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Self-directed learning through gamification of mathematics lessons: Literature review and a framework for enactment

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ABSTRACT
Gamification, or the use of game-like elements for classroom instruction, has been the subject of study by educators. Currently there are no frameworks for enacting gamification in the mathematics classroom. In this paper, we propose a framework for facilitating Mathematics teachers to implement gamification in classrooms. Through gamification, we target to nurture self-directed learners of Mathematics. Our proposed framework is based on a systematic literature review conducted by the researchers. In our proposal, we map the use of various elements of gamification at each phase of learning through to Merrill's First Principles of Instruction and van Hiele's Model of Geometric Thinking. An exemplar of a lesson on geometry with the use of the framework is also presented.

INTRODUCTION

Educators have lamented that priorities of teaching and learning mathematics are focused on excelling in the high-stake national examinations for students. For example, the high-stake national examinations could have resulted in mathematics lessons not being enacted according to the true spirit of problem-solving in Singapore, for which problem solving is the heart of the mathematics curriculum (e.g., Toh et al., 2011). Such a situation might result in students' lack of motivation in learning mathematics (e.g., Lim, 2010).

A lesson that focuses almost exclusively on examinations is usually teacher-centric, and that teachers "depend heavily on textbooks and instructional materials and provide students with a significant amount of worksheets and homework" (Deng & Gopinathan, 2016, p. 459). This seems to be incongruous to inculcating the joy of learning mathematics. As Kenan (2018, p. 37) suggests, "fostering the joy of learning in mathematics can be achieved only if teachers first have the mindset that students are the centre of learning".

The use of game-like elements for classroom instruction, known as gamification, has been the subject of study by researchers (e.g., Karamert & Kuyumcu, 2021; Sailor et al., 2017). We believe that gamification can be introduced to address the lack of motivation among students, especially the low achievers, in the learning of Mathematics among school students. In this paper, by conducting a systematic literature review, we attempt to answer the research question: How can gamification be integrated into classroom geometry lessons to nurture self-directed learners? Further, there is no available framework for enacting a mathematics lesson using gamification. In addressing this gap, we propose a framework for gamification in geometry lessons.

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METHODS

In this study, a systematic literature review was conducted to ensure that the key aspects of the research question were covered thoroughly. The keywords "self-directed learning", "self-directed learning in mathematics", "definition of gamification", "gamification elements", "limitations of gamification" were used to source for existing education literature. We also included the keywords "learning and teaching geometry" for our search since we focus our study on plane geometry.

Both Google Scholar and the ERIC (Education Resources Information Center) Databases were used to search for articles with the above keywords. Citations on these articles were also tracked to other papers which cited the paper to allow to trace for follow-ups of a particular study. The literature review process covered 62 articles published between 1975 to 2023. For self-directed learning, learning in classrooms and motivation and goals, a total of 20 articles were reviewed. For gamification, nine articles were reviewed for its definition, 16 articles for its elements and nine articles for its limitations. Out of the nine articles reviewed for the definition of gamification, six included concrete examples of lessons regarding gamification. For van Hiele's Model of Geometric Thought, one article was reviewed.

Self-directed learning

According to Maslow's Hierarchy of Needs (Maslow & Lewis, 1987), cognitive needs rank as one of the three highest levels of needs. This is consistent with the Self-Determination Theory (SDT) (Ryan & Deci, 2017) which identifies autonomy, relatedness and competence as three innate psychological needs of human beings. When these needs are fulfilled, it is more likely that learners in the classrooms become more self-motivated and independent in learning (Fredricks et al., 2004).

Independent learning, or self-directed learning, is the process in which students set their own learning goals and plan the process of achieving those goals (e.g., Metallidou & Vlachou, 2017; Knowles, 1975). Students also make independent academic decisions and engage themselves in activities to achieve these goals. Self-directed learning has been defined as the acquisition of knowledge by individuals for themselves through utilising available resources and taking charge of his own learning, without the help of others (Herlo, 2017; Din et.al, 2016). In self-directed learning, teachers serve as facilitators of learning, rather than transmitters of knowledge. Students are given the autonomy to choose their own learning resources and strategies, and they actively participate in self-assessment based on the learning objectives they have set for themselves (Knowles, 1975).

Self-directed learners in mathematics usually exhibit dedication, curiosity and independence in the learning of the subject (Bishara, 2020; Sumantri & Satriani, 2016). They also tend to use their social interactions with their peers and teachers to work on their mathematical problem-solving skills and develop higher mathematical abilities (Bishara, 2020). Furthermore, self-directed learners take responsibility for their own learning (Khiat, 2017; Tan & Koh, 2014; Bagheri et. al., 2013) and display the initiative to monitor their own progress in meeting the learning objectives. They also tend to make connections between different disciplines and form relationships between formal and informal education (Khiat, 2017; Tan & Koh, 2014; Bagheri et. al., 2013). Additionally, they are highly inclined to engage in problem identification and are constantly searching for new perspectives of thinking and assigning meaning to what they have learnt (Bishara, 2020).

Classroom teaching

Merill (2002) proposed the application of the First Principles of Instruction in the classrooms. The First Principles of Instruction consists of five principles: the problem-centred principle, activation principle, demonstration principle, application principle and the integration principle.

The problem-centred principle is enacted when teachers engage their students in solving real-world problems. The activation principle comes into play when teachers use review or a quiz to activate existing knowledge as a foundation of new knowledge. Students are introduced to new knowledge via the demonstration principle. The newly acquired knowledge is then applied to solve problems to enhance the learning process, which is known as the application principle. Through the integration principle new knowledge is integrated into the learner's schema.

Types of motivation and goals

According to the SDT, an intrinsically motivated student will likely engage in learning for its inherent satisfaction (Ryan & Deci, 2017). Researchers found that students who are given the autonomy to learn in classrooms are more likely to be intrinsically motivated, more curious and more likely to be challenged by their own learning (Deci et.al, 1981). On the other hand, students who learn in a controlled environment tend to lose the desire to learn or learn less effectively (Utman, 1997).

According to the 2x2 Model of Achievement Goals (Elliot & McGregor, 2001; Pintrich, 2000), students potentially have four types of goals: performance-approach, performance-avoidance, mastery-approach and mastery-avoidance goals (Table 1). Mastery-approach goals are likely to lead to increased engagement (Harackiewicz et al., 1997; 2000) and achievement in academia (Linnenbrink-Garcia et al., 2008), while performance-avoidance goals are not likely to be beneficial to learning outcomes (Maehr & Zusho, 2009).

Gamification

Gamification is defined as the deliberate and planned incorporation of game-like elements into a non-game environment, with the intention to increase students' motivation to learn by making the learning process interesting (Karamert & Kuyumcu, 2021). Gamification does not equate to game-based learning, that is, using a game to learn concepts (e.g., Leong & Toh, 2021). It is a pedagogical approach that is "reflective of a thoughtful approach to integrate characteristics of games into learning through an intentional approach" (Smith, 2018, p. 39). Many researchers have shown that the inclusion of gamification in mathematics classrooms can help to channel students' focus towards the learning of mathematics (e.g., Karamert & Kuyumcu, 2021; Yanuarto & Hastinasyah, 2023), and allow learners of different abilities to participate effectively at their own pace (e.g., Sezgin et al., 2018; Huang & Soman, 2013).

From a psychological perspective, the aims of gamification include internalising extrinsic motivation of students - for students to become intrinsically motivated and self-directed in their own learning - and providing feedback for students of their learning as they progress (e.g., Karamert & Kuyumcu, 2021; Xi & Hamari, 2019; Sailer et al., 2017). Similar to a man who manages to successfully cross a river does not need the raft which he has built earlier to cross the river, the ultimate goal of gamification is for students to not further rely on gamification once they have become self-regulated in their own learning (Nicholson, 2013).

Elements of gamification

There are three possible types of feedback that can be incorporated into gamification - granular feedback, sustained feedback and cumulative feedback. Granular feedback means providing feedback directly to students' actions (Lo & Hew, 2018; Sailor et al., 2017). This can be executed by giving reward points to students when they complete tasks which are representative of a students' learning progress in a gamified environment (Werbach & Hunter, 2015; Sailor et al., 2013).

Sustained feedback involves the tracking of students' progress over time (Lo & Hew, 2018; Sailor et al., 2017) and can be implemented by having a progress bar or a levelling system where the points that the students gain contribute to the progress bar or level (Sailor et al, 2013). Cumulative feedback involves assessing a cumulation of students' actions throughout the course of learning (Rigby & Ryan, 2011; Lo & Hew, 2018). This can be achieved by having a leaderboard to track the points in which each individual student has accumulated. The leaderboard is a form of comparative feedback representing students' ranking in a gamified environment (Codish & Ravid, 2014), with their successes based on a designated set of criteria (Costa et al., 2013). Studies on the effects of gamification have also shown that feedback in the form of points and rewards, immediate feedback and leaderboards could result in greater academic achievement and engagement, satisfaction and enjoyment (Chan et al., 2017) as well as motivation (González et al., 2016).

Acknowledgement in the form of badges or skill levels encourages engagement and provides a clear indication to students of their achievements (Toda et al., 2019). Badges are representations of students' achievement and merit that students acquire in a gamified environment (Werback & Hunter, 2015; Anderson et al., 2013). This will likely lead to increased motivation, engagement (Ding et al., 2017) and academic achievement (Pechenkina et al., 2017) in students.

Table 1
Model of Achievement Goals

	Mastery →	Performance
Approach ↑	**Mastery-approach goals:** Learners with mastery-approach goals focus more on the understanding of what they are learning.	**Performance-approach goals:** Learners with performance-approach goals focus on performing better than others.
Avoidance	**Mastery-avoidance goals:** Learners with mastery-avoidance goals focus more on not losing skills or competence.	**Performance-avoidant goals:** Learners with performance-avoidance goals focus on not looking incompetent relative to others.

Another possible gamification approach is the use of narrative or storyline. Providing a narrative or storyline for students to follow as their learning progresses could also encourage greater engagement in learning (Toda et al., 2019), thereby enabling students to experience meaningful engagement (Rigby & Ryan, 2011).

Other possible gamification elements that can be implemented for classroom instruction include the use of objectives, avatars and time pressure. Objectives promote engagement and motivation as it guides students' learning (Toda et al., 2019). Avatars are pictorial representations of students in a gamified environment (Werbach & Hunter, 2015), and they offer the players a freedom of choice in the gamified environment. This feature indeed fuels their need for autonomy in learning (Annetta, 2010; Peng et al., 2012). Lastly, time pressure allows for students to experience a stressful yet exhilarating environment, and can be used to promote engagement, academic achievement (Spires & Lester, 2016) and motivation (Toda et al., 2019).

Negative responses to gamification

According to Cook (2013), the deliberate incorporation of game-like elements must be seamless. Much thought and intentionality are required to be able to reflect the true characteristics of games while achieving the learning objectives, and to reduce any possible drawbacks. Arnold (2014) proposed that by incorporating badges and points systems in a classroom environment does not necessarily mean that the learning environment is gamified. Hence, designing a curriculum that encompasses gamification can be time consuming and must be done with much deliberation to attain the educational objectives intended (Hanus & Fox, 2015).

Other researchers claimed that excessive use of badges, levels, etc., can lead to students being overdependent on extrinsic motivation, allowing intrinsic motivation to falter in comparison (e.g., Hamari, 2013; Deterding, 2011). Students may be more inclined to achieve more points or a higher level, rather than focusing on what they are learning (Hamari, 2013), resulting in the superficial application of gamification.

Gamification could also lead to “zombification” (Conway, 2014), which is the irrational quest for external rewards. This could also contribute to unnecessary academic stress if students are excessively dependent on it, or recognise their worth by their points, progress and rankings (Juul, 2013; Stott & Neustaedter, 2013). Haaranen et al. (2014) cautioned that some students may become distracted by the interactive elements of the game instead of the content being taught.

The van Hiele’s model

The van Hiele’s model proposes five levels of understanding geometry: Visualisation, Analysis, Informal Deduction, Deduction and Rigour – with the abstractness of geometrical concepts increasing at each level (Crowley, 1987). The model made three assumptions about geometric thought: Firstly, geometric thought is hierarchical. Students must proceed from one lower level to the next higher level in order to function well at a certain level. Secondly, a student's progress through the levels is more dependent on the content and method of instruction rather than age (property of advancement). There is no method of instruction that allows students to skip any level of geometric thought and methods of instruction that facilitate a student's progress is preferred. Thirdly, "the inherent objects at a particular level become the objects of study at the next level of geometric thought" (Crowley, 1987, p. 4). For example, at the first level of visualisation, students may only perceive the form of a geometric figure and its properties are not discussed. Once students proceed to the second level of analysis, students become conscious of the properties of the figure (property of extrinsic and intrinsic). Each level of geometric thought corresponds to its own set of linguistic symbols and their relationships (Crowley, 1987). Consequently, a relation that is classified to be true at one level may be modified at another level. The use of appropriate language and relations is extremely important once students hit the level of analysis. It is noteworthy that the match between teachers' instruction and students' level of geometric thought is crucial. If the instruction is more advanced than that of the students' level, the student may not be able to effectively follow the thought process, leading to the students' stagnation at a certain level. On the other hand, the students' learning will not progress if the instruction is less advanced than the students' level.

FINDINGS AND DISCUSSION

Some key findings

Based on the literature review conducted, we synthesized key learning points in the implementation of gamification in our classrooms to nurture self-directed learning in geometry. Firstly, the teacher has to correctly ascertain the level of Van Hiele's geometric thought that students are working at in order to utilise methods of instruction (mismatch) and language (linguistics) that are suitable to the students' level. The teacher needs to sequence the materials and topic of study, deliberately choosing what to teach and what not to, according to the level of geometric thought (extrinsic and intrinsic). This is based on the assumption of a good knowledge of students' level of geometric thought.

In addition, gamification elements could be used to encourage students to set mastery-approach goals instead of performance-avoidance goals. For example, instead of giving badges for hitting a certain number of points, the badge could be used for rewarding skills based on achieving various mathematical competencies (e.g., such as distinguishing acute and obtuse angles or identifying similar and congruent figures).

We propose that in order to utilise gamification as a pedagogical approach in the classrooms, the teacher must be able to incorporate game-like elements seamlessly. The teacher must be able to pick and choose suitable elements for different class profiles.

In answering our research question to integrate gamification to nurture self-directed learnerswe propose that the teachers align their practices in the classroom to the First Principles of Instruction (Merrill, 2002) and van Hiele's phases of l learning (Crowley, 1987), as shown in Table 2.

Table 2
Application of van Hiele's phases of learning and first principles of intruction in classrooms

First Principles of Instruction	van Hiele's Levels of Learning	Application in Classrooms
Problem-centred Principle: Teachers should allow students to engage in solving real-world problems.	**Inquiry/Information:** At this stage of learning, students are to make observations and ask questions about some geometrical figures while teachers introduce level-specific vocabulary (Hoffer, 1981). This is done in order for the teacher to assess students' prior knowledge and to determine which level of geometric thought that the student is working on.	The teacher can apply both the problem-centred principle and inquiry/information phase by giving students real-life problems with regard to geometrical figures - asking them to make observations and pose questions.
Activation Principle: Teachers activate existing knowledge as a foundation of new knowledge via a review or a quiz.	**Directed Orientation:** At this stage of learning, the students will learn more about the topic of study by being exposed to materials that have been deliberately sequenced by the teachers - in order to slowly reveal the characteristics of geometrical figures suitable for their level of geometric thought. These materials include well-designed short tasks to achieve the learning objective.	The teacher can practise applying activation principle and directed orientation by asking students to recall what they know to explain new concepts that are deliberately sequenced.
Demonstration Principle: New knowledge is being introduced to students and teachers to clarify any doubts that students have.	**Explication:** Students at this phase of learning are encouraged to express their thoughts about the geometrical figures that they have observed. Teachers then act as the facilitator of knowledge - guiding students to use accurate and appropriate language to describe	Both are being applied when the students use their new knowledge to express their opinions on geometrical concepts. Teachers should also guide students' thinking and clarify any doubts that they have at this point of learning.

	geometrical concepts. The purpose of this phase is to allow students to discover their level's system of relations as mentioned in the property of linguistics.	
Application Principle: New knowledge is being applied to solve problems.	**Free Orientation:** At this phase of learning, students will encounter more complex and open-ended tasks. In this way, students will be able to gain experience in finding their own methods in solving problems (Crowley, 1987). By immersing students in investigative tasks, the relationships between the objects of study become obvious to them (Crowley, 1987).	The teacher can apply both the principle and the phase of learning through allowing students to explore novel situations and using their new knowledge to try to solve them.
Integration Principle: New knowledge is being integrated into the learner's schema - teachers should adopt an approach that purposefully draws together knowledge, skills, attitudes and values from within or across subject areas to develop a more powerful understanding of key ideas.	**Integration:** Lastly, the students integrate what they have learnt into their current schema by reviewing and summarising what they have learnt during the lessons. Teachers can assist this process of learning as well.	At this stage, the teacher can allow new knowledge into the students' schema through summaries and reviews.

Table 2. Application of van Hiele's Phases of Learning and First Principles of Instruction in Classrooms

Addressing the potential drawbacks of gamification

In addressing the arguments presented by Cook (2013), Arnold (2014), and Hanus and Fox (2015), we propose a framework to guide teachers in picking and choosing the elements of gamification to facilitate teachers to seamlessly integrate gamification into classroom instruction. Our framework also addresses the concerns of Hamari (2013), Conway (2014) and Haaranen et al. (2014) as our model involves the integration of learning principles such as van Hiele's phases of learning (Crowley, 1987) and Merrill's First Principles of Instruction, instead of relying solely on the principles of gamification, which can potentially result in the superficial application of gamification as argued by Hamari (2013). In this way, we can ensure that elements of gamification in mathematics classrooms are implemented deliberately and suitably at the different phases of learning, with the intention of internalising students' extrinsic motivation.

We propose the facilitation of the setting of mastery-approach goals instead of performance-avoidance goals via the application of objectives, badges and skill levels. Furthermore, students will also be able to monitor their own progress of learning via the elements of granular, sustained and cumulative feedback.

A Framework for Gamification

In succinctly presenting our answer to our research question above, we propose a framework for gamification as shown in Table 3. We next elaborate on the six gamification elements shown in Table 3.

Narrative/Storyline

Narrative/storyline allows for the other elements of gamification to be introduced and integrated seamlessly into the classrooms. Narratives can help to tie together all the other elements and the different phases of learning into one coherent context. Teachers could choose to adopt different storylines for different classes to suit their inclination. Narratives and storylines may be used throughout all the five stages of learning.

Objectives

Explicitly explicated objectives provide students milestones to work with a sense of purpose and direction to work towards these objectives. The objectives set should be coherent with the setting of mastery-approach goals, instead of performance-avoidance goals. Objectives may be used throughout all the five stages of learning.

Avatars

Avatars are crucial for students to be immersed in the narrative/storyline. Teachers can choose to utilise different avatars for different classes. Similarly, avatars may be used throughout all the five stages of learning.

Feedback

The point system can serve to reward the students with points and contribute to the progress bars. The accumulation of points can also allow students to earn badges and increase their levels of competency. With the points system in place, the leaderboards may also be implemented to increase engagement from students as they compete with their peers. Through the point system, progress bars and leaderboards, students will receive granular, sustained and cumulative feedback. Feedback may be used from the Activation & Directed Orientation stage to the Integration stage.

Badges & Skill Levels

Badges and skill levels are the physical manifestation of students' engagement in demonstrating, applying and integrating new knowledge into their scheme. These badges and skill levels should be aligned to the setting of mastery goals. Badges and skill levels may be used from the Demonstration & Explication to the Integration stage.

Time Pressure

Time pressure can be included to excite the students with the learning process when they are applying their newly acquired knowledge. Time pressure can be in the form of short quizzes on

Table 3
Framework for gamification in classrooms

<table>
<tr><th>Phases</th><th colspan="6">Suitable Gamification Elements</th></tr>
<tr><td>Problem-Centred & Inquiry/ Information</td><td rowspan="5">Narrative/ Storyline</td><td rowspan="5">Objectives</td><td rowspan="5">Avatars</td><td></td><td></td><td></td></tr>
<tr><td>Activation & Directed Orientation</td><td rowspan="4">Feedback</td><td></td><td></td></tr>
<tr><td>Demonstration & Explication</td><td rowspan="3">Badges & Skill Levels</td><td rowspan="2">Time Pressure</td></tr>
<tr><td>Application & Free Orientation</td></tr>
<tr><td>Integration</td><td></td></tr>
</table>

concepts that they have previously learnt. Time pressure may be used throughout the Demonstration and Explication, and the Application and Free Orientation stage.

Demonstration of the framework in designing a lesson

We exemplify an application of our proposed framework (Table 3) through the use of a lesson plan (Table 4) for the topic on Congruence and Similarity at the secondary two (age 14) level in the Singapore Mathematics Secondary 2 Ordinary Level Syllabus (MOE, 2019). The lesson plan (Table 4) allows students to experience the five phases of learning (Table 3). Each phase of learning is accompanied by elements of gamification in accordance with the framework.

In enacting the lessons, students are assumed to have prior knowledge on the properties of triangles, including the angle sum of interior angles in a triangle. They are also expected to be able to identify similar and congruent triangles, list properties of similar triangles, and discern and explain the differences between similarity and congruence by the end of the lesson.

Table 4
Framework for gamification in classrooms

Phase of Learning	Lesson Development	Gamification Elements
Problem-Centred & Inquiry/ Information	**Introduction of Similarity and Congruence:** Before the introduction of the concepts of congruence and similarity, teachers first introduce two scenarios that show the importance of similarity and congruence in real-life applications. This helps in piquing the interest of students and relating to what they observe in real-life. Teachers should also seek for students to note down the following: a. What are some possible mathematical concepts I can utilise to solve this problem? b. Are these the most appropriate concepts to use to solve this problem?	**Avatars:** Avatars should have been implemented from the start of the semester or topic. For the context of this lesson plan, assume that students in the classroom have chosen the Zombie theme where students are survivors in a post-apocalyptic world. **Narrative/Storyline:** Scenario 1: Today, you have been tasked by the guards to head out of the safe zone to identify new buildings that can be used as a sentry point. In a world without cameras or batteries, you are only given glass lenses to figure out the exact height of the buildings. How would you do it with mathematical concepts? Scenario 2: On the way out, you also came across a signal tower that is dilapidated and needs to be reinforced. You found some triangle-shaped blocks, but you are not sure how to go about reinforcing the tower. How would you do it with mathematical concepts?
	Introduce Lesson Objectives: After the scenarios have been presented, teachers then make the learning objectives for the lesson explicit by introducing them to the students. At the end of the lesson, students will need to check if they have succeeded in meeting the lesson objectives.	**Objective:** Objective 1: By the end of this lesson, I should be able to determine whether two triangles are congruent or similar. Objective 2: By the end of this lesson, I should be able to describe the properties of similar triangles.

Activation & Directed Orientation	**Recall the Properties of Triangles:** At this point of the lesson, teachers are to ask students to recall what they have learnt about the properties of the triangle. After they have listed down what they have learnt, seek for them to explain what they understand about the terms similar and congruent triangles.	**Granular Feedback - Point System:** As students attempt to answer questions and share their answers for the practices, teachers should practise providing granular feedback via verbal feedback as well as the point system. Points can also be given when students do well in their homework or tests. This point system will directly reward desirable students' actions in the classrooms and motivate them more and others around them to do likewise. These points will contribute towards the progress bars of each individual student. **Sustained Feedback - Progress Bars:** The progress bar is a form of sustained feedback that can be used to monitor students' learning. As points accumulate, students progress through the bar. Once students have reached a certain level of progress, they will be awarded with badges and skill levels that represent their competency in the topic. Teachers can calculate the points accumulated by students every lesson to track their learning progress. **Badges and Skill Levels:** Badges and skill levels represent students' competency in the learning of the topic. The badges and skill levels presented to students should encourage the setting of mastery-approach goals. Some examples of such badges and skill levels for the topic of similarity and congruency are as shown below: *"Skill Level: Congruency and Similarity - Level 1 to 3"*
Demonstration & Explication	**Introduce Similar Triangles** Teachers then introduce the concept of similarity to students. Teachers can choose to either (i) give the students some examples of similar triangles and seek for them to deduce the properties of similar triangles, or (ii) provide them the conditions for triangles to be similar, and then seek students to come out with some examples of similar triangles. Teachers are to also provide feedback in order to guide students' thinking when they are working on this task. **Introduce Congruent Triangles** The concept of congruence will then be introduced to students. Similarly, teachers can choose to either (i) give the students some examples of congruent triangles and seek for them to deduce the properties of congruent triangles, or (ii) provide them the conditions for triangles to be congruent, and then seek students to come out with some examples of congruent triangles. Teachers are to also provide feedback in order to guide students' thinking when they are working on this task.	

Application & Free Orientation	**Practice Questions:** Teachers then provide students with a worksheet and seek for them to identify the groups of identifying groups of similar triangles and groups of congruent triangles based on what they have previously learnt. Students also get to try their hands on the proving of similarity and congruence between triangles. Teachers should provide immediate feedback to correct any mistakes and answer any doubts that students may have. **Deduction of Differences between Similarity and Congruence:** After students have been exposed to enough questions, teachers can seek for students to try to deduce the commonalities and differences between similar and congruent triangles. As students are sharing their answers, teachers are to slowly shape and direct their thinking towards the suggested answers. Along the way, teachers may also share their feedback on students' answers to allow them to understand how their answers and thought process can be improved.	*"Badge: Novice in Congruency and Similarity" (upon achievement of Skill Level: Congruency and Similarity Level 1* *"Badge: Proficient in Congruency and Similarity" (upon achievement of Skill Level: Congruency and Similarity Level 2* *"Badge: Master of Congruency and Similarity" (upon achievement of Skill Level: Congruency and Similarity Level 3* **Time Pressure:** Time pressure can also be added as an element of gamification to increase excitement and thrill in the classrooms. Teachers can provide time pressure when assigning students practice questions (with accuracy of answers as a priority) or when seeking students to deduce the commonalities and differences between the concepts of similarity and congruence.
Integration	**Summary:** To conclude the topic on similarity and congruences, teachers can go through a quick review on the concepts thought in class, as well as the procedures for the proving of similar and congruent triangles. **Homework:**	**Cumulative Feedback - Leaderboards:** Teachers can compute the total number of points that students have accumulated throughout the course of the semester and reward students at the top of the leaderboard. This also helps to track a students' level of competency based on a set criterion across the semester.

	As a form of formative assessment, teachers can allow students to work on the two scenarios that were given at the start of the class as a form of self-directed learning. The possible solutions for the scenarios can be discussed in the following lesson to provide students with a closure to the topic.	

CONCLUSIONS

The idea of using game-like elements in the mainstream classrooms has been described by mathematics teachers of low achieving mathematics students (Toh & Lui, 2014), although the word "gamification" was not articulated, nor was there a concerted effort to design or implement gamification in the mathematics classroom. We believe that gamification is suitable for all students, and not necessarily restricted to low achieving ones, and that proper implementation of gamification in mathematics classrooms can positively impact students' attitudes and behaviour towards learning mathematics. Ultimately, the goal of gamification is to internalise extrinsic motivation provided by game-like elements into students' own intrinsic motivation. The use of gamification as a pedagogical approach should be eventually removed once the students have become self-directed learners in Mathematics.

This study has not been trialled in authentic classroom environments where other realistic factors that facilitate or inhibit teaching and learning could have influenced the enactment of the lesson. However, through this study, we hope to inspire empirical studies on the impact on student learning through the use of gamification in authentic mathematics classrooms.

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