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Covariational reasoning of field dependent prospective mathematics teacher in solving covariance problem

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ABSTRACT

This study aims to analyze the coordination and construction aspects of covariational reasoning of field dependent students' prospective mathematics teachers in solving covariation problems. This research is qualitative research with the subject of research being field dependent students' prospective mathematics teachers of STKIP PGRI Jombang. The instruments in this study are the main instrument (the researcher himself) and supporting instruments in the form of GEFT, Covariation Test, and interview guidelines. Data analysis uses data reduction steps, data presentation, and conclusions. Based on the results of the study, the subjects coordinate the magnitude of variable changes and determine the pattern of change for linear covariation problems based on calculations from the results of the representation of covariation task-based problems to compile their change patterns, for linear students using students' own understanding without doing calculations. In the construction aspect, the subject constructs through the representation of the relationship of two variables into a graph by determining the coordinate axis with known variables and drawing a graph from the results of calculating the pattern of change in the relationship of two variables. From the depicted graph, the direction of the chart can be determined. The resulting graph corresponds to a given covariation task-based problem.

INTRODUCTION

Reasoning is fundamental in learning mathematics. Reasoning is part of a person's thought process (Ghavam & Soghra, 2011; Janssen et al., 2021; Koklu, 2007). One of the goals in the process of learning mathematics is the ability to reason and solve problems. Problem understanding and reasoning are cognitive components that students must have in the problem-solving process (Mukuka et al., 2020; Norqvist et al., 2019). One concept in mathematics that requires reasoning in understanding it is function. Understanding the concept of functions is closely related to graphs, which are the basis for understanding the concept of functions in calculus. (Carlson dkk., 2002) stated that students are still lacking in the ability to interpret charts. Students have difficulty representing and interpreting changes in function graphs. (Koklu, 2007) states that students tend to think procedurally about changes in function that hinder their reasoning ability. (Carlson dkk., 2002) Defining cognitive activity that involves coordinating two kinds of quantities by paying attention to the change of one quantity to another is called covariational reasoning.

Covariational reasoning originally emerged as a theory expressed by Jere Confrey in the late 1980s and by Patrick Thompson in the late 1990s. Both describe coordination as the basis of covariational reasoning about dynamic relationships of events. Confrey described covariational reasoning as a discrete approach that focuses on changing the value of a variable. Meanwhile,

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Table 1.
Mental action framework of covariation (Carlson, 2002)

Mental Action	Description of Mental Action
Mental Action 1 (AM 1)	Coordinate the value of a variable against changes in other variables
Mental Action 2 (AM 2)	Coordinate the direction of variable changes against changes in other variables
Mental Action 3 (AM 3)	Coordinate the magnitude of changes from variables to changes in other variables
Mental Action 4 (AM 4)	Coordinates average speed with uniform increments in input variables
Mental Action 5 (AM 5)	Coordinate instantaneous speed with continuous changes in *independent variables* for the entire *domain*

(Thompson, 1998) describe continuous covariational reasoning with a focus on measuring object properties and continuous simultaneous change.

Covariational reasoning can be developed through dynamic event modeling (Gil & Gibbs, 2017). Students need to be confronted with problems related to the concept of dynamic events involving the relationship between two variables. Dynamic events are events that describe changes in the value of a variable that cause changes in the value of other variables (Basu & Panorkou, 2019; Kertil, 2020). The identification of covariational reasoning on field-dependent cognitive styles involves visual ability when constructing graphs on covariation tasks. Visualization is an important tool in solving mathematical problems. This study aims to analyze the coordination and construction aspects of covariational reasoning of field dependent students' prospective mathematics teachers in solving covariation problems.

Covariational reasoning

Reasoning has an important role in the thought process. Furthermore, reasoning is part of the logical and analytical thinking process in solving a problem. Reasoning is part of the thinking hierarchy which includes basic thinking, critical thinking, and creative thinking but does not include remembering (Mukuka et al., 2020; Thuneberg et al., 2018). Mathematics and mathematical reasoning are interrelated. Reasoning is the basis of mathematics. Mathematical reasoning is one of the necessary and important abilities to be mastered by students (Berndt et al., 2021; Janssen et al., 2021; Schlatter et al., 2021). Reasoning ability occurs when a person thinks about a problem or solves a problem. Through reasoning, students are expected to see that mathematics is a reasonable and logical study.

Thompson (1998) states that covariation as an approach to solving problems in mathematics is a function as a more formally intuitive alternative. In function learning, there are two kinds of approaches, namely correspondence and covariation. Carlson et al (2002) It defines covariational reasoning as a cognitive activity that involves the coordination of two types of quantities relating to the ways in which the two quantities change with each other. Meanwhile, covariational reasoning is a mental activity in coordinating two quantities (independent variable and dependent variable) related to the way a change in one quantity causes a change in the other quantity. Carlson et al. (2002) Develop a covariation framework that describes five mental actions and five levels of covariational reasoning. The description of the five mental actions involved in covariational reasoning is presented in the following Table 1.

The ability of covariational reasoning is derived from the level of covariational reasoning given, that is, their support for actions related to the level of mental covariational reasoning. According to Carlson et al, the levels of covariational reasoning are Level 1 (L1) Coordination, Level 2 (L2) Direction, Level 3 (L3) Coordination Quantity, Level 4 (L4) Rate of change, and Level 5 (L5) Instantaneous rate. In addition, some experts define covariational reasoning as follows Table 2. According to experts in previous studies, covariational reasoning is defined in Table 3.

Table 2.
Definition of covariational reasoning according to some experts

Year	Source	Definition
1994	Confrey & Smith	Covariation involves coordinating the values of two variables
1997	Slavit	Covariational reasoning is the ability to analyze, coordinate, and understand the overall relationship of changes in quantity
1998	Saldanha&Thompson	Covariational reasoning has been shown to be an important ability to interpret, describe, and represent dynamic function events. Coulombe and Berenson construct definitions, and the ideas that have been discussed to describe the concept of covariation involve aspects of: 1. **Identify** two data sets. 2. **coordinate** patterns from data to see increase, decrease, or constant relationships. 3. **Connect** two patterns to establish a custom connection between data values. 4. **generalize** links to predict values from unknown data
2002	Carlson *et al*	Covariational reasoning is defined as a cognitive activity that involves **the coordination** of two quantities that relate one to the other
2007	Onder Koklu	Covariational reasoning is an important reasoning ability in **analyzing**, **interpreting**, and **representing** forms of change in the continuous change of dynamic events
2018	Micah Kurzer	Covariational reasoning includes three stages of perfect drawing of each rate of change defined in the section constructing a change in quantity in an image, coordinating two images changing with each other, forming a simultaneous image of two-quantity covariance

Table 3.
Covariational reasoning indicators in resolving covariation problems

Aspects of Covariational Reasoning according to researchers	Indicator
Coordinate	Determine the pattern of change between two variables and determine their direction. Determine the magnitude of change in one variable when viewed from another variable
Constructing	Represent the relationship between variables in general in a graph

Field dependent cognitive style

Cognitive style refers to the individual's preferred way of processing information. Based on differences in psychological aspects, cognitive styles consist of two parts, namely field independent and field dependent. Individuals with field-dependent cognitive styles are more global in nature where individuals are more focused on the environment as a whole or dominated by the environment. Individuals with field-dependent cognitive styles have difficulty separating incoming information from the contextual environment and are more likely to be influenced by external cues and become less effective at absorbing. The relationship between cognitive style and reasoning ability in general can be traced through relationships based on working memory capacity (Gojkov et al., 2013; Pektaş, 2014). Efficiency in receiving information becomes an individual difference in reasoning that requires new structures of representation.

Identification of covariational reasoning in field-independent and field-dependent cognitive styles involves visual ability when constructing graphs on covariation tasks (Ummah, 2020). Covariational reasoning involves the visual ability to construct graphs on covariation's task-based problems. Measurement of visual-spatial ability can be seen from the cognitive styles of

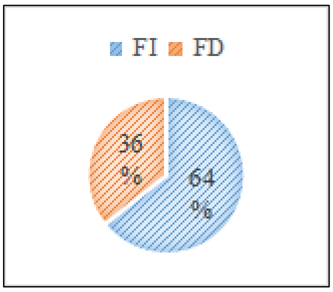
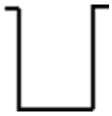


Figure 1. GEFT Yield Percentage

Table 4.
Covariation Task

Take a look at the following bottle image and solve the problem below!

1. It is known that the base of the bottle is square with a side length of 4 units and a bottle height of 10 units as shown on the side.
   The bottle will be filled with water constantly (a lot of running water per unit time is the same).
   a. Determine the height of the water in the bottle if the volume of water introduced into the bottle is 16 units. Explain your answer.
   b. Determine the height of the water in the bottle if the volume of water introduced into the bottle is 32 units. Explain your answer.
   c. What volume of water should be put in the bottle so that the bottle is full? Explain your answer.
   d. Explain the relationship between the volume of water inserted and the height of the water in the bottle and draw the graph.



1. It is known that the base of the bottle is square with a side length of 4 units, the top of the bottle is also square with a side length of 2 units and a bottle height of 10 units as shown on the side.
   The bottle will be filled with water constantly (a lot of water flows per unit time equally).
   a. Determine the height of the water in the bottle if the volume of water introduced into the bottle is 16 units. Explain your answer!
   b. Determine the height of the water in the bottle if the volume of water introduced into the bottle is 32 units. Explain your answer!
   c. What volume of water should be put in the bottle so that the bottle is full? Explain your answer!
   d. Explain the relationship between the volume of water inserted and the height of the water in the bottle and draw the graph!

2. If the shape of the bottle is like the picture on the side, draw a graph that shows the water level is a function of the amount of water that is constantly filled into the bottle. Give your reasons!

field independent and field dependent (Pithers, 2006). Visual-spatial abilities are needed in graphic construction which is one aspect of covariational reasoning. This is in line with research Ummah (2020) which states that differences in covariational reasoning are found when identifying information in the problem-solving process based on covariation tasks in subjects with different cognitive styles. In addition, graphic representations of subjects with different cognitive styles also showed different results in solving covariation tasks.

METHODS

The type of research used in this study is qualitative research. (Downs, 1990; Hamilton & Finley, 2020) states that qualitative research deals with qualitative phenomena related to quality or variety. The subject of this study was a mathematics teacher candidate student in the Mathematics Education Study Program of STKIP PGRI Jombang who had a field dependent cognitive style based on the lowest score from the GEFT test results. The results of the GEFT are as follows Figure 1

The subject is given a covariation problem in the form of a bottle problem. After that, subjects were interviewed to confirm their thought process. The instrument of the covariation problem is as follows Table 4

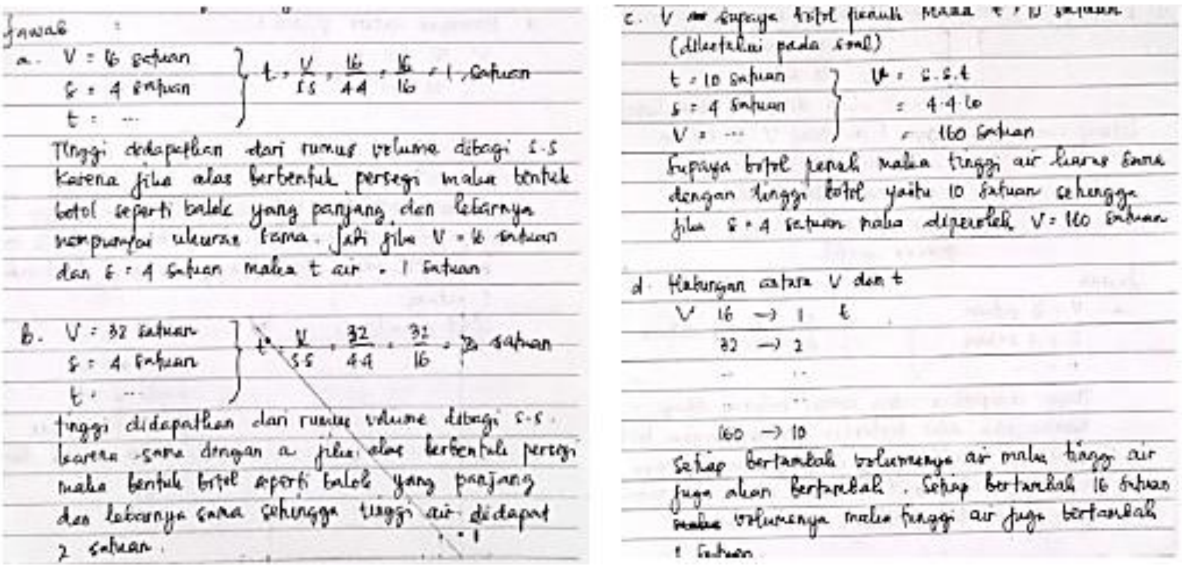


Figure 2. Subject's Answer on the Coordinating Aspect (Number 1)

FINDINGS

Coordination Aspect

The subject determines the magnitude of the change in height and volume of water through calculations using the formula of blocks whose length and sides are the same (Figure2). The subject explained that with every constant addition of water to the bottle, the water level will also increase. The pattern of height changes when water is added constantly, then the height changes constantly as well. This is as written in the interview as follows:

P : *How do you solve question number 1?*
S : *Here you are asked to find the water height when the volume is 16 units, the side length was 4 units, then the height is equal to the volume divided by the area of the base.*
P : *How wide is the base?*
S : *16 units of Bu because the base is square, the area of the side×.*
P : *So obtained how high?*
S : *The height is obtained 1 unit.*
P : *Why you use the formula?*
S : *Now if you imagine this bottle is shaped like a block whose length and width are the same so that the height was obtained from the volume of the beam divided by the sides×*
P : *Now what about question b?*
S : *For problem b it is the same find the height, but the known volume is 32 units and sides the base is 4 units*
P : *So how high is obtained for the question b?*
S : *Height is obtained from the volume divided by the area of the base to meet 2 units.*
P : *You get from which formula?*
S : *Same ma'am as the one just now.*
P : *Now for the c, you are asked what to look for?*
S : *For the c was looking for volume so that the bottle was full.*
P : *How do you solve it?*
S : *I think the bottle will be full if the water level is equal to the bottle height.*
P : *So how high is the water?*
S : *Yes, the water level should be 10 units*
P : *Then the next step how?*
S : *Since what is demanded is volume, then the volume is equal to the side x side x high*
P : *What formula is Mbak?*
S : *The formula of the beam just now, but the length and width are the same, that is, units* $s = 4$.
P : *Then get what volume so that the bottle is full?*
S : *In order for a full bottle to get a volume of 160 units.*
P : *Now continue to the d, what is asked?*
S : *For which d relationship between volume and height.*
P : *How is it related?*

S : *In my opinion, every time the volume of water increases, the water level will also increase. Now if at number 1 every increase of 16 units of volume, then the water level will also increase by 1 unit.*

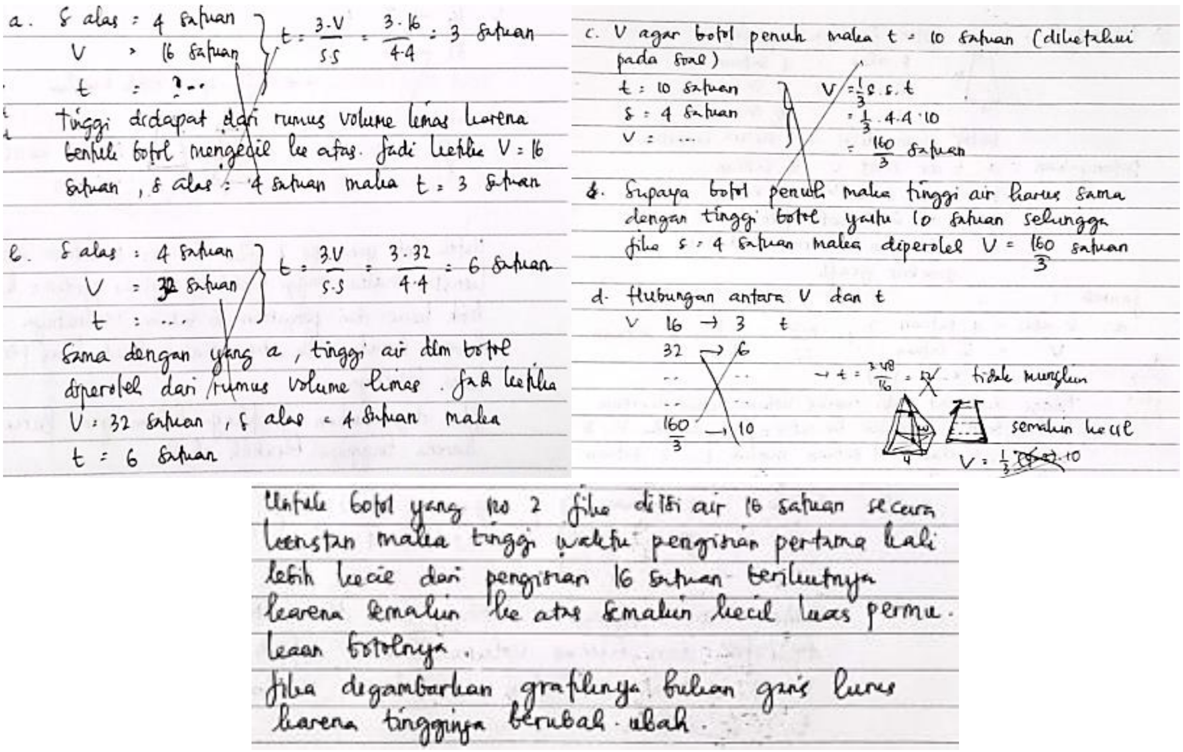


Figure 3. Subject's Answer on the Coordinating Aspect (Number 2)

The subject determines the magnitude of the change in height or volume of water using his understanding of the known shape of the bottle because the subject cannot determine the height or volume of the bottle in point number 2 (figure 3). The subject explained that with every constant addition of water to the bottle, the water level will also increase. The shape of the bottle that is getting more and more narrowed affects the pattern of changes in water height. When filled with water constantly in a bottle whose shape is getting narrower, the change in height will not be the same. This is as written in the interview as follows:

P : *How do you solve question number 2, how come there are so many scribbles here, what does this mean Mbak?*
S : *Okay, ma'am, at the beginning I imagined that bottle number 2 is in the form of a pyramid then I did it for a problem that when the volume is 16 units, the height is obtained 3 units.*
P : *Where are 3 units obtained from?*
S : *From the pyramid volume.*
P : *Why did you cross it out there, it meant wrong or how?*
S : *Nope, ma'am, after I did the b then the c and finally the c problem when I connected the volume and height here, I just understood when the bottle was filled with water then the height change was not the same.*
P : *Why?*
S : *This is the number 2 bottle is tilted.*
P : *Tilt how? Instead of upright the bottle?*
S : *Yes, italic what I mean is that the shape is getting smaller and smaller.*
P : *What does it have to do with water volume and water height?*
S : *Yes, when filled with a constant volume, the height will not be the same because the shape of the bottle is not straight up.*
P : *Well then how high is the water when the volume is 16 units and 32 units earlier?*
S : *That's ma'am, I'm confused, if I use the volume of the pyramid, then when the volume of 48 units is more than 10 units high, it is more than the height of the bottle.*
P : *Then what volume should be used?*
S : *I don't understand, what I understand is the shape of the bottle that narrows at the top and it affects the height of the water in the bottle.*
P : *Meaning for a, b, and c how was it for its height and volume?*
S : *I can't find yet.*

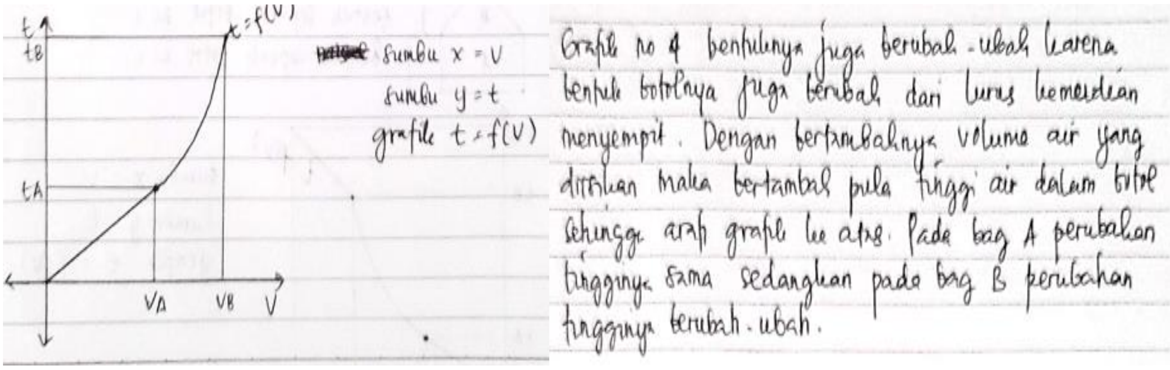


Figure 4. Subject's answer on the coordinating aspect (number 3)

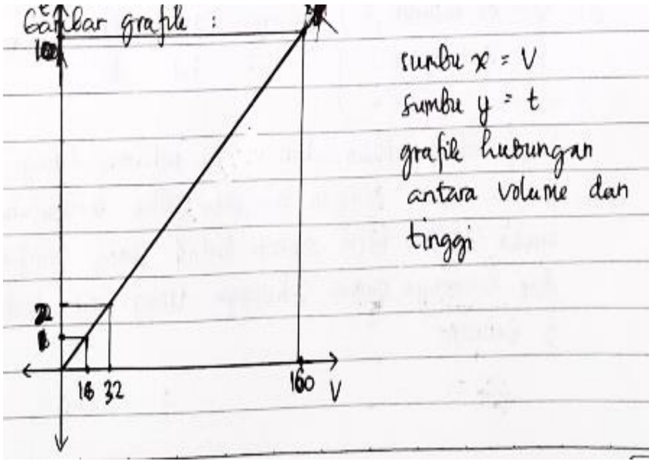


Figure 5. Subject's Answer on the Constructing Aspect (Number 1)

Based on the parts of the bottle or jug made, the Subject formulates the height change of each part of the bottle or jug which will then be used in constructing the graph (figure 4). The subject explained that when a straight bottle or jug the height change will be fixed and for a bottle or jug that is tilted in shape then the height change is not the same. This will affect the graphic shape of the bottle or jug. This is as written in the interview as follows:

P	:	*What is the direction of the chart you are creating?*
S	:	*In my opinion, the direction remains upward because every time the volume of water is added to the bottle, the height will also continue to increase*
P	:	*How does the height change on the bottle?*
S	:	*The straight yes, the height is constant when filled with a constant volume of water.*
P	:	*What about the narrowed?*
S	:	*If it is narrowed, it means that the height changes*
P	:	*Why be fickle?*
S	:	*Yes, because the surface area is not the same, it means that the more upward it gets smaller so that the graph is a curved line as I drew there.*
P	:	*Approximately what caused the shape of the chart to change like that?*
S	:	*In my opinion, it depends on the shape of the bottle, ma'am, to draw the graph:*
P	:	*So, what is the relationship between volume and height of water in bottle number 4?*
S	:	*In my opinion, it depends on the shape of the bottle too, ma'am. So indeed, if all bottles are added water constantly, the height must increase, but in the case of bottle number 4, the height change is not the same in each part of the bottle.*
P	:	*What does it mean?*
S	:	*Yes, if the bottle is straight, the size of the change is fixed, but if the bottle is tilted, the height change is not the same.*

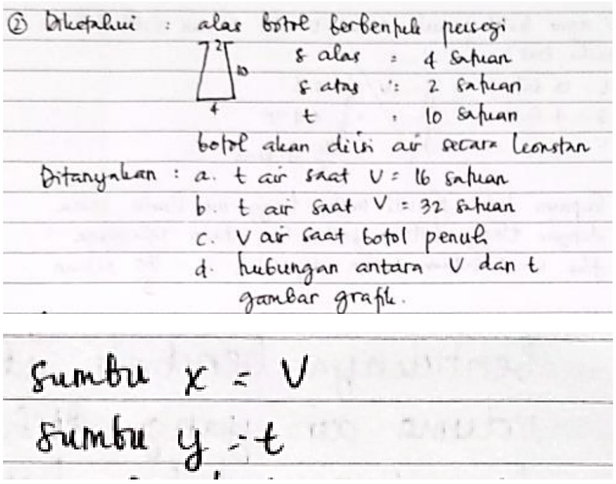


Figure 6. Subject's Answer on the Constructing Aspect (Number 2)

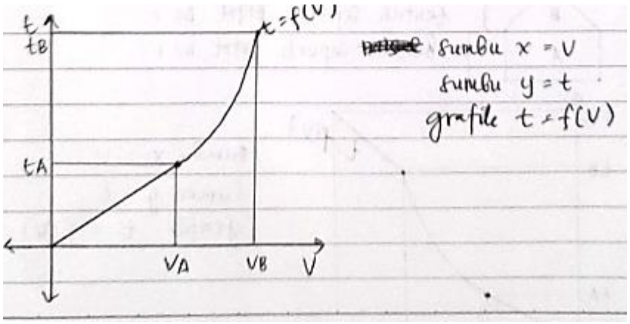


Figure 7. Subject's Answer on the Constructing Aspect (Number 3)

Construction Aspects

The subject represents the relationship of the specified variable, namely and by connecting the dots from the previous question so that a graph in the form of a straight line is obtained (figure 5). The direction of the chart that is made is always pointing up because every time water is added constantly, the height will also increase. The subject also explained that when the bottle is full and water is added again, the graph will be a horizontal straight line because the water level after it is full will remain the same as the height of the bottle. This is as written in the interview as follows:

P : *How do you draw the graph??*
S : *If drawing a graph, first axis x here is V and axis y is t Then I enter the relationship between V and t just now when* $V = 16$ *unit then* $t = 1$ *unit, when* $V = 32$ *unit then* $t = 2$ *unit.*
P : *Then what is the shape of the graph?*
S : *The shape of the graph is a straight line.*
P : *Why?*
S : *Yes, because of the relationship earlier, every time you add the amount of water in Kan, the height will also change in Kan..*
P : *What is the direction of the chart?*
S : *The direction of the chart is upwards.*
P : *Why?*
S : *As each volume increases in height, it will also increase.*
P : *If the volume is 200 units, what will happen to your bottle and chart?*
S : *If the volume is 200 units of water in the bottle, it will spill ma'am.*
P : *What about the graphics?*
S : *Here was the height of the bottle 10 units, so if the volume is 200 units even though the water is spilled but on the graph the water height will still be 10 units.*
P : *How would you describe on the graph if it remained 10 units?*
S : *This means that the graph is a straight line that is horizontal.*

The subject represents the relationship of the specified variable, that is, and through the understanding of the shape of the bottle, so that a graph is obtained in the form of a curved line, the direction of the graph made is always pointing up, because every time water is added constantly, the height will also increase (figure 6). The subject also explained that when the bottle is full and water is added again, the graph will be a horizontal straight line because the water level after it is full will remain the same as the height of the bottle. This is as written in the interview as follows.

P : *Try for number 2, what is the information in question number 2?*
S : *In question number 2, it is known that the shape of the bottle then the base remains square. Here there is a base side length of 4 units, the top side length of 2 units, and the height is 10 units the same bottle will be filled with water constantly.*
P : *Okay, what else is known from question number 2?*

S : *I think that's it.*
P : *What to ask about the question?*
S : *Water height when the volume is 16 units, then water height when the volume is 32 units, then the volume of water when the bottle is full, and the relationship between volume and height and draw a graph.*
P : *How do you solve question number 2, how come there are so many scribbles here, what does this mean Mbak?*
S : *Okay, ma'am, at the beginning I imagined that bottle number 2 is in the form of a pyramid then I did it for a problem that when the volume is 16 units, the height is obtained 3 units.*
P : *Now if you can't find, how to draw the graph?*
S : *First, as number 1 earlier, I suppose the axis x as V and y as t Then I draw the graph like a curved line.*

The subject represents the relationship of the specified variable i.e. and through a graphic image constructed through the parts that the Subject makes in the shape of his bottle or jug (figure 7). The difference in the shape of the graph is influenced by the shape of the bottle, which causes a big change in height is not fixed. The direction of the chart that is made is always pointing up because every time water is added constantly, the height will also increase. The subject also explained that when the bottle or jug is full and water is added again, the graph will be a straight line. This is as written in the interview as follows:

P : *How do you solve it?*
S : *The shape of bottle number 4 is straight at the bottom, ma'am, so it looks like the shape of bottle number 1 then the top is narrowed so that it looks like bottle number 2 earlier.*
P : *How do you create the graph?*
S : *Yes, initially the axis as then the axis as then I draw a graph based on the number 1 earlier, when the shape is straight, it will be a straight line and the narrowed one is a curved line$xVyt$.*
P : *What is the direction of the chart you are creating?*
S : *In my opinion, the direction remains upward because every time the volume of water is added to the bottle, the height will also continue to increase*
P : *Approximately what caused the shape of the chart to change like that?*
S : *I think it depends on the shape of the bottle, ma'am, to draw the graph.*
P : *If you fill the water again, what about the shape of the graph?*
S : *I think it's the same as the previous bottles, the height will be the same as the height of the bottle so the graph is straight horizontally.*

DISCUSSION

In the coordinating aspect, the field dependent subject determines the amount of change in water height to the volume of water given or vice versa through calculations and understanding of the shape of the bottle. After knowing the magnitude of the change of the specified variable, the two subjects determined the pattern of change in water height to see the relationship between the two variables increased, decreased or even fixed. This is in line with opinion (Saldanha & Thompson, 1998; Carlson et al, 2002) which involves the coordinating aspect of changes in two variables in covariation reasoning. The results of this study on the coordinating aspect show that the subject has not constructed the multiplication of objects based on known variables as a result of uniting the measurable properties of simultaneously changing quantities This contradicts the definition of covariational reasoning according to Thompson & Carlson (2017) states that the level of covariational reasoning includes coordination involving the multiplication of objects. The difficulty of the field dependent subject in determining the magnitude of the height change through the calculation of the pyramid formula that is truncated at the end is because the subject does not understand the pyramid formula that is photographed at the end so that it cannot determine the magnitude of the height change in the bottle represented into the pyramid space build. This corresponds to the character of the dependent field which is more global.

In the construction aspect, the subject can present the variable relationship on the covariation problem into a graph. In line with opinion (Johnson et al., 2017; Kertil, 2020) which involves the aspect of constructing covariational reasoning, namely constructing changes in variables in an image and forming a simultaneous picture of the covariance properties of two variables. The subject begins the process of constructing a graph, e.g. the coordinate axis with variables that have been labeled from the covariation problem, determining the points, and connecting the points into a graph of dynamic functions. In the combination of linear and nonlinear covariation problems, researchers combine graphs obtained from previous problems to obtain the form of dynamic event graphs. From the graph depicted, the subject can explain the magnitude of the change and the pattern of change in the relationship between two variables and the direction of the graph made based on the reason. However, the subjects in the study used their way of thinking only for graphic construction without distinguishing between chunky or smooth. Contrary to Castillo-Garsow's opinion, Moore dkk (2013) which characterizes two different images of change associated with covariational reasoning: chunky and smooth. So also, according to Thompson & Carlson (2017) which develops a level of covariational reasoning involving chunky and smooth. If it is associated with cognitive styles in constructing function graphs, visual abilities are needed when building graphs on covariation problems.

The results of this study illustrate the covariational reasoning of mathematics teacher candidate students in terms of cognitive styles that only apply to field-dependent subjects so that these results cannot be generalized. In addition, aspects of covariational reasoning are also limited to aspects of coordinating and constructing so that they can be developed through other aspects of covariational reasoning. In order for all aspects of covariational reasoning to develop well in students in mathematics education, it is necessary to often give students assignments that can measure all aspects in the form of a varied covariation problem.

CONCLUSIONS

The essence of covariational reasoning is on the aspects of coordination and construction. Cognitive style affects covariational reasoning because the coordination and construction aspects require visualization skills in graphing the relationship between two variables. Prospective teacher students with field dependent cognitive styles coordinate large changes in variables and determine patterns of change for linear covariation problems based on calculations from the results of representations of covariation task-based problems to compile patterns of change, for linear students use students' own understanding without doing calculations. In the constructing aspect, the subject represents the relationship of two variables into a graph by determining the coordinate axis with known variables and drawing a graph from the results of calculating the pattern of change in the relationship of two variables. From the depicted graph, the direction of the chart can be determined. The resulting graph corresponds to a given covariation task-based problem.

This study has limitations on aspects of indicators used and a review of cognitive styles used according to aspects of psychology. Thus, it is still open to further research using broader aspects of indicators in looking at covariational reasoning and reviewing different cognitive styles. In addition, the instruments used by researchers are still limited to positive covariations that result in the relationship between the two variables equally increasing, further research can be carried out with instruments in the form of negative or zero covariations. In solving the problem of covariation, researchers do not use APOS theory (Action, Process, Object, Schema), which is a theory about mental construction and about the level of understanding of concepts where the level of understanding of each person's concept varies according to the type of personality possessed, making it possible for further researchers to associate this with APOS theory.

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