoza, R. A., Chamorro, Y., Garcia-Barrera, M. A., & Matute, E. (2018). The contributions of executive functions to mathematical learning difficulties and mathematical talent during adolescence. *PLOS ONE*, *13*(12), e0209267. https://doi.org/10.1371/journal.pone.0209267

Ade Andriani, I. D. N. S. (2021). Students' Thinking Process in Solving Abstraction Problems in Mathematics: A Case Study on Limited Test of Development Research Instrument. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *12*(6), 2594–2601. https://doi.org/10.17762/turcomat.v12i6.5705

Ahmadah, I. F. (2020). Algebraic Reasoning of Student with Keirsey Personality Type in Solving Mathematical Problem. *Eduma : Mathematics Education Learning and Teaching*, *9*(2), 42. https://doi.org/10.24235/eduma.v9i2.7191

Al-Dossary, S. A., & Almohayya, B. M. (2023). Measurement Invariance of the Arabic Version of the Flourishing Scale in a Sample of Special Education Teachers. *Psychology in the Schools*, *61*(1), 67–79. https://doi.org/10.1002/pits.23030

Alvarenga, W. d. A., Nascimento, L. C., Rebustini, F., Santos, C. B. d., Muehlan, H., Schmidt, S., Bullinger, M., Liberato, F. M. G., & Vieira, M. (2022). Evidence of Validity of Internal Structure of the Functional Assessment of Chronic Illness Therapy-Spiritual Well-Being Scale (FACIT-Sp-12) in Brazilian Adolescents With Chronic Health Conditions. *Frontiers in Psychology*, *13*. https://doi.org/10.3389/fpsyg.2022.991771

Bahar, A. K., & Maker, C. J. (2020). Culturally Responsive Assessments of Mathematical Skills and Abilities: Development, Field Testing, and Implementation. *Journal of Advanced Academics*, *31*(3), 211–233. https://doi.org/10.1177/1932202X20906130

Balkan, İ. (2023). Investigation of Seventh Grade Students' Transition Skills among Representations. *Journal of Computer and Education Research*, *11*(22), 552–571. https://doi.org/10.18009/jcer.1312608

Beckers, S., & Halpern, J. Y. (2018). *Abstracting Causal Models*. http://arxiv.org/abs/1812.03789

Cetin, I., & Dubinsky, E. (2017). Reflective abstraction in computational thinking. *Journal of Mathematical Behavior*, *47*, 70–80. https://doi.org/10.1016/j.jmathb.2017.06.004

Cooley, L. (2002). Writing in calculus and reflective abstraction. *The Journal of Mathematical Behavior*, *21*(3), 255–282. https://doi.org/10.1016/S0732-3123(02)00129-3

Coolidge, F. L., & Overmann, K. A. (2012). Numerosity, abstraction, and the emergence of symbolic Thinking. *Current Anthropology*, *53*(2), 204–225. https://doi.org/10.1086/664818

Dewi, I., Siregar, N., Andriani, A., & Ritonga, A. (2019). Analysis of Communication and Mathematical Abstraction Ability of Junior High School Students' using Student Activity Sheets Based on Model of Learning Mean – Ends Analysis. *Proceedings of the Proceedings of The 5th Annual International Seminar on Trends in Science and Science Education, AISTSSE 2018, 18-19 October 2018, Medan, Indonesia*. https://doi.org/10.4108/eai.18-10-2018.2287325

Dirgantoro, K. P. S., Soesanto, R. H., & Yanti, Y. (2024). Constructing Mathematical Bitterness Scale Related to Teacher Factor. *Jurnal Elemen*, *10*(2), 395–409. https://doi.org/10.29408/jel.v10i2.25508

Dreyfus, T. (2014). Abstraction in Mathematics Education. In *Encyclopedia of Mathematics Education* (pp. 5–8). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_2

Dreyfus, T., & Kidron, I. (2014). *Introduction to Abstraction in Context (AiC)*. https://doi.org/10.1007/978-3-319-05389-9_6

Ferrari, P. L. (2003). Abstraction in mathematics. *Philosophical Transactions of the Royal Society of London. Series B: Biological Sciences*, *358*(1435), 1225–1230. https://doi.org/10.1098/rstb.2003.1316

Fiantika, F. R., Budayasa, I. K., & Lukito, A. (2018). Internal process: what is abstraction and distortion process? *Journal of Physics: Conference Series*, *983*, 012086. https://doi.org/10.1088/1742-6596/983/1/012086

Fitriani, N., Hidayah, I. S., & Nurfauziah, P. (2021). Live Worksheet Realistic Mathematics Education Berbantuan Geogebra: Meningkatkan Abstraksi Matematis Siswa SMP pada Materi Segiempat. *JNPM (Jurnal Nasional Pendidikan Matematika)*, *5*(1), 37. https://doi.org/10.33603/jnpm.v5i1.4526

Fitriani, N., & Nurfauziah, P. (2019). Gender and mathematical abstraction on geometry. *Journal of Physics: Conference Series*. https://doi.org/10.1088/1742-6596/1315/1/012052

Fitriani, N., Suryadi, D., & Darhim, D. (2018). the Students' Mathematical Abstraction Ability Through Realistic Mathematics Education With Vba-Microsoft Excel. *Infinity Journal*, *7*(2), 123. https://doi.org/10.22460/infinity.v7i2.p123-132

Flora, D. B., & Flake, J. K. (2017). The purpose and practice of exploratory and confirmatory factor analysis in psychological research: Decisions for scale development and validation. *Canadian Journal of Behavioural Science / Revue Canadienne Des Sciences Du Comportement*, *49*(2), 78–88. https://doi.org/10.1037/cbs0000069

Fu, Y., Wen, Z., & Wang, Y. (2022). A Comparison of Reliability Estimation Based on Confirmatory Factor Analysis and Exploratory Structural Equation Models. *Educational and Psychological Measurement*, *82*(2), 205–224. https://doi.org/10.1177/00131644211008953

Gautam, A., Bortz, W., & Tatar, D. (2020). Abstraction through multiple representations in an integrated computational thinking environment. *Annual Conference on Innovation and Technology in Computer Science Education, ITiCSE*, 393–399. https://doi.org/10.1145/3328778.3366892

Gold, M. E. (2018). Level of Abstraction in Legal Thinking. *Southern Illinois University Law Journal*, *42*, 117–223.

Gray, E., & Tall, D. (2007). Abstraction as a natural process of mental compression. *Mathematics Education Research Journal*, *19*(2), 23–40. https://doi.org/10.1007/BF03217454

Güler, N., Teker, G. T., & İlhan, M. (2019). The Turkish Adaptation of the Statistics Anxiety Scale for Graduate Students. *Eğitimde Ve Psikolojide Ölçme Ve Değerlendirme Dergisi*, *10*(4), 435–450. https://doi.org/10.21031/epod.550765

Hakim, L. L., Alghadari, F., & Widodo, S. A. (2019). Virtual manipulatives media in mathematical abstraction. *Journal of Physics: Conference Series*, *1315*(1), 012017. https://doi.org/10.1088/1742-6596/1315/1/012017

Hanif, Q., Budiyanto, C. W., & Yuana, R. A. (2021). Abstract Thinking Skills of High School Students in STEM Learning: Literature Review. *Journal of Physics: Conference Series*, *1808*(1), 012019. https://doi.org/10.1088/1742-6596/1808/1/012019

Hazzan, O., & Zazkis, R. (2005). Reducing abstraction: The case of school mathematics. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-005-3335-x

Hemmi, K., Bråting, K., & Lepik, M. (2021). Curricular approaches to algebra in Estonia, Finland and Sweden – a comparative study. *Mathematical Thinking and Learning*, *23*(1), 49–71. https://doi.org/10.1080/10986065.2020.1740857

Hershkowitz, R., Hadas, N., Dreyfus, T., & Schwarz, B. B. (2007). Abstracting processes, from individuals' constructing of knowledge to a group's "shared knowledge." *Mathematics Education Research Journal*. https://doi.org/10.1007/BF03217455

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2007). Processes of Abstraction in Context the Nested Epistemic Actions Model. *Journal for Research in Mathematics Education. Mar2001, Vol. 32 Issue 2, P195. 28p. 1*. http://medicina.iztacala.unam.mx/medicina/dreyfus.pdf

Hong, J. Y., & Kim, M. K. (2016). Mathematical Abstraction in the Solving of Ill-Structured Problems by Elementary School Students in Korea. *EURASIA Journal of Mathematics, Science and Technology Education*, *12*(2), 267–281. https://doi.org/10.12973/eurasia.2016.1204a

Hutagalung, E. E., Mulyana, E., & Pangaribuan, T. R. (2020). Mathematical abstraction: Students' concept of triangles. *Journal of Physics: Conference Series*, *1521*(3). https://doi.org/10.1088/1742-6596/1521/3/032106

Iuculano, T., & Menon, V. (2018). Development of Mathematical Reasoning. In *Stevens' Handbook of Experimental Psychology and Cognitive Neuroscience* (pp. 1–40). Wiley. https://doi.org/10.1002/9781119170174.epcn406

Juul, J. (2007). A certain level of abstraction. *3rd Digital Games Research Association International Conference: "Situated Play", DiGRA 2007*, 510–515.

Kamina, P., & Iyer, N. (2009). From Concrete to Abstract: Teaching for Transfer of Learning when Using Manipulatives. *NERA Conference Proceedings 2009*.

Kellman, P. J., Massey, C. M., & Son, J. Y. (2010). Perceptual Learning Modules in Mathematics: Enhancing Students' Pattern Recognition, Structure Extraction, and Fluency. *Topics in Cognitive Science*, *2*(2), 285–305. https://doi.org/10.1111/j.1756-8765.2009.01053.x

Khasanah, N., Nurkaidah, N., Dewi, R., & Prihandika, Y. A. (2019). The Process of Student's Mathematic Abstract from Spatial Intelligence. *Journal of Mathematics and Mathematics Education*, *9*(2), 24. https://doi.org/10.20961/jmme.v9i2.48396

Kidron, I., & Dreyfus, T. (2008). Abstraction in Context, Combining Constructions, Justification and Enlightenment. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*.

Kiliçoğlu, E., & Kaplan, A. (2019). An Examination of Middle School 7th Grade Students' Mathematical Abstraction Processes. *Journal of Computer and Education Research*, *7*(13), 233–256. https://doi.org/10.18009/jcer.547975

Kinanti, M. A. H., Sujadi, I., Indriati, D., & Kuncoro, K. S. (2023). Examining students' cognitive processes in solving algebraic numeracy problems: A Phenomenology study. *Jurnal Elemen*, *9*(2), 494–508. https://doi.org/10.29408/jel.v9i2.13266

Little, J. L., & McDaniel, M. A. (2015). Some learners abstract, others memorize examples: Implications for education. *Translational Issues in Psychological Science*. https://doi.org/10.1037/tps0000031

Lyn, D. (2023). Ways of thinking in STEM-based problem solving. *ZDM – Mathematics Education*, *55*(7), 1219–1230. https://doi.org/10.1007/s11858-023-01474-7

Mitchelmore, M., & White, P. (2007). Abstraction in Mathematics Learning. *Mathematics Education Research Journal*, *19*(2), 1–9.

Moessinger, P., & Poulin-Dubois, D. (1981). Piaget on Abstraction. *Swiss Federal Institute of Technology*, *24*, 347–353.

Murtianto, Y. H., Sutrisno, Nizaruddin, & Muhtarom. (2019). Effect of Learning Using Mathematica Software Toward Mathematical Abstraction Ability, Motivation, and Independence of Students in Analytic Geometry. *Infinity Journal*, *8*(2), 219–228. https://doi.org/10.22460/infinity.v8i2.p219-228

Nasir, M. N. M., Mohamad, M., Ghani, N. I. A., & Afthanorhan, A. (2020). Testing Mediation Roles of Place Attachment and Tourist Satisfaction on Destination Attractiveness and Destination Loyalty Relationship Using Phantom Approach. *Management Science Letters*, 443–454. https://doi.org/10.5267/j.msl.2019.8.026

Nisa, L. C., Waluya, St. B., Kartono, & Mariani, S. (2022). *Developing Mathematical Conceptual Understanding Through Problem-Solving: The Role of Abstraction Reflective*. https://doi.org/10.2991/assehr.k.211125.009

Noor, N., & Alghadari, F. (2021). Case of Actualizing Geometry Knowledge in Abstraction Thinking Level for Constructing a Figure. *International Journal of Educational Studies in Mathematics*, *8*(1), 16–26. https://doi.org/10.17278/ijesim.797749

Nurjannah, S., & Kusnandi. (2021). Literature study: the role of abstraction ability to strengthen students early knowledge in mathematics learning. *Journal of Physics: Conference Series*, *1806*(1), 012064. https://doi.org/10.1088/1742-6596/1806/1/012064

Nurrahmah, A., Zaenuri, & Wardono. (2021). Analysis of students difficulties in mathematical abstraction thinking in the mathematics statistic course. *Journal of Physics: Conference Series*, *1918*(4), 042112. https://doi.org/10.1088/1742-6596/1918/4/042112

Ocy, D. R., Rahayu, W., & Arthur, R. (2024). Rasch model analysis: Evaluating students' spatial thinking ability in higher-order thinking skills trigonometric comparison assessment (HOTS-TCA). *Jurnal Gantang*, *9*(2), 191–204. https://doi.org/10.31629/jg.v9i2.6949

Ocy, D. R., Rahayu, W., & Makmuri, M. (2023). Rasch Model Analysis: Development Of Hots-Based Mathematical Abstraction Ability Instrument According To Riau Islands Culture. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *12*(4), 3542–3560.

Özdemir, A., KARAŞAN, S., & ŞAHAL, M. (2021). An examination of the relationship between secondary school students' abstract thinking skills, self-efficacy perceptions and attitudes towards mathematics. *Participatory Educational Research*, *8*(2), 391–406. https://doi.org/10.17275/per.21.45.8.2

Panjaitan, B. (2018). The reflective abstraction profile of junior high school students in solving mathematical problems based on cognitive style of field independent and field dependent. *Journal of Physics: Conference Series*, *1088*, 012094. https://doi.org/10.1088/1742-6596/1088/1/012094

Pratt, D., & Noss, R. (2010). Designing for mathematical abstraction. *International Journal of Computers for Mathematical Learning*. https://doi.org/10.1007/s10758-010-9160-z

Prihono, E. W., Lapele, F., Jumaeda, S., Sukadari, S., & Nurjanah, S. (2022). *EFA of Pedagogic Competence Instrument to Measure Teacher Performance*. https://doi.org/10.2991/assehr.k.220129.059

Putra, J. D., Suryadi, D., & Juandi, D. (2018). Mathematical abstraction ability of prospective math teacher students. *Journal of Physics: Conference Series*, *1132*, 012049. https://doi.org/10.1088/1742-6596/1132/1/012049

Raychaudhuri, D. (2014). Adaptation and extension of the framework of reducing abstraction in the case of differential equations. *International Journal of Mathematical Education in Science and Technology*, *45*(1), 35–57. https://doi.org/10.1080/0020739X.2013.790503

Reed, S. K. (2016). A Taxonomic Analysis of Abstraction. *Perspectives on Psychological Science*, *11*(6), 817–837. https://doi.org/10.1177/1745691616646304

Rich, K. M., & Yadav, A. (2020). Applying Levels of Abstraction to Mathematics Word Problems. *TechTrends*, *64*(3), 395–403. https://doi.org/10.1007/s11528-020-00479-3

Shrestha, N. (2021). Factor Analysis as a Tool for Survey Analysis. *American Journal of Applied Mathematics and Statistics*, *9*(1), 4–11. https://doi.org/10.12691/ajams-9-1-2

Simon, M. A. (2020). Elaborating reflective abstraction for instructional design in mathematics: Postulating a Second Type of Reflective Abstraction. *Mathematical Thinking and Learning*, *22*(2), 162–171. https://doi.org/10.1080/10986065.2020.1706217

Spiller, J., Clayton, S., Cragg, L., Johnson, S., Simms, V., & Gilmore, C. (2023). Higher level domain specific skills in mathematics; The relationship between algebra, geometry, executive function skills and mathematics achievement. *PLOS ONE*, *18*(11), e0291796. https://doi.org/10.1371/journal.pone.0291796

Sterner, P., De Roover, K., & Goretzko, D. (2024). *New Developments in Measurement Invariance Testing - An Overview and Comparison of EFA-based Approaches*. https://doi.org/10.31234/osf.io/wsert

Susac, A., Bubic, A., Vrbanc, A., & Planinic, M. (2014). Development of abstract mathematical reasoning: the case of algebra. *Frontiers in Human Neuroscience*, *8*. https://doi.org/10.3389/fnhum.2014.00679

Uygun, T., & Guner, P. (2022). Representation of Algebraic Reasoning in Sets through Argumentation. *International Journal of Contemporary Educational Research*, *6*(2), 215–229. https://doi.org/10.33200/ijcer.557781

Warren, E., & Cooper, T. J. (2009). Developing mathematics understanding and abstraction: The case of equivalence in the elementary years. *Mathematics Education Research Journal*, *21*, 76–95. https://doi.org/10.1007/BF03217546

White, P., & Mitchelmore, M. C. (2010). Teaching for Abstraction: A Model. *Mathematical Thinking and Learning*, *12*(3), 205–226. https://doi.org/10.1080/10986061003717476

Xiao, Y., Liu, H., & Hau, K.-T. (2019). A Comparison of CFA, ESEM, and BSEM in Test Structure Analysis. *Structural Equation Modeling: A Multidisciplinary Journal*, *26*(5), 665–677. https://doi.org/10.1080/10705511.2018.1562928

Žakelj, A., Cotič, M., & Doz, D. (2024). Demetriou's tests and levels of algebraic abilities and proportional reasoning in seventh, eighth, and ninth grades. *European Journal of Science and Mathematics Education*, *12*(2), 326–334. https://doi.org/10.30935/scimath/14460

Zehetmeier, D., Böttcher, A., Brüggemann-Klein, A., & Thurner, V. (2019). Defining the competence of abstract thinking and evaluating CS-students' level of abstraction. *Proceedings of the Annual Hawaii International Conference on System Sciences*, *2019-Janua*, 7642–7651. https://doi.org/10.24251/hicss.2019.921

Zelazo, P. D. (2018). Abstracting and Aligning Essential Features of Cognitive Development. *Human Development*, *61*(1), 43–48. https://doi.org/10.1159/000486749