

A task design based review on eye-tracking studies within mathematics education

Marcelo Bairral^{1*}, Gilles Aldon²

¹Federal Rural University of Rio de Janeiro, Brazil

²University of Lyon, France

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ABSTRACT

This paper presents a based review on Eye-Tracking (ET) studies in mathematical problem-solving. ET method provides a promising channel for educational researchers to connect learning outcomes to cognitive processes. Due to the advent of digital technologies, eye tracking studies are increasingly growing in different fields and in mathematics education. The main principle of eye tracking is that our gaze and our focus of attention are connected. If the task provided by the teacher is one of the ways to analyze cognitive processes, we wonder how ET studies are dealing with load cognitive on task design. We captured 39 papers, 23 of which were more specifically analyzed, based on mathematical proposed tasks in three intertwined strands: mathematical problem-solving, affective aspects and reasoning and proofs. We observed an expressive amount of low demand tasks on ET papers, the studies are still mainly based on numerical issues and focused less on geometry topics, and few of them are integrating digital technology on the provided tasks. Our didactical contribution is to claim for more high cognitive demand tasks regarding mathematical processes of thinking and geometrical learning.

INTRODUCTION

Our cognition is blended, embodied, and socially situated (Barsalou, 2008). In this paper we have focused on a part of our body: the eyes. Eye movements are related to a subject's cognitive process, suggesting that the direction of human gaze and the focus of attention are connected, and visual attention can indicate levels of competence in intellectual tasks (Garcia Moreno-Esteva, Kervinen, Hannula, & Uitto, 2020). Eye tracking (ET) studies have a very old origin in connection with the invention of photography. For instance, in the field of psychology we have found the book of Gray (1917) who focused on reading ability; the book explained different methods already used to capture eye movements and gave examples of eye movement recordings (Figure 1). After these initial studies, few studies were carried out and it was the advent of a new technology that rekindled interest in the study of eye movements for learning purposes. In mathematics education, with the advancement of digital technologies, from 2014 onwards, the number of published studies using different eye-tracking devices significantly increased (Strohmaier, MacKay, Obersteiner, & Reiss, 2020).

There are a few literature reviews based on eye-tracking method in different fields and focusing on different time intervals for collecting studies as well (Beach & McConnel, 2019; Deng & Gao, 2022; Holmqvist et al., 2023; Lai et al., 2013; Lilienthal & Schindler, 2019; Mock, Huber, Klein, & Moeller, 2016; Perttula, 2017; Rosch & Vogel-Walcutt, 2013; Strohmaier, MacKay, Obersteiner, & Reiss, 2020). Two of them are in the mathematics education field. Lilienthal and Schindler (2019) focused on papers published in the proceedings of The International Group for the Psychology of

*Corresponding author: mbairral@ufrj.br

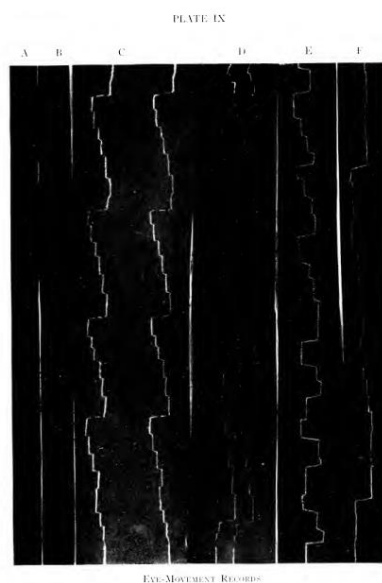


Figure1. Eye movements records (Gray 1917, p. 89)

Mathematics Education (PME) of the last ten years. Strohmaier et al. (2020) reviewed 161 eye-tracking studies published between 1921 and 2018 to assess what domains and topics were addressed, how the method was used, and how eye movements were related to mathematical thinking and learning. We decided to elaborate our review starting from Strohmaier et al. (2020) as it is an exhaustive review that included other publication contexts and investigated a large variety of research areas on mathematics education.

As mathematics education researchers, we agree with Campbell et al. (2009) that we develop cognitive frameworks about mathematical thinking to help better understand matters concerning learning, instruction, and assessment. Nevertheless, we consider that in the cognitive models we must also address didactical issues for classrooms. Our didactical contribution with this article is to claim for more cognitive results regarding learning, mathematical processes of thinking, competences etc., in connection with the collections of data of eye movements, fixation times, gazes, and other parameters of eye-tracking. According to Hannula et al. (2019) we need to study mathematical behavior in ecologically valid ways. But how does research consider the role of the task in ecological learning process? One of the most crucial challenges in eye-tracking research is to properly link eye movements to these assumed underlying cognitive processes (Strohmaier et al., 2020). Based on such cognitive challenge, we wonder how studies in eye-tracking in mathematics education are dealing with cognitive aspects, mainly task design, particularly 1) What can we infer regarding the task demand on eye-tracking in mathematics problem solving papers? 2a) What educational level and main subjects focus on the studies? 2b) What can we observe in terms of topic or mathematics domains approached in the studies? 2c) What kind of tasks are being designed?

In examining the mathematical tasks offered to learners, we therefore restricted ourselves to studying publications presenting mathematical tasks and placing learners in a problem-solving context. We focused on the type of provided task and not its impact on learning. In other words, we did not analyze the studies, nor the learning outcomes indicated by them. We focused on the task statements.

We captured 39 papers, 23 of which were more specifically analyzed, based on mathematical proposed tasks in three intertwined strands: mathematical problem-solving, affective aspects and reasoning and proofs. We didn't classify or make judgments on each described paper. We focused only on the proposed tasks and analyzed them with the concept of task-demand (Stein & Lane 1996) to provide new insights concerning task design in ET futures studies, as well as our research project. We are not evaluating or comparing the papers based on the provided task. Besides the different literature review based on eye-tracking methodology, this article shows the amount of research

being conducted and the diversity of technology and apparatus arrangements used to capture data using this methodology.

Theoretical Framework

Eye-tracking enriching mathematical understanding.

Human cognition is embodied, grounded in the human body, and in its location in space and time (Andrà et al., 2009). In the same way that motions with graphical calculators with sensors (Borba & Scheffer, 2004) and other sensory-motor coordination (Anna Shvarts, Alberto, Bakker, Doorman, & Drijvers, 2021), 3D virtual representation (Zhou, Li, & Bian, 2020) and augmented reality (Paulo, Pereira, & Pavanelo, 2020; Zhou et al., 2020), and touchscreen devices (Assis & Bairral, 2022; Sinclair & de Freitas, 2014), eye-tracking movement integrate our sensory-motor (Nemirovsky, Kelton, & Rhodehamel, 2013) and our gaze behavior in its situational nature (Haataja, Moreno-Esteva, Toivanen, & Hannula, 2018). Eye tracking technology is used to understand individuals' non-conscious, moment-to-moment processes during video-based learning. It's a kind of non-verbal way to gather data and a specialized data analysis program to examine the positions and movements of an individual's eyes as it happens (Deng & Gao, 2022 n).

Eyes are part of the body, and we have known for a long time that human cognition is embodied (Glenberg, 2008; Radford, 2014; Wilson & Golonka, 2013). Understanding what a person pays attention to in a cognitive task such as a recognition, identification, or classification, allows the researcher to better understand the task solving process (Garcia Moreno-Esteva, Kervinen, Hannula, & Uitto, 2020). So it is not surprising to find in the literature that the relationship between eye-tracking and cognitive performance recognition seems to be a shared hypothesis (Andrá et al., 2015; Andrzejewska & Stolińska, 2016; Haataja et al., 2018; Şimşek, Uygun, & Güner, 2020); observing the parameters of eye movements, the fixation time, the dwelling time, the glance duration or the gaze and even the pupil dilation and blinks (Andrzejewska & Stolińska, 2016) all witness the way learners are reading and interpreting texts, graphic, drawings or screens and, in turn, how they learn or reflect on a given task.

Longer fixations are associated with more effortful cognitive engagement with a task: a higher mean fixation duration on a given location indicates a higher level of cognitive engagement with the information at that location (Hodds, Alcock, & Inglis, 2014). In the case of task integrating the use of GeoGebra suggested that the long fixations are related both to instrument manipulation and cognitive processes (Hannula, Toivanen, & Garcia Moreno-Esteva, 2019). According to the authors, these results suggest that using a digital tool increases the amount of both automatic scanning fixations and long fixations related to more elaborated processing. However, the qualitative examination of the long gazes in context suggests that a significant amount of these long fixations are related to interacting with GeoGebra, for example when selecting an option from a drop down menu or using mouse to place an object in the coordinate system.

For instance, concerning ET focusing on reading when the input is text, dwell time and number of fixations have values that make text close to formulas (namely, quite low), but fixation duration is similar to graph (low). The authors infer that text in the input is attended for a shorter time, both in terms of duration and in terms of the number of times the students come to see it (Andrá et al., 2015). Another example regarding fixations showed that tables concentrated students' attention on the dependent variable data, whereas diagrams distributed students' attention evenly across the numeric and visual elements of the task (Xolocotzin, Inglis, & Medrano, 2020). The authors claim that an interpretation of results from a cognitive load framework would have been problematic and that diagrams should have produced less functional answers because they have more information and require more cognitive resources than tables. Research by Soares, Lukasova, Carthery-Goulart, and Sato (2021) presented studies showing differences between people with high- or low-performance strategies during a reasoning task. Although they observed low performance students had difficulties in both reading and specific mathematics contents, they claim that the grade does not necessarily represent performance. According to them ET can facilitate interpreting the most prominent explanation for students' difficulties and, without this equipment, some teachers would not have identified such problems.

From a socio-constructivist perspective, interactions in the classrooms, both among students and students with teacher, participate in learning and teaching. Group work plays an important role in engaging students in reflection by allowing them to help each other (Dillenbourg, 1999; Kanav et al. 2009; Johnson et al. 2014). In this context, the use of ET could be helpful to better understand the interactions in the classroom as well as the effect of the teacher's gaze on the students' learning process. Haataja et al. (2018) study through ET the amount of eye contacts which carries with it high dwell times of face-targeted gazes. Teachers can talk about different ways of interacting with students facing the same mathematical problem (Soares et al., 2021). But they can also analyze their students' visual attention in problem-solving processes to make a prediction about their difficulties and help their students increase their problem-solving performance and academic achievement (Şimşek, Uygun, and Güner (2020).

Eye-tracking method provides a promising channel for educational researchers to connect learning outcomes to cognitive processes (Lai et al., 2013) and in agreement with Wilson and Golonka (2013) research programs based on embodiment cognition should follow - as a first step - a task design analysis. Since most results on the paper in our review are mainly focused on the eye-movements, we could add reflections concerning the demand of the task to enrich the studies on eye tracking. Considering the advance of digital technology and the devices to gather a great amount and detailed data from gaze we think it could be important to reflect about the kind of proposed task.

Cognitive task demand in eye-tracking studies

Tasks are mediating strategies for teaching and learning mathematics, and the way in which they are designed and used for learning and knowledge production is considered by researchers (Watson & Ohtani, 2021). In mathematics education, the concept of task is broad and covers a wide variety of activities: lists of exercises, construction of geometric objects using dynamic geometric environment, construction of handling models of geometric solids, providing examples of definitions or statements, problem-solving, carrying out an experiment or investigation, games, etc. (Bairral, 2021).

The relationship between teaching and learning that links dimensions of instructional tasks with gains in student learning outcomes was examined using a conceptual framework based on cognitive task demand (Mary Kay Stein & Lane, 1996). According to the researchers, the greatest student gains on a performance assessment consisting of tasks that require high levels of mathematical thinking and reasoning were related to the use of instructional tasks that engaged students in the "doing of mathematics" or the use of procedures with connections to meaning. In addition, student performance gains were greater for those sites whose tasks were both set up and implemented to encourage the use of multiple solution strategies, multiple representations, and explanations. Whereas student performance gains were relatively small for those sites whose tasks tended to be both set up and implemented in a procedural manner and that required a single solution strategy, single representations, and little or no mathematical communication (Mary Kay Stein, Engle, Smith, & Hughes, 2008, M. K. Stein, Smith, Henningsen, & Silver, 2000).

Based on the framework of Stein and Smith, (1998) and of Stein, Smith, Henningsen, and Silver, (2000), In this document, we have associated the two levels of cognitive demand for the proposed tasks. Speaking of "level" does not imply comparing the tasks in terms of better or worse, easy or difficult but rather to point out the mathematical concepts and procedures involved in the tasks. The levels serve only to distinguish the cognitive load of the tasks, and not to personify abilities and understandings (Stein & Smith, 1998). In other words, levels don't refer to classifying better or worse learning, but the possibility of mathematical thinking processes which could be achieved when solving a task. It's not a question of thinking in terms of levels of learning, but of the cognitive load that involves time consumption and complex thinking. (Moutsios-Rentzos & Stamatis, 2015). Demand implies reasoning, emerging strategies, and different kinds of representations (graphical, pictorial, table, algebraic, geometrical etc.)

Table 1
Cognitive demands and/or thinking processes in mathematics tasks

Demand	Level	Kind	Brief description
High	2	Doing mathematics	The use of complex, non-algorithmic thinking to solve a task in which there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example. "Doing mathematics" processes are often likened to the processes in which mathematicians engage when solving problems.
	1	Use of procedures with connections to concepts, meaning, and/or understanding.	The use of a procedure in a manner that maintains and/or develops deep levels of understanding of mathematical concepts and ideas. Although students follow a suggested pathway through the problem, the pathway tends to be a broad, general procedure that has close connections to underlying conceptual ideas as opposed to a narrow algorithm that is opaque with respect to underlying concepts.
Low	2	Use of Procedures without connections to concepts, meaning, and/or understanding.	The use of a procedure to solve a problem with no attention to why or how the algorithm works. Usually, the procedure is a well-rehearsed algorithm with limited, if any, connection to underlying mathematical ideas. Little cognitive effort is required for successful completion of the task because the algorithm is either specifically modelled prior to task implementation or its use is evident based on prior instruction, experience, or placement of the task.
	1	Memorization	Either reproducing previously learned facts, rules or definitions or the process of committing facts, rules, or definitions to memory. Such tasks are non-ambiguous and have little or no connection to concepts or meaning.

Source: From Stein and Lane (1996, p. 58-59)

METHODS

We decided to focus our review on mathematical problem-solving (PS) and task design with digital technology because this is one of our research field interests; there is a group at IREM of Lyon (called DREAM group) focused on PS, and in state schools in Rio de Janeiro there is a school subject called Problem-Solving. Such courses demand pedagogical contribution for teachers and curriculum developers, mainly about mathematical tasks, and integration of digital resources. Although in mathematics education literature there are some differences between tasks and problems, in this article we didn't consider such conceptual singularity. The steps we followed in our review of papers using eye-tracking are:

1. Mathematical problem-solving studies.
2. Affective variables studies.
3. Studies of joint attention.
4. Reasoning and proof studies.
5. Papers from cited references on the four steps or randomly, but only papers based on problem solving.

At first, we were interested in analyzing tasks provided in mathematical problem-solving studies only based on Strohmaier et al.'s (2020) review, but the results inspired us to generate the following 4 steps. This was not a linear and predictable process. In other words, the initial interest drove the selection considering issues related with problem-solving such as affectivity (step 2), joint attentions (step 3) and proving (step 4). The steps 1, 2 and 4 are based on papers from Strohmaier et al. (2020). To refine our analytical procedures, we must explain the criteria of inclusion and exclusion of any articles in our study (Randolph, 2009).

Criteria of inclusion and exclusion of paper

In all five steps, we didn't consider or classify the studies according to the eye-tracking device used and/or regarding the theoretical framework adopted. The main exclusion criterion was when the article didn't present a provided mathematics task. Articles in our review are based on eye-tracking studies using explicitly the whole mathematics tasks or part of them. In that case, part of the task presented allows us to infer about the task demand, for example, in Campbell et al. (2009) and Espino, Santamaría, Meseguer, and Carreiras (2005) studies, or in a paper we saw the provided tasks in the analysis section (Şimşek et al., 2020). If the paper comes from the same research project or team – using the same task – we have considered the article which contains the proposed task (Salminen-Saari et al., 2021). When the paper presents mathematics and non-mathematics tasks, we consider only the mathematical one (Werner & Raab, 2014).

According to Strohmaier et al. (2020) reviewing problem solving was examined in four studies (2%), investigating relations between eye movements and objective/subjective task difficulty (Andrzejewska & Stolińska, 2016), insight problem solving (Knoblich, Ohlsson, & Raney, 2001), and the association of body movements and problem solving processes, assuming effects of embodied cognition (Werner & Raab, 2014), and the use a collaborative problem-solving task to investigate a teacher's attention during scaffolding (E. Haataja, Moreno-Esteva, Toivanen, & Hannula, 2018).

Since affective factors represent an important aspect of learning throughout mathematics problem solving, we decided to analyze the two studies linking eye movements to mathematics-specific affective variables, namely mathematical self-concept (Strohmaier et al., 2017) and mathematics anxiety (Hunt et al., 2015). The provided tasks are only indicated in the last study.

Since only one (Haataja et al. 2018) of the six papers found in steps 1 and 2 were from mathematics education, we thought that finding other related papers from their research team to know more about the problem provided by them could be helpful in our task analysis. On step 3 we found eight more papers from Hannula's research team focusing on joint attention. Six of them refer – explicitly on paper or in some reference – to a problem of four cities¹ and we supposed in M. S. Hannula, Toivanen, and Garcia Moreno-Esteva (2019) that they also used the same problem. We use all the papers related to the cable cities problem and recurring on Heyd-Metzuyan et al. (2023) because it presents detailed students' solutions during various stages of the lesson.

Inspired by papers from the third step based on the four cities, which provided us insight about the kind of task provided and considering such type of task is usually proposed on studies in proofing, we decided on step 4 to look back in Anselm R. Strohmaier et al. (2020) now focusing on the twelve papers concerning reasoning and proof. In this strand we excluded the paper from Panse, Alcock, and Inglis (2018) because it includes the same tasks as those in Inglis and Alcock (2012). Concerning the two-paper gathered on proportional reasoning, we consider Abrahamson, Shayan, Bakker, and van der Schaaf (2015) because it presents detailed design of the task shortly commented on Duijzer, Shayan, Bakker, Van der Schaaf, and Abrahamson (2017). Papers concerning reasoning and proof, in this strand we excluded the paper from Panse, Alcock, & Inglis (2018) because it includes the same tasks as those in Inglis & Alcock (2012). Concerning the two-paper gathered on proportional reasoning, we consider Abrahamson, Shayan, Bakker, and van der Schaaf (2015) because it presents detailed design of the task shortly commented on Duijzer, Shayan, Bakker, Van der Schaaf, and Abrahamson (2017).

Finally, on **step 5**, we selected randomly other papers that focused on eye-movements and still referring to mathematical problem solving which explicitly show the task or part of it, mainly a mathematical task, even if the focus was not strictly linked to learning or teaching mathematics such as Andrzejewska and Stolińska (2016). In Kosko (2022) we encounter more details about the task shortly commented in Kosko, Austin, and Zolfaghari (2023).

Based on such criteria of inclusion and exclusion, we generated [table 1](#) with the description of the number of papers included or not in our review concerning task demand analysis. All studies are important in the theoretical framework and for future analysis.

¹ The students were asked to find the shortest possible way to connect four cities located at the vertices of a square. This problem is the four-point version of the Steiner tree problem.

Table 2
Detailed steps description of our task-based review

Domain	The aim of study	Author(s)	Included (I) and not included (E) paper
Step 1: Problem solving studies (4 papers)			
Mathematical problem solving	To test three specific implications of these hypotheses against eye movements recorded while participants solved matchstick arithmetic problems.	Knoblich et al. (2001)	I
	To analyze the association of body movements and problem-solving assuming effects of embodied cognition.	Werner and Raab (2014)	I
	To investigate relations between eye movements and difficulty.	Andrzejewska and Stolińska (2016)	I
	To investigate the teacher's attention during scaffolding a collaborative problem-solving task.	Haataja et al. (2018)	E
Step 2: Affective variables studies: anxiety and self-concept (2 papers)			
Affective variables	To measure a range of eye movements of undergraduate students in response to performance on an arithmetic verification task (mathematics anxiety).	Hunt et al. (2015)	I
	To analyze the influence of mathematical self-concept on university students' eye movements in reading mathematical PISA items in a mathematical and a problem-solving context.	Strohmaier et al. (2017)	E
Step 3: Studies from joint attention (more 8 papers)			
Mathematical problem solving using a task adapted from Steiner point	To investigate the relation between a teacher's scaffolding intentions and his gaze behavior.	Eeva Haataja, Garcia Moreno-Esteve, et al. (2019)	E
	To explore the frequency of teacher-student eye contacts and their connection to teachers' scaffolding intentions.	Eeva Haataja, Toivanen, Laine, and Hannula (2019)	E
	To examine student eye movements during collaborative geometry problem solving using GeoGebra.	M. S. Hannula et al. (2019)	E
	To examine joint attention in collaborative mathematical problem solving.	Salminen-Saari et al. (2021)	I
	To investigate six teachers' gaze patterns when they are giving task instructions for a geometry problem in four different phases of a mathematical problem-solving lesson.	Maatta et al. (2021)	E
	To compare and analyze two research settings that use the latest video technology to capture classroom interactions in mathematics education.	Markku S. Hannula et al. (2022)	E
	To analyze a particular group that was ineffective in their problem-solving process.	Heyd-Metzuyanin et al. (2023)	E

Table 2 (Continued)

Domain	The aim of study	Author(s)	Included (I) and not included (E) paper
Task concerning the observation of characteristics of two bird species in biology education	To discuss a network model to represent the gaze tracking data in a way that is meaningful for the study of students' biodiversity observations.	Garcia Moreno-Esteva, Kervinen, Hannula, and Uitto (2020)	E
Step 4: Reasoning and proof studies (14 papers)			
Reading and comprehension of proofs	To present comparison of the proof validation behavior of beginning undergraduate students and research active mathematicians.	Inglis & Alcock, (2012)	I
	To report three experiments demonstrating that a simple booklet containing self-explanation training, designed to focus students' attention on logical relationships within a mathematical proof, can significantly improve their proof comprehension.	Hodds et al. (2014)	I
	To present an intervention designed to help undergraduates comprehend mathematical proofs.	Roy, Inglis, and Alcock (2017)	I
	To contribute to the debate about whether expert mathematicians skim-read mathematical proofs before engaging in detailed line-by-line reading.	Inglis & Alcock (2018)	E
	To investigate possible differences among experts and novices in attention allocation, in cognitive demand and in the mathematical reading process.	Panse et al. (2018)	E
Proportional reasoning	To describe how the coordination of action and perception stimulated students' progression from additive to multiplicative solution strategies.	Duijzer et al. (2017)	E
	To investigate eye-gaze behaviors during engagement in solving tablet-based bimanual manipulation tasks designed to foster proportional reasoning.	Shayan, Abrahamson, Bakker, Duijzer, and van der Schaaf (2017)	I
Probabilistic reasoning	To adopt consistent strategies in these Bayesian reasoning problems and investigated the nature of these strategies. In two experiments, one laboratory-based and one internet-based, each participant completed 36 problems with factorially manipulated probabilities	Cohen and Staub (2015)	E

Table 2 (Continued)

Domain	The aim of study	Author(s)	Included (I) and not included (E) paper
Logic	To examine the role of statistical numeracy for effects of such fallible but adaptive inferences on choice behavior.	Fleig, Meiser, Ettlin, and Rummel (2017)	E
	To examine the effects of the difficulty of the problem (simple versus complex problems) and the type of figure (figure 1 or figure 4) on the time course of processing categorical syllogisms.	Espino, Santamaría, Meseguer, and Carreiras (2005)	I
	To discuss the differences in the two classes (expert and non-expert), both with respect to the correctness of responses to the problems and the structure of the scanning and identification of important components within the problem.	Kim, Pollanen, Reynolds, and Burr (2018)	I
	To examine different combinations of representations (text, formula, graphic) in the field of propositional logic.	Ott, Brünken, Vogel, and Malone (2018)	I
	To analyze the relationship between practiced monitoring activities and performance.	Cohors-Fresenborg, Kramer, Pundsack, Sjuts, and Sommer (2010)	I
Functional thinking	To investigate how mathematics graduates and engineering undergraduates studied sequential tables of values with the aim of deriving a function (limited to linear and quadratic function).	Crisp, Inglis, Mason, and Watson (2011)	I
Step 5: Other studies on problem solving (11 papers)			
Mathematical problem solving	To seek a better understanding of cognitive processes associated with geometrical image-based learning.	Campbell et al. (2009)	I
	To highlight the ongoing process of making sense of mathematical representations during problem solving.	Andrà et al. (2009)	E
	To examine how different mathematical representations of the same mathematical object are attended to by students.	Andrá et al. (2015)	I
	To investigate the students' thinking about 'change' and 'compare' problems, as indicated by their eye movements. _	Moutsios-Rentzos and Stamatis (2015)	I

Table 2 (Continued)

Domain	The aim of study	Author(s)	Included (I) and not included (E) paper
	To analyze the communicative elements that help the tutor-student dyad to sustain joint attention within a teaching-learning activity and explores the progression from joint visual attention to joint mental attention during the collaboration.	Shvarts (2018)	I
	To determine whether geometric skills of the children in rural preschools are at the same level as those of their peers in urban preschools.	Nazaruk and Marchel (2019)	I
	To propose a model for mathematics achievement considering the mediating role of eye tracking measurements in the relationship between problem solving performance and mathematics achievement	Şimşek, Uygun, and Güner (2020)	I
	To investigate the effectiveness of tables and diagrams for supporting covariational reasoning amongst elementary students.	Xolocotzin, Inglis, and Medrano (2020)	I
	To explore the teachers' view of ET videos recorded while students solved mathematical problems (number line and operation using money representation).	Soares, Lukasova, Carthery-Goulart, and Sato (2021)	I
	To examine whether, and how, holographic representations of practice affected PSTs' professional noticing of children's fraction reasoning.	Kosko (2022)	I
	The report on an initial exploration of the relationship between these two constructs using eye-tracking technology and the PCK fractions measure.	Kosko et al. (2023)	I
Total	Amount of captured papers	39	
	Amount of included papers in the research question concerning task demand	23	

FINDINGS

The results are grouped to answer the two research questions, **RQ1** concerning the task demand on eye-tracking in the selected papers, and **RQ2** about the educational level and main subjects focus on the studies, mathematics domains approached, and task-design.

RQ1: About our inference regarding the task demand in selected papers.

We are not presenting many details of data collecting settings, considering them to categorize the task. We are interested in focusing only on the type of the provided mathematical task. Some tasks could be of low demand but depending on the classroom dynamic, the semiotic mediation and interaction among teacher and students, they could make another version of the task arise (based on the first) which could be in a higher level of demand. To avoid comparing the studies we decided not

to consider the dynamic for collecting data. In other words, we did not analyze the task demand related to the data collection setting.

In the amount of analyzed papers we found 4 high demand tasks level 2 and 3 level 1. In the low level we found 8 tasks in each level. It is a significant amount of low demanding tasks.

High demanding task level 2

Tasks provided in HD1 and HD2 normally do not give the solutions or procedures explicitly. They also explore different representations and mathematical thinking processes, not only based on memorization or shape identification. The tasks captured in this level aim to observe regularities, reading statements with self-explanation, contrasting, and using different ways of interpreting mathematical concepts and some tasks are not usual in the classroom.

Synthesis of the results (HD-L2)

In the different studies, dwell and time duration were used with the hypothesis that dwelling indicates a particular attention significant to a cognitive effort on a particular point. In the case of reading proofs, dwell time is used to compare the way students read and understand what they read. It allows a better cognitive engagement of students trained to a specific behavior in one case (Hodds et al. 2014); and it allows one to better understand the difference of behavior when reading a mathematical proof on a screen or in a textbook (Roy et al. 2017). The possible students' failures with proofs are due not to some inherent intellectual incapacity. Eye-tracking results indicated that undergraduate students do have at least some of the skills and understanding they need to read proofs effectively, and that a light-touch intervention can lead to better mobilization of these skills and thus to considerably better proof comprehension. They argue that 'transition to proof' courses should incorporate self-explanation training. On the other hand, eye-tracking allows us to compare the students' engagement regarding the support of the reading. Even if authors reported that it is risky to evaluate learning innovations using only students' feedback. The two papers from Haataja et al. (2018) and Hannula et al. (2019) are using eye-tracking to better understand the interactions between students in a group work or between teacher with his/her students; results showed that teacher's gaze behavior is of situational nature. In the analysis of the events, the teacher either initiated eye contact without students' response, where he did not seek eye contact at all, or where he established successful eye contact with students. The amount of eye contact carries with it high dwell times of face-targeted gazes. Nevertheless, the studies do not explain the reasons for the small amount of eye contact. However, we can suggest that it relates to an already established teacher-student relationship and/or the novelty brought about by the gaze tracking glasses on the teacher's face.

In the comparison of solving a task with pen and pencils and with the help of a DGS, it appears that using technology influenced the duration of gaze fixations: slightly more short fixations, less medium length fixations, and clearly more long fixations. A more detailed analysis suggested that the long fixations are related both to instrument manipulation and cognitive processes. These results suggest that using a digital tool increases the amount of both automatic scanning fixations and long fixations related to more elaborated processing. However, the qualitative examination of the long gazes in context suggests that a significant amount of these long fixations is related to interacting with GeoGebra, for example when selecting an option from a drop-down menu or using a mouse to place an object in the coordinate system.

High demanding task level 1

HD1 tasks mainly use the procedure in a way that maintains a deep level of understanding when students follow a suggested path to solve the task, either because it comes from their course or because it is natural at this level of teaching. Three articles have been selected in this category and are analyzed below.

Synthesis of the results (HD-L1)

In these three papers eye-tracking is used to better understand how learners can construct relations between objects or between representations of an object. In that sense the papers consider a semantic interpretation of eye movements based on mathematical understanding of objects, say the concept of function or the concept of multiple representation of a mathematical object. Authors

indicate that eye-tracking cannot present the whole story but gives information that can be crossed with mathematical or semiotics reflections. Findings describe how learners behave but do not provide insight into the reasons for their behavior. The question is asked in Crips et al. (2011): "what determines what strategy a participant adopts when tackling such problems?" (p. 39).

Visual attention of learners is correlated with dialogues and gestural expressions (Shvarts 2018) which can be interpreted as a joint mental attention. Eye movements do not follow but rather anticipate the gesture along the side of the triangle thus joining visual and gestural expressions in anticipating perception. The two participants may acquire partial independence: joint mental attention may supersede joint visual attention thus freeing the visual system for other tasks and serving future independence of the tutor's and the student's activity.

When the input is text, dwell time and number of fixations have values that make text close to formulae (namely, quite low), but fixation duration is like graph (low). We can infer that text in the input is attended for a shorter time, both in terms of duration and in terms of the number of times the students come to see it. The degree of understanding of graphs or formulae is not the same which can indicate that students would prefer to consider a formula or a text instead of a graph to find information. It would also indicate that a necessary and deep teaching of reading a graph is important for a good understanding of mathematical concepts.

Low demanding task level 2

The main difference between tasks from low demand 1 (LD1) and low 2 (LD2) is that in LD2 the participants had the opportunity to explain or make comments about their answers. Beyond solving the task on the screen, the study provides other resources (interview, questionnaire, self-monitoring, audio, or small video) to better understand subjects' comprehension.

The tasks captured in level 2 aim to explore procedures without connections or tasks based on visual perception and identification of shapes or statements on screen. We found the following mathematical topics: addition and subtraction, proofreading, placing numbers on a real line, calculation using money representation, algebraic manipulation, and some usual classroom problems.

Synthesis of the results (LD-H2)

Eye-tracking in these articles is used generally to highlight learners' understanding of a particular task. Some articles compare the behaviors of different learners monitoring activities and indicate that the willingness for monitoring is deeply anchored in a person (Cohors-Fresenborg et al., 2010); Soarès et al. (2021) compare eye-movements of low and high performers looking at the eye gaze interpreted in terms of difficulty or specific attention for students. Some other articles compare through eye-tracking, the behaviors of learners and experts, (Inglis & Alcock, 2012) who compared undergraduate students and mathematicians when looking at the validity of a proof; Soarès et al. (2021) also compare students' behavior with the teacher's predictions, and they highlight the benefits for teachers of observing pupils' behavior through the study of eye-movements. The relationship between eye-movements and fixation times or eye paths and the cognitive efforts being made is emphasized in most of the papers. Espino et al. (2005) and Ott et al. (2018) consider the understanding of logic sentences, the first part of the sentence show that eye-tracking technique confirmed the predictions of the model theory of reasoning, which posits that the figure (the way and the order one presents the possible solution, for instance, A-B or B-A) operates in the integration of information to construct the models; the second take profits of gaze behavior and gaze shifts to understand how multiple representations (text, formula, graphics) were prioritized. Moutsios-Rentzos et al. (2015) are considering eye-movements of grade 1 students when they think and study the correlation of these eye-movements with activation of brain hemispheres; Shayan et al. 2017 foster the development of proportional reasoning through eye and hand movements, considering the close relationship between mathematical thinking and embodiment. Which is also the purpose of Werner and Raab (2014) who consider eyes movements as a particular case of body movements in the process of solving a problem.

Table 3
Overview of high demand level 2 papers.

Author(s)	Subjects	Proposed task(s)
High demand level 2		
Hodds et al. (2014)	76 mathematics undergraduates. All participants were studying mathematics in 3-year single- or joint- honors degree programs	From the proofs of number properties, students must show their understanding; two groups, a control group and a group trained to self-explanation are compared regarding comprehension of proofs.
Roy et al. (2017)	43 undergraduate students	Proofs designed for beginning real analysis. Students studied an e-Proof, or a standard written proof and their comprehension was assessed in both immediate and delayed tests.
Salminen-Saari et al. (2021)	Four 15–16-year-old students within a 22 student 9 th class.	Geometry problem to find the shortest possible way to connect four imaginary cities with electrical cable, located at the vertices of a square. Appendix 1
Xolocotzin et al. (2020)	57 students in Grade 4, 5 and 6	There were 12 tabular tasks and 12 diagrammatic tasks. In each type of task, there were four items involving sums, four items involving subtractions, and four items involving multiplications. The same as in the previous study, each task required the identification of a relationship between two quantities, and selecting one of four response options: functional, recursive, first instance, or random.

Table 4
Overview of high demand level 1 papers.

Author(s)	Subjects	Proposed task(s)
High demand level 1		
Crisp et al. (2011)	16 participants (8 mathematics graduates and 8 first year undergraduate engineering students)	Eight generalization problems of varying difficulties given a table of values. Appendix 2
Andrá et al. (2015)	46 undergraduate students recruited based on their different backgrounds in mathematics	An inequality sentence with 5 areas of interest (statement) is given.
A. Shvarts (2018)	Four student-tutor pairs took part in the research. School level is not given. No <i>a priori</i> analysis of the task.	The task is proposed in two stages: first, it is a question of highlighting a curve based on a geometric property (locus of points equidistant from a point and a line) and then of finding the functional relationship associated with this curve.

The second aspect corresponds to the methods used, which are all laboratory methods, in which selected learners are observed solving problems. Quantitative methods measure fixation times, gazes, dwell times when qualitative ones focus on describing the eye-movements, but both interpret the data in terms of cognitive engagement and compare results with complementary perspective to didactic or psychological analyses.

Table 5
Overview of low demand level 2 papers

Author(s)	Subjects	Proposed task(s)
Low demand 2		
Espino et al. (2005)	University students	sylogistic reasoning statements Appendix 3
Cohors-Fresenborg, Kramer, Pundsack, Sjuts, and Sommer (2010)	10th to 13th graders, differing in their mathematical performance	equivalence algebraic statement
Inglis and Alcock (2012)	18 first-year undergraduate students studying either single-honors or joint-honors mathematics, and 12 academic mathematicians, all from university	One proof about integration and many in number field.
Werner and Raab (2014)	Seventy-two participants (from 19 to 29 years) without knowledge background information on the paper.	the water-jar problem (experiment 2)
Moutsios-Rentzos and Stamatis (2015)	Grade 1 (6 years old)	six-word arithmetic problems:
Shayan et al. (2017)	elementary- and vocational-school students (9-15 years) participated in individual task-based clinical interviews	Task-oriented sensorimotor interactions with a tablet device.
Ott, Brünken, Vogel, and Malone (2018)	146 university students	25 problems from the field of elementary propositional logic
Soares et al. (2021)	19 participants from 5th grade in a primary school, and teachers	Two Multiple choice questions: place number on the line and arithmetic operation using currency.

Low demanding tasks level 1

The tasks captured in this level are mainly focused on memorization, verification of some result, computation using numbers and without context, finding known results, identification or perception concerning some geometric object or shape. Tasks captured on this level include previously learned facts, rules or definitions or the process of committing facts, rules, or definitions to memory. In this set of studies, the ways to explore subject answers by the task, their explanations, or ways to solve the impasses are not clearly presented or discussed on paper. Most of the provided mathematical tasks are based on visual stimuli.

Synthesis of the study results (LD-H1)

Numerous papers in this category address issues of psychological nature, linking mathematics performance and anxiety (Hunt et al., 2015, Şimşek et al., 2020), or mathematics perceptions and skills (Knoblich et al., 2001, Campbell et al., 2009) or mental effort and task difficulty (Andrzejewska and Stolińska, 2016). But Kosko and al. (2023) are focusing on teacher's gaze on specific students. Eye tracking is used as a medium allowing to better understand the links (links between two sensibilities or links supporting interactions), but with rather little interest in the mathematical tasks themselves; for example, in Kosko et al. (2023) the task is barely mentioned, and we must refer to another article to understand what it's all about (Kosko, 2022). Knoblich et al. (2001) show that

elements of a mathematical sentence are not considered in the same way, which leads to indications about the possible difficulties of students in problem-solving. Campbell et al. (2009) focus on identifying physiological manifestation of perceptual shift which allows researchers to gain knowledge of the neural correlates of additional subjective cognitive experiences that are relevant for image-based reasoning in geometry-shifts of attention, for example, or conceptual-verbal identification of a figure. Hunt et al. (2015) as well as Şimşek et al. (2020) take profit of measurements of fixation duration, fixation dispersion, saccade duration and blink duration to conclude that eye-tracking can provide information and serve as an indicator to interpret the students' difficulty in problem-solving or in image-based reasoning in geometry shifts of attention or conceptual-verbal identification of a figure. Andrzejewska and Stolińska (2016) argue that longer fixation times can be linked either to a confrontation to a difficulty or a greater involvement in the exploration of a problem. Kim et al. (2018) through the study of mathematical symbolism and the way students at different levels and with different knowledge can tackle it, show that eye-tracking can give important feedback to conceive and design mathematical software. The study of Nazaruk et al. (2019) concerns acquisition of mathematical skills for preschools' pupils using different representations of geometric figures and everyday life objects such as a house, a dog, etc. Eye tracking benefits are considered in terms of method of catching the children's behavior in front of these figures. There is also in numerous papers an attempt of correlation between the time of task completion, the accuracy of the answers and the dwell time. And, even if, "tracking eye movements does not actually give direct knowledge of thought processes (and problem-solving technique)." (Kim et al. 2018), the method appears in the papers of this category to interpret the psychological state of learners faced with solving a problem.

RQ2: About the educational level and main subjects focus on the studies, mathematics domains approached, and task-design.

To answer RQ2 we have drawn up the table of Appendix 5 which gives summarized indications as to 1) the level of the students concerned, 2) whether the article is aimed more at the study of teaching or learning (from the pupil's or the teacher's point of view) and 3) in which area of mathematics the proposed task appears. We summarize our findings concerning the papers in eye-tracking studies as following:

Educational level and subjects

- Twenty-nine papers addressed students' issues and seven focused on teachers.
- Fifteen over thirty-eight papers concern students at a university level when twelve concern students of secondary classes.

Mathematics domain

- Many of the studies focused mainly on numbers and arithmetic.

Task design

- Many of the studies provided multiple-choice tasks.
- Geometric tasks mainly based on the identification process.
- Proof tasks are mainly based on numbers field.
- One study commented about the challenging and cognitive demands of the proposed task.
- Five studies integrated digital technology on the provided tasks.

Table 6
Overview of low demand level 1 papers

Author(s)	Subjects	Proposed task(s)
Low demand level 1		
Knoblich et al. (2001)	24 undergraduate students from 18 to 29 years selected only who were familiar with Roman numerals.	Matchstick arithmetic insight problems. Appendix 4
Campbell et al. (2009)	Not presented	Necker cube
Hunt, Clark-Carter, and Sheffield (2015)	78 undergraduate psychology students	80 two-digit addition problems, e.g., $23+29=52$
Andrzejewska and Stolińska (2016)	52 students in the lower secondary school. Among them a group above average aptitude in sciences and other formed a group that was called non-competition students.	Tasks in mathematics physics, computer science and biology. MATH1 task
Kim, Pollanen, Reynolds, and Burr (2018)	Twenty upper-year and graduate mathematics students (class: expert) and eighteen science (non-mathematics) students (class: non-expert) volunteered	14 mathematical questions, all at approximately an 11th to 12th grade (secondary school) level. The questions ranged from True/False to "Find the Error. The problems included set theoretic, function, and matrix notation, as well as several common algebraic and arithmetic logical errors.
Nazaruk and Marchel (2019)	352 preschool-age children (5 to 7 years old) residing in cities and the countryside	observing a picture/scene, geometric shapes, and various objects on screen computer
Şimşek, Uygun, and Güner (2020)	381 7th grade students	multiple-choice and matching geometry test
Kosko et al. (2023)	future elementary mathematics teachers	$6 : \frac{3}{4} = 4 : \frac{3}{4}$ (p. 5)

In our sample of papers, only seven directly involve teachers (Haataja et al., 2019, Maata et al. 2021, Hannula et al. 2022, Soares et al. 2021, Kosko, 2022, 2023); some are also considering the contribution of the study for teacher development even if the main target of the study is students' behavior (Şimşek et al., 2020) or school curriculum (Nazaruk & Marchel, 2019). On the other hand, Professional Content Knowledge appears in the result of students' devoted studies (Kosko et al., 2023).

Fifteen over thirty-eight papers concern students at a university level (undergraduate) when twelve concern students of secondary classes (3 of high school, grade 10-13, seven of low secondary grades (grade 6 to 9) and three of primary and preschool level, some straddle the line between low secondary and primary or low and high secondary. The nine others do not give information about the learners' grade. Twenty-nine of the papers address students' issues and seven are more focused on teachers. The two last addresses both viewpoints.

Concerning mathematics domains and topics our findings coincide with Strohmaier et al. (2020); many of the papers we analyzed relate mainly to numbers and arithmetic, and we identified a variety of other topics: Algebra (2 papers), Function (2), Geometry (3), Logics (3), Number (7), Probability (1), Proportionality (3), Proof (5), Pre-algebra (1).

Contrarily to Strohmaier et al. (2020) we identified methodological details in eye-tracking studies. Although the researchers make clear the data collection setting and the reason why the proposed task is used according to research goals, it was curious to find just one paper author's comment about challenging and cognitive demands of the proposed task (Moutsios-Rentzos & Stamatis, 2015), our interest in this article. However, in some of them the cognitive singularities and relevance of the proposed situation was presented (Andrá et al., 2015; Campbell et al., 2009; Salminen-Saari et al., 2021; Shvarts, 2018).

Many of the studies provided multiple-choice tasks. Since eye tracking seemed particularly beneficial for studying processes rather than outcomes (Strohmaier et al. (2020) we classify most of multiple-choice tasks as low cognitive demand because we had no detailed information concerning subjects' improvements on mathematical reasoning, development of emerged procedures or other different kinds of revealed mental representation.

We know ET data collection settings need many technologies and human sources, including ethics and non-invasive ways of using devices. In most of the papers, it seems that identification tasks on the screen (multiple choice) take an important role in task design in eye-tracking studies; it could be due to equipment and the possibility to gather data with the gaze, reading and identification tasks could be easier to observe shifts and stimuli in brain record with eye-tracking equipment (Campbell et al., 2009), or the idea of changing geometry approach using task mainly based on identification process.

Since eye-tracking has a more visual input, it is understandable that tasks mainly based on observation or identification on screen take more place in the task design of such research. On the other hand, it's curious because this type of visualization task is not often used in dynamic geometric environment research, which usually provides task-design exploring and constructing geometric procedures (Assis & Bairral, 2022), or justifying and proving processes (Mariotti, 2000). In proving analyzed studies three of them provided tasks mainly based on numbers filed (integer, prime, rational, divisibility etc.) and two exploring function topics (Roy et al. 2017; Panse et al. 2018). The five captured studies focusing on reading and comprehension of proofs approach: proof validation (Inglis & Alcock, 2012), explanation and self-explanation training (Hodds, Alcock & Inglis 2014), distribution of attention and inferred processing demand (Roy, Inglis & Alcock (2017), reading behavior and skimming (Inglis & Alcock 2018), and attention allocation (Panse et al. 2018).

Since ET studies involve many digital technologies and nowadays the integration of technological resources is demanding on mathematics content, we decided also to analyze whether the studies integrate technology on the provided tasks. We assume integration as protagonist, as a process to think and solve the task differently. Only five studies integrated digital technology on the provided tasks; two of them focused on geometric content (Hannula et al. 2019; Shayan et al. 2017). Roy et al., 2017 provided e-Proof with audiovisual attempts to consider the dimensions of proof discussed in their theoretical framework and making explicit how they define learning to investigate the effects of e-Proofs on students' reading behaviors. Crisp et al. (2011) provided tasks related with research based on students' strategy to deriving a function, Shvarts (2018) elaborated a computer-based interactive activity for disclosing a parabola, and Hannula et al. (2019) inserts GeoGebra in a task adapted from Steiner point. Shayan et al. (2017) implemented tasks to foster proportional reasoning in a tablet of bimanual interaction problems in which the user must place a left-hand finger and a right-hand finger on the screen at the same time and move both fingers simultaneously to receive a particular visual feedback goal, a green coloration either of the background or of objects they are manipulating.

DISCUSSION

In this article, we analyzed how studies in eye-tracking in mathematics education are dealing with cognitive aspects. Starting from Strohmaier et al. (2020) we captured 39 papers, focusing our analysis on task-design in problem-solving studies. We identified an expressive amount of low demand tasks on ET papers, the studies are still mainly based on numerical issues and focused less on geometry topics, and few of them integrating digital technology on the provided tasks.

Our final remarks are based on two main arguments from ET studies; (i) we need to study mathematical understanding in ecologically valid ways (Hannula et al. (2019), and (ii) the interpretation of eye movements should be based on a reasonable assumption of what eye movements measure and what cognitive processes these measures reflect (Strohmaier et al., 2020). For doing this, we organized our final reflection in two intertwined strands: task-design and cognitive demand, and methodological issues to achieve cognitive processes integrating digital technology. Finally, we conclude with some suggestions about future research using eye-tracking in terms of interactions and learning.

Concerning task-design and cognitive demand

In an ecological system (e.g. a class), the provided task is an essential point that needs to be carefully considered. The task design and the subject's performance are not neutral and form part - among other things - of semiotic mediation and have an influence on cognitive requirements, data collection and results. This means that teachers and students, and even researchers, play an important role in the quality of teaching-learning processes and not only in metacognitive monitoring activities (Cohors-Fresenborg, Kramer, Pundsack, Sjuts, & Sommer, 2010).

Reducing the typology to two levels (low or high) is probably not fine enough, even if it is already a challenge to determine the boundaries between cognitive task requirements. We rely on Stein's framework to categorize the tasks provided in the selected papers. Difficulty in solving certain tasks could not be related only to prior knowledge, but rather to the dynamics of data production and the uncomfortable or intrusive use of eye vision equipment.

Since ET captured papers didn't relate task design with cognitive load, we are aware it could be a limitation of our analysis. The cognitive task demand framework (Stein & Lane, 1996; Stein & Smith, 1998; Stein et al. 2000) was neither based on ET studies nor even included digital technologies on the criteria of task identification. For future research we suggest the possibility of creating a typology of task demand with description including technological issues. In this sense this paper can contribute with such view.

Another point of questioning is that such a framework mixing gaze observation and problem-solving does not contain many tasks based on reasoning and proof processes, particularly in geometry. When we decided to create the fourth stage by considering reasoning and proof studies, we thought, given that this is a subject with a high cognitive load, that we would find more tasks with high cognitive demand, but this is not the case. In addition to observing that the geometry program needs even more attention from professional developers and researchers, we assert that the content of our paper and the provision of such analysis could enrich this model.

Concerning methodological issues to achieve cognitive processes integrating digital technology

In some papers we have the idea that technology resources are only added in data collection settings without reflection about cognitive changes and so on. On the other hand, in some studies we can observe how researchers are improving their way of gathering data (Roy et al., 2017), for instance, designing tasks using digital resources. Although we identified detailed information regarding data collection setting, we think that studies could provide details (aims, affordances, constraints etc.) of the proposal tasks relating with the analysis and learning gains etc.

If we are interested in promoting other learning aspects (besides identification, observation etc.) we must pay attention to the type of the designed task. Hence, results of study are not separated from this didactical decision. We're not saying we can't provide undemanding tasks, but we need to be aware of the situation in which they may or may not be more appropriate. As eye tracking is a multidisciplinary field, we need to consider the focus of study between psychology, neuroscience, mathematics education and so on. But it is necessary to cross-analyze with other frameworks (didactic, psychological, pedagogical, etc.) to establish the link between ET parameters and cognitive functions. Another important point is to consider tasks in a digital environment which considers more than the effective task but also the technology as a tool which changes the task-design and the mathematics content and related procedures.

In terms of methodology, numerous papers are using both quantitative and qualitative analysis which participate in this cross analysis which brings to eye-tracking research a more cognitive insight highlighting different ways of learning within the whole research.

Educational contributions and future analysis

Cognitive and meta-cognitive strategies (Cohors-Fresenborg et al., 2010) are linked with eye-tracking parameters (Shayan, Abrahamson, Bakker, Duijzer, & van der Schaaf, 2017). Most of the papers are considering eye-tracking as an individual tool, allowing to describe and understand the glance when a learner is confronted with a task. But some of them consider eye-tracking as a meaning to better describe and understand the non-verbal interactions between students and between students and teacher. These remarks lean authors to consider as possible a statistical treatment of data (duration of gaze, number of blinks etc.) which leads to a quantitative methodology based on parameters of eye-tracking. Papers coming from mathematics education crossed this quantitative analysis with a qualitative one, mostly using tools of task analysis and record of dialog and gestures. So far, we could classify the use of eye tracking:

1. individual eye-tracking to describe the functioning of learners,
2. individual eye-tracking to better understand the cognitive functioning of learners when faced with a task,
3. eye-tracking to describe interactions in a group of students,
4. eye-tracking to better understand the effects of interactions on the process of task understanding and problem-solving.

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Appendixes – Example of analyzed tasks.**Appendix 1 (High demand – Level 2)**

Salminen-Saari et al. (2021, p. 776)

The students were asked to find the shortest possible way to connect four cities located at the vertices of a square (Fig. 5).

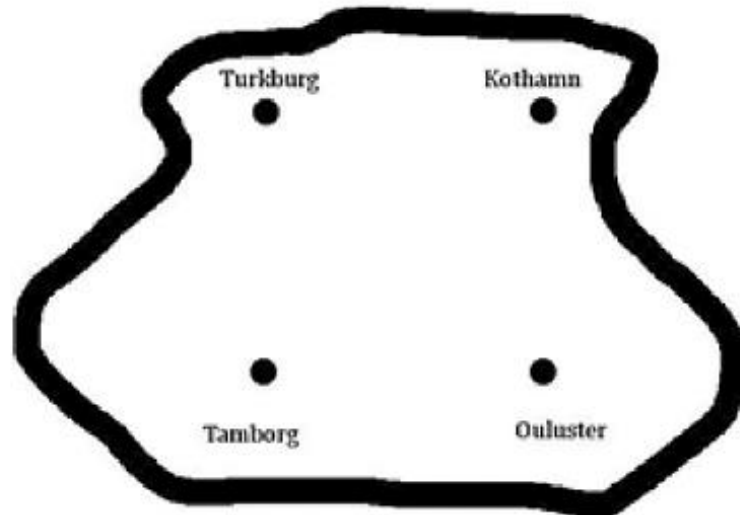


Fig. 5 The illustration shown on the whiteboard to pose the problem

Appendix 2 (High demand – Level 1)

Crisp, Inglis, Mason & Watson (2011, p. 1).
Example of a generalisation task used.

<i>n</i>	<i>m</i>
1	3
2	8
3	15
4	24
5	35
6	48

Appendix 3 (Low demand – Level 2)

Espino, Santamaría, Meseguer & Carreiras (2005, p. B1-B2, table 1 - B4)

This paper examines the time-course of processing during syllogistic reasoning. This kind of reasoning is produced from pairs of categorical premises such as:

*All Athletes are Brokers
All Brokers are Catalans*

where a valid conclusion is “All Athletes are Catalans”. Depending on the quantifier used, there are four kinds of premise and conclusion:

- *All A are B (A)*
- *Some A are B (I)*
- *No A are B (E)*
- *Some A are not B (O)*

The letters in parentheses are the traditional abbreviations for each kind of proposition. The arrangement of terms in the pair of premises determines the figure of the syllogism. There are four figures:

1 2 3 4
B-A A-B B-A A-B
C-B C-B B-C B-C

A is the extreme-term in the first premise (e.g. Athletes); B is the middle-term, that appears repeated in both premises (e.g. Brokers), and C is the extreme-term in the second premise (e.g. Catalans).

Table 1
Example of the materials used in the experiment (translated from Spanish)

<i>Figure 4 and simple problem</i>	
AA-4	IA-4
All/politicians are Italian./	Some/politicians are Italian./
All/Italians are geographers./	All/Italians are geographers./
<i>Figure 4 and complex problem</i>	
EA4	IE4
No/geographer is Italian./	Some/politicians are Italian./
All/Italians are politicians./	No/Italian is a geographer./
<i>Figure 1 and simple problem</i>	
AA-1	AI1
All/Italians are politicians./	All/Italians are politicians./
All/geographers are Italian./	Some/geographers are Italian./
<i>Figure 1 and complex problem</i>	
AE-1	EI1
All/Italians are politicians./	No/Italian is a politician./
No/geographer is Italian./	Some/geographers are Italian./

The slashes represent the segmentation used for analyses.

Appendix 4 (Low demand – Level 1)

Knoblich, Ohlsson, and Raney (2001, p. 1002)

