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Using international large-scale assessment *for* learning: Analyzing U.S. students' geometry performance in TIMSS

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ABSTRACT
The data from international large-scale assessments, such as The Trends in International Mathematics and Science Study (TIMSS), is often designed and used as an assessment *of* learning rather than an assessment *for* learning. This research employed TIMSS 2011 data, focusing on the geometric performance of fourth-grade students in the United States, to demonstrate how large-scale assessments can be utilized qualitatively to identify students' learning challenges, providing valuable insights to inform geometry teaching practices. Using van Hiele's levels of geometric thinking as a guiding framework, the 24 released geometry items were analyzed to identify students' struggle points in geometry nationwide. Our analysis revealed that students performed well on tasks that were visual, contextually grounded, and had clearly defined manipulative expectations.. However, students needed further support to solve items requiring a deep understanding of geometrical concepts, such as finding the perimeter of a square. Additionally, students excelled at solving tasks involving the identification of relative positions of objects on a map and recognizing lines of symmetry in regular shapes. However, they struggled with measurement-related tasks. Our analysis also identified a range of item features that might cause difficulties that impacted the way students responded to these items. Instructional implications for elementary mathematics education are discussed.

INTRODUCTION

Geometry (used inclusively, which includes measurement) is essential in itself and fundamental for other mathematics topics (Common Core State Standards Initiative, 2010; Lee & Lee, 2021; Lowrie & Logan, 2018; Yao, 2020), as well as pivotal significance for students participating in the modern workplace, especially in STEM fields (Cheng & Mix, 2014; Mix & Cheng, 2012; Tian et al., 2022; Wai et al., 2009). Tian et al. (2022) found that strong spatial skills in fourth grade could improve their chance of choosing STEM college fields after controlling math achievement and motivation, verbal achievement and motivation, and family background.

However, results from large-scale assessments such as the Trends in International Mathematics and Science Study (TIMSS) and National Assessment of Educational Progress (NAEP) often show that U.S. students consistently lagged in geometry as compared to other mathematical content domains and their peers in other countries (Bokhove et al., 2019; Chen et al., 2021; Kloosterman & Lester, 2007; Provasnik et al., 2012). In general, research asserts that U.S. students struggle with geometrical topics (e.g., Fielker et al., 1979; Hershkowitz, 1987; Lehrer et al., 1998;

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Sinclair & Bruce, 2015; Wallrabenstein, 1973), which motivates mathematics educators and researcher to identify students' struggle points to inform more responsive geometry education (e.g., Clements & Sarama, 2011; Fielker et al., 1979; Sinclair & Bruce, 2015).

Many qualitative studies have been conducted to assess students' geometry learning, which can be used *for* students' learning. This type of assessment is referred to as an assessment *for* learning, in which evidence about students' knowledge, understanding, and learning difficulties is used to inform teachers' instruction (Earl & Katz, 2006; Small, 2019). However, these results could not be generalized to a larger population, not to mention nationwide. In other words, qualitative studies can help to identify students' potential struggle points in learning geometry. Still, to what extent it represents the national population is still being determined, which limits us from providing federal suggestions and explore cultural patterns.

To address this issue, a large-scale assessment is needed. However, analyses of large-scale assessments (e.g., TIMSS) are often used as an assessment *of* learning (e.g., Chen et al., 2021; Shapira-Lishchinsky & Zavelevsky, 2020) rather than assessing *for* learning. Assessment *of* learning is often referred to as summative assessment and serves for the grading or ranking purposes (Earl & Katz, 2006; Harlen, 2007), which barely concerns specific pedagogical implications for enriching students' learning opportunities. Using TIMSS as an assessment *of* learning, researchers help the countries or regions to locate their mathematics education performance in the global context, which is especially useful for policymakers, administrators, and researchers to ascertain policy and curriculum reforms (Furner & Robison, 2004; Schmidt & McKnight, 1998; Schmidt et al., 2007).

But using TIMSS as an assessment *of* learning fails to provide fine-grained guidelines for supporting mathematics teachers' instructional decision-making. For example, Provasnik et al. (2012) highlighted U.S. fourth grader's mathematics performance in TIMSS 2011 data that "At grade 4, the United States was among the top 15" (p. iii) and "in comparison with other education systems, U.S. 4^{th} graders performed better on average in number and data display than in geometric shapes and measures" (p. 14). While these results provide valuable insights into U.S. students' performance in geometry within a global context, they are insufficient for mathematics teachers or curriculum developers to implement meaningful changes to enhance students' geometry learning opportunities. Simply knowing that U.S. fourth graders performed relatively poor in geometry does not offer enough guidance forteachers to adjust their instruction, especially without detailed information on how—and possibly why—students performed in each specific subtopic of geometry.

Therefore, researchers should aim to strike a balance that allows them to understand students' learning challenges on a broader scale, applicableto a national population. One approach is to use qualitative methods to analyze student performance on large-scale assessment items. This analysis can help identify specific subtopics in geometry that students find most or least challenging and uncover the underlying reasons (Liu, 2019). By doing so, educators and teachers accross the country can be better informed and provide appropriate instructions to support students' geometric development. As such, this study leverages the affordance of TIMSS data, which, in addition to making inferences related to the assessment *of* learning, examines data with a lens of assessment *for* learning. The TIMSS items were analyzed to learn how to improve students' geometry learning opportunities in effective and efficient ways. Specifically, content analysis was used to zoom in on the 24 released fourth-grade geometry items from TIMSS 2011. Meanwhile, the van Hiele levels of each item were analyzed to help identifying U.S. students' areas of strengths and weaknesses in geometry. This study aims to answer the following two central research questions:

RQ1: What can be learned from the geometric items that U.S. students performed differently, with a focus on understanding the features of these items *for* supporting learning?

RQ2: How do U.S. students perform on each geometry subtopic, and what insights can be gained from the variations in performance regarding assessment *for* learning?

Learning difficulties of geometry and measurement

Research found that students' struggle to understand geometric concepts, reasoning, and problem-solving in early grades is associated with their unpreparedness to learn learn other topics such as fractions (Liu & Jacobson, 2022) and abstract geometrical concepts and proof in high school (Clements & Battista, 1992). This calls for teachers to identify the areas where students struggle during the primary years so that they can create meaningful learning experiences for students to develop solid understanding of geometry (Zhou et al., 2021). Previous literature found that younger students' geometric learning challenges cluster in the following three areas: shape classification, measurement, and transformation.

In geometry, shape classification is a complex area for students (e.g., Aslan & Arnas, 2007; Bernabeu et al., 2021; Clements et al., 1999; Guncaga et al., 2017; Walcott et al., 2009). Aslan and Arnas (2007) studied 3- to 6-year-old children's recognition of geometric shapes and found that they performed better in identifying the typical forms of shapes (e.g., square with a horizontal base) and were distracted by the orientation or skewness of the shapes. Similarly, Bernabeu et al. (2021) investigated how 3rd-graders understand the concept of polygon and found students' difficulties in classifying polygon due to their ability to identify relevant (side length, angle measure) and non-relevant (color or orientation of shape) attributes. Non-relevant attributes such as symmetry, concavity/convexity, number of sides, parallelism, and length of the sides in the triangles interrupted students' understanding of definitions of a class of polygons, making it hard for students to recognize attributes that determine a class of figures (Bernabeu et al., 2021). In Walcott et al.'s (2009) analysis of an item that asked students to list the similarities and differences between the parallelogram and rectangle in the NAEP 4th grade assessments in 1992 and 1996, they found that only 11% of 900 responded satisfactorily. Researchers conjectured that students' understanding of shapes might be constructed based on their limited empirical experiences with (e.g., observing, touching, working with) shapes' attributes.

Measurement is another challenging area for students (e.g., Kamii & Kysh, 2006; Kim et al., 2017; Sevgi & Orman, 2020). Most students use measurement instruments or apply formulas (e.g., the perimeter of a square is four times the side length) to get the answer without understanding the conceptual underpinnings (Clements & Battista, 1992). For instance, students identify the square unit as the standard unit of area measurement and consider the concept of area as the number of square units (Barrett et al., 2017). There is a possibility that students might fail to solve tasks when the unit is triangular and rectangular, indicating difficulties related to space-covering or space-filling properties, which are critical aspects of understanding the concept of area (Barrett et al., 2017; Cullen & Barrett, 2020).

Researchers (e.g., Mammarella et al., 2012) observed that transformation was more challenging than shapes and measurement for students. Students needed help in understanding angles (Clements, 2004; Devichi & Munier, 2013; Uttal, 1996). They tended to believe that a right angle must orient to the right, and one ray of an angle must lie horizontally (Devichi & Munier, 2013; Fuys & Geddes, 1984; Mitchelmore, 1998). Changing an angle's orientation challenged children's reorganization of different types of angles (Devichi & Munier, 2013; Izard & Spelke, 2009). Researchers (e.g., Izard & Spelke, 2009; Spelke et al., 2010) hypothesized that students' understanding of geometric topics, such as transformation, might be culturally mediated (e.g., an urban environment provided more opportunities for learning right angles). All these studies suggest that students struggle with conceptually understanding geometrical ideas, which could contribute to their performance in large-scale assessments (like TIMSS).

Van Hiele's level as the analytical framework

Pierre Marie van Hiele and his wife, Dina van Hiele-Geldof (1984), identified five levels of thought in geometry. These original five levels are called the van Hiele levels. Mayberry (1983) described the five levels: Level 0 (visualization) indicates the ability to recognize figures by their appearance without perceiving their properties. Level 1 (analysis) entails identifying figures' essential properties and seeing the shapes' classes. At this stage, students do not see the relationships between the properties of diverse shapes and are likely to comprehend that all the properties of the shapes are of the same significance. Level 2 (informal deduction): Learners perceive the relationships between properties of different figures, create meaningful definitions, and understand class inclusions based on properties. Level 3 (deduction), learners comprehend the meaning of necessary and sufficient conditions for a geometric figure to understand the role of axioms and definitions and be able to construct mathematical proofs. Level 4 (rigor): Students understand the formal aspects of deduction and can function in non-Euclidean systems. Each of these levels has its characteristics and network of relations. A learner must master the level to achieve the higher level, emphasizing the importance of sequential and connected instruction as new ideas are developed based on prior knowledge of interrelated topics (van Hiele, 1984; van Hiele-Geldof, 1984). A learner's progression from one level to the next level depends on the learner's experiences with geometrical ideas.

Since the 1980s, researchers have validated the effectiveness of van Hiele levels in describing students' development of geometric thinking (Fuys & Geddes, 1984; Mayberry, 1983; Wang & Kinzel, 2014). Others used van Hiele levels as an analytic framework to investigate whether the geometry curriculum aligns with the development of students' geometric progression (e.g., Dingman et al., 2013; Newton, 2011; Zhou et al., 2022). This paper used van Hiele levels to reflect the geometric thinking required to solve a specific item. Given that TIMSS items addressed a range of topics, a modified version of the van Hiele model was adopted, as offered by the Ohio Department of Education (2018), which provided a detailed description of each of the levels concerning different sub-concepts covered in geometry curriculum (i.e., two- and three-dimensional shapes, transformations/location, and measurement [length, area, and volume]).

METHODS

Data source and gathering procedure

This study utilizes large-scale assessment data (using TIMSS 2011 data as a showcase) and employs descriptive statistics and content analysis methods (Erlingsson & Brysiewicz, 2017) to address the research questions. In 2011, 369 schools and 12,569 fourth graders in the U.S. participated in TIMSS (Provasnik et al., 2012). Drawing U.S. fourth graders' performance data from the TIMSS 2011, this study focuses on the information related to geometric shapes and measurement (hereafter referred to as fourth-grade geometry). Specifically, among the 54 geometric items, the 24 released items were analyzed (the latest publicly released items, which can be accessed via https://nces.ed.gov/timss/pdf/TIMSS2011_G4_Math.pdf). Half of these items were multiple choices; the other was constructed responses. Information from each item was used, including item number, item label, cognitive domain assigned by TIMSS (including knowing, applying, reasoning), overall correct percent of U.S. students, and international average (Foy et al., 2013). To better understand students' common incorrect choices on the multiple-choice items, Information from "TIMSS 2011 Released Items with Percent Statistics-Fourth Grade" was used (refer to https://timssandpirls.bc.edu/timss2011/international-database.html for more details).

Data analysis

To answer the first research question: *What can be learned from the geometric items that U.S. students performed differently, with a focus on understanding the features of these items for supporting learning?* Three levels of coding have been done. Firstly, each item was coded for van Hiele levels (The Ohio Department of Education, 2018), and then the items were categorized into three groups. Coding for the van Hiele levels was used to determine the cognitive challenge of each item as stated in the trajectory of geometric progression defined by van Hiele's theory. To establish coding truthfulness, one coder completed the first round of coding independently and then shared the codes with others. There were discrepancies in the codes assigned to six items by coders, and a consensus was attained (Saldaña, 2021).

Once the items were coded for van Hiele levels, this information and the details extracted from TIMSS 2011 on students' performance data were compiled. This performance data gave us the percentage of students performed each item. This information was compiled into three categories - *strong* (S) when the U.S. students have a correct percentage of more than 70%, and *moderate* (M) and *weak* (W) for percentages between 50%-70% and below 50%, respectively. By comparing the U.S. students' correct percent with the international average, U.S. students' performance was further justified by making claims that U.S. students performed weakly in this item but stronger than the national average.

Afterward, each item was carefully read to assess which geometrical concepts were foregrounded in the item. For this process, each coder took turns defining the list of attributes related to each item. The commonalities and discrepancies between coders' observations were discussed until a consensus was reached (Saldaña, 2021).

To answer the second research question, *How do U.S. students perform on each geometry subtopic, and what insights can be gained from the variations in performance regarding assessment for learning?* A content analysis method (Erlingsson & Brysiewicz, 2017) was followed. Each coder individually coded each item to discern the critical geometric topics as captured in the item. These codes were discussed as a team, and seven specific topics- relative position, line of symmetry, transformation, angle size, connections between 2D-3D shapes, 2D shapes, and measurement- were identified within these 24 items. These geometric topics concerning students' performance data and how students performed on each subtopic of geometry were compiled.

Ethical considerations

Since the data used in this study is derived from a secondary source and is publicly accessible, no Institutional Review Board (IRB) consent was needed. The dataset is publicly available at [https://nces.ed.gov/timss/pdf/TIMSS2011_G4_Math.pdf]. The nature of the data and its public accessibility eliminate the need for individual ethical approval for this analysis.

FINDINGS

U.S. students' geometric learning strengths and challenges

The first research question is to infer the features of the geometric items for which the U.S. students performed differentially. The international average of correct responses for these items ranged from 15% to 78%, whereas this percentage ranged between 13% and 92% for the U.S. students. Results showed that U.S. students performed strongly (more than 70% accuracy) on nine items, moderately (accuracy percent ranging between 50-70%) on 11 items, and weakly (accuracy percent less than 50%) on five items. The data indicates that U.S. students performed strongly on the items that were either structured in daily contextualized problems or for those that explicitly stated a manipulative expectation (draw, mark, rotate, et al.). Contrarily, they performed moderately for the items that needed students to conceptualize or make connections between geometric ideas. The U.S. students performed poorly for items involving measurement, or that required a deep conceptual geometric understanding (e.g., line of symmetry). A more detailed description of each grouping category is given below.

Items show students' strength

Table 1 shows that among nine strong-performing items (accuracy percent between 78% to 92%), four items focused on the two-dimensional shape and relative position. These items challenged students cognitively from two major domains: knowing (n = 5) and applying (n = 4). Three items were coded as level 0 on van Hiele levels, and six were coded as level 1. By comparing the U.S. correct percentage with the International Average, the U.S. correct percentage is much higher for these nine items.

In general, the items in the strong category showed that U.S. students performed well on items with explicit details on mathematical manipulation. This claim holds true based on the key descriptors captured from these nine items, like 'rotate,' 'put,' 'order,' 'move,' 'draw,' 'make,' 'tell,' and 'mark.' These items were also designed in contextual settings, i.e., one can relate them to daily situations, such as relating to everyday problem solving like navigating locations in maps (e.g., S2, S8, S9), grounding in a situation similar to video games (e.g., S4), and everyday household items like clocks (e.g., S1).

Items shows students' potential

Table 2 shows the ten items in the Moderate category (accuracy percent ranging between 50-70%). This category includes items with a range of geometrical concepts, like sizes of angle (M5 and M9), lines of symmetry (M2 and M8), rotation (M3), and reflection (M6). Similar to the Strong category, the items in the Moderate category focused on two cognitive domains: knowing (n = 6) and applying (n = 4). Most of these items are coded under van Hiele's level 1 (Analysis, n = 9), suggesting that these items have higher cognitive demand. The U.S. correct percent for these ten items is also higher than the International Average.

The items in the Moderate category were more challenging than the items in the Strong category. Most of these items expected students to conceptualize or make connections between geometric ideas. For instance, students were asked to execute 1/2 turn of a given shape/figure (M3),

Table 1
Item information for the strong category

Item Code	Item Number & Information	CD	VH	IA	US
S1	M031159 Rotate the shape 1/4 clockwise	K	L1	64	78
S2	M051064 B Put an X where Troy lives	A	L0	64	78
S3	M031109 Ordered angles by size	K	L1	63	78
S4	M031088 Move in the board game	A	L1	68	80
S5	M031093 Which dotted line is of symmetry	K	L1	54	80
S6	M041327 Draw the line of symmetry	A	L1	47	80
S7	M031083 Pieces of cardboard to make shape	K	L1	69	83
S8	M041160 A Tell the position of the shop	K	L0	76	87
S9	M041160 B Mark Lucy's house	A	L0	78	92

Note for all the Tables. Item code, S: Strong, M: Moderate, W: Weak; CD: Cognitive Domain (K: Knowing, R: Reasoning, A: Applying); VH: van Hiele level (L0: Level 0, L1: Level 1; L2: Level 2); IA: International Average; US: US correct percent (in increasing order).

Table 2
Item information for the moderate category

Item Code	Item Number & Information	CD	VH	IA	US
M1	M041148 Statement about 3-D figures	K	L1	32	50
M2	M051015 Complete Jay's shape	A	L1	42	51
M3	M031071 Position of shape after a ½ turn	K	L1	43	52
M4	M041143 Identify shapes in the picture	K	L1	53	53
M5	M031325 Angle between 90 and 180 degrees	A	L1	31	53
M6	M041328 Draw reflection of the triangle	A	L1	53	56
M7	M051064A Complete the table	K	L0	50	63
M8	M051123 Lines of symmetry complex figure	K	L1	43	66
M9	M041329 Which one is a right angle	K	L1	64	68
M10	M041158 number of boxes in the stack	A	L1	63	69

draw an angle between 90 and 180 degrees (M5), identify a right angle (M9), count the number of boxes in the stack (some visible and some invisible; M10). Few items in this Moderate category required students to draw or write responses (e.g., M1, M2, M5, M7).

Items reflect students' learning challenges

Table 3 shows the five items in the Weak category (accuracy percent less than 50%). These items spanned through the geometrical topics: measurement (n = 3), 2D shape (n = 1) and 2D-3D relation (n = 1); cognitive domains: reasoning (n = 3) and applying (n = 2); and van Hiele's levels: level 1 (Analysis, n = 3) and level 2 (Informal Deduction, n = 2). These items were challenging for the fourth graders compared to the International Average, especially for four items (W1, W2, W3, and W5). The item wording shows that these items require a deep understanding of the related concepts or procedures. For example, measuring the length of a string (W2), figuring out the perimeter of common shapes (W5), and connecting 2-D and 3-D shapes (W3).

U.S. students' learning on specific geometric topics

The second research question concerns how U.S. students perform on specific geometric topics. The 24 items were grouped into seven groups, and the following sections discuss each topic area in detail.

Relative position

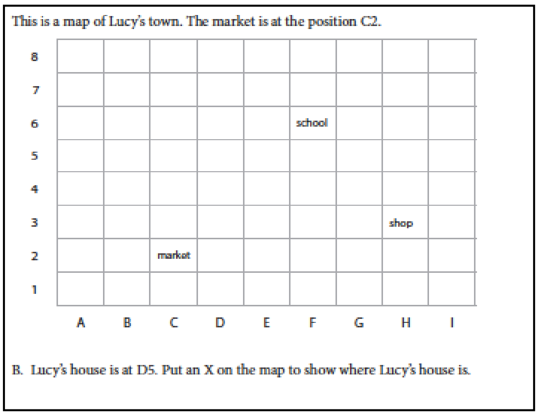
The data in Table 4 show that U.S. students performed well in finding relative positions in cartesian coordinates. Their correct percentage on these five items ranges from 63% to 92%, much higher than the international average. While analyzing the items, it is interesting that U.S. students' correct percent dropped rapidly from Item S9 to S8, although the contexts of these two items are the

Table 3
Item information for the weak category

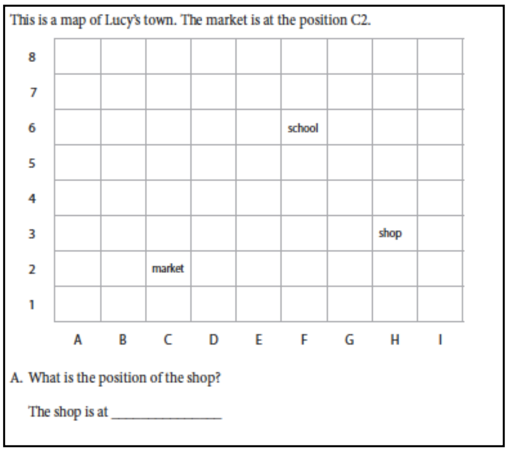
Item Code	Item Number & Information	CD	VH	IA	US
W1	M041284 Identify the shapes	R	L1	15	13
W2	M031004 Length of string pulled straight	A	L1	29	20
W3	M041265 Relating net with its 3-D figure	R	L1	37	34
W4	M031297 Shaded area in square centimeters	A	L2	30	38
W5	M041155 How far does Ruth walk	A	L2	50	43

Table 4
Information of the items related to relative position

Item Code	Item Information	CD	VH	IA	US
S9	M041160B Mark Lucy's house	A	L0	78	92
S8	M041160A Tell the position of the shop	K	L0	76	87
S4	M031088 Move in the board game	A	L1	68	80
S2	M051064B Put an X where Troy lives	A	L0	64	78
M7	M051064A Complete the table	K	L0	50	63



Item S9: M041160B Mark Lucy's house



Item S8: M041160A Position of the shop

Figure 1. Itemon relative position

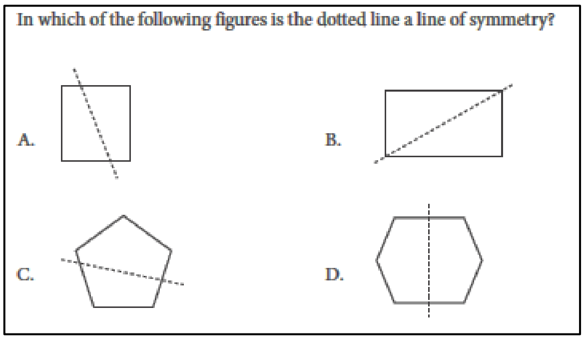
same (see Figure 1; both items offer a map of Lucy's town). The difference is that S9 provides the position (D5) of a building (Lucy's house) and asks students to mark the position on the map, while S8 posit students to tell the position of a building (Correct Responses include H3, (H, 3), 3H, (3, H) or other equivalents)) on the map. These results suggest that U.S. students were familiar with the relative position but needed more directions about reporting a subject's position based on the map.

Line of symmetry

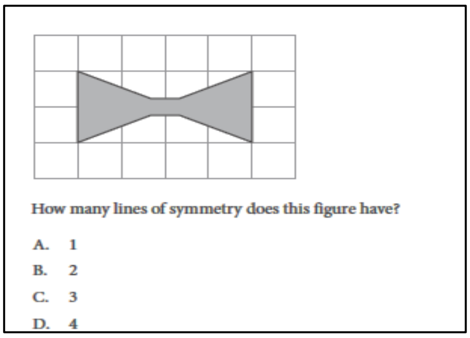
Table 5 shows that U.S. students performed strongly on the items related to the line of symmetry compared to the International Average. U.S. students can improve their understanding by solving tasks involving multiple steps and complex shapes. The comparison of the items (Figure 2) substantiates this point. Both items S5 and M8 (Figure 2) fall under the cognitive domains of knowing. 80% of U.S. students identified that option D shows a line of symmetry on the hexagon correctly (S5), while only 66% answered M8 correctly. These results indicated that though students know the concepts of symmetry, they need help to use this knowledge for complex shapes. After zooming in on students' selected options for these two items, data shows that most students answered diagonal(s) as a line(s) of symmetry. Around 14% of students chose option B in S5, suggesting they considered diagonals a symmetry line. Also, 24.2% of U.S. students thought the shape in M8 had four symmetry lines (option D), which might be because these students not only count the horizontal and vertical lines of symmetry on the shape but also count its two diagonals as lines of symmetry. The students who counted diagonals as lines of symmetry might consider that the line of symmetry is a line that divides a shape into two equal parts, regardless of whether these two parts

Table 5
Information of the items related to line of symmetry

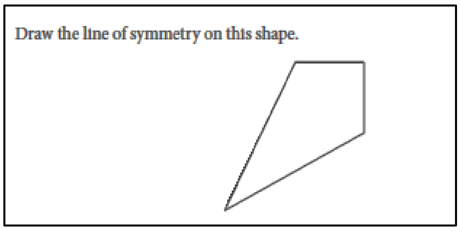
Item Code	Item Information	CD	VH	IA	US
S5	M031093 Which dotted line is of symmetry	K	L1	54	80
S6	M041327 Draw the line of symmetry	A	L1	47	80
M8	M051123 Lines of symmetry complex figure	K	L1	43	66
M2	M051015 Complete Jay's shape	A	L1	42	51
Item Code	Item Information	CD	VH	IA	US



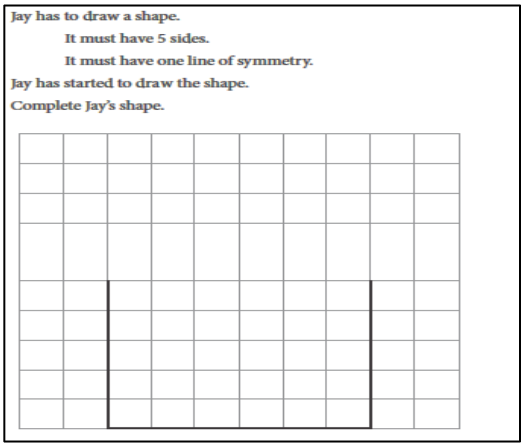
Item S5: M031093 Which dotted line is of symmetry



Item M8: M051123 Lines of symmetry complex figure



Item S6: M041327 Draw the line of symmetry



Item M2: M051015 Complete Jay's shape

Figure 2. Two pairs of items related to line of symmetry

can be folded along into matching parts.

Regarding the second pair of items (S6 and M2 in Figure 2), both require students to draw lines to complete a task; however, students' performance on M2 was lower than the S6 (see Table 5). It might be because the item wording for M2 was more complex than S6. S6 directly asked them to draw a line of symmetry. However, M2 requires students to create a shape that meets two criteria: have five sides and one line of symmetry. Our analysis reveals that the U.S. students generally were good at the line of symmetry on simple shapes (S6 and S5) but needed help to apply this knowledge to solve the more complicated problems (M8 and M2).

Transformation

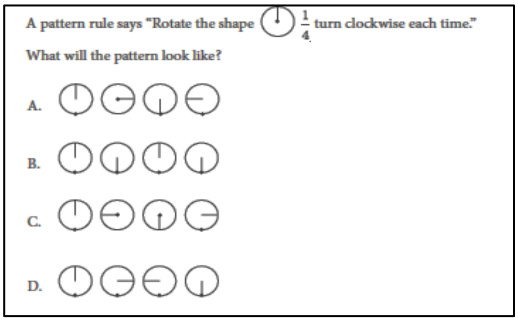
U.S. students performed relatively strongly on transformation items (see Table 6), especially on the two rotation items (Item and Item M3 ranked in the top 10). Figure 3 shows the pair of items assessing students' knowledge about rotation. 78% of U.S. students correctly rotated a circular shape 1/4 clockwise (S1), while only 52% correctly identified the position of a flag-like shape after a 1/2 turn (M3). International averages also suggest that M3 is more challenging than S1. One possible reason is that students might be more familiar with clock-like shapes (as given in S1), while the flag-like shape is rare for them daily (M3). Less familiarity with shapes might contribute to their performance on these items.

Table 6
Information on the items on transformation

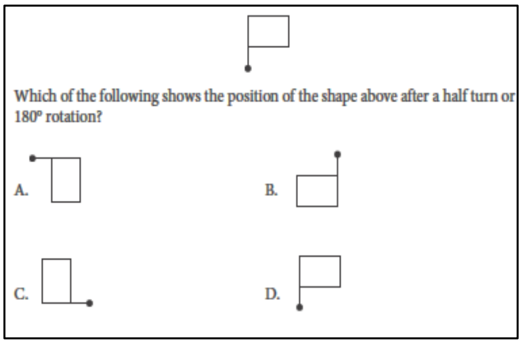
Item Code	Item Information	CD	VH	IA	US
S1	M031159 Rotate the shape 1/4 clockwise	K	L1	64	78
M5	M041328 Draw reflection of the triangle	A	L1	53	56
M3	M031071 Position of shape after a 1/2 turn	K	L1	43	52

Table 7
Information on the angle size related items

Item Code	Item Information	CD	VH	IA	US
S3	M031109 Ordered angles by size	K	L1	63	78
M9	M041329 Which one is a right angle	K	L1	64	68
M4	M031325 Angle between 90 and 180 degrees	A	L1	31	53

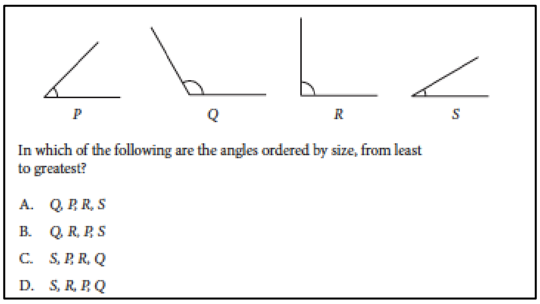


Item S1: M031159 Rotate the shape 1/4 clockwise

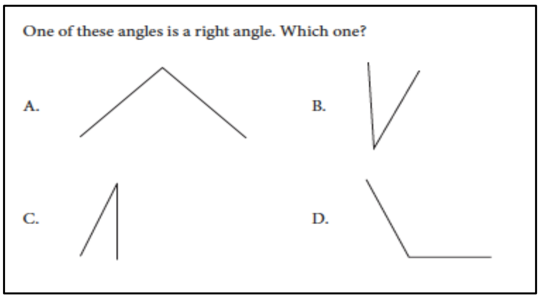


Item M3: M031071 Position of shape after a 1/2 turn

Figure 3. Item pair on rotation



Item S3: M031109 Ordered angles by size



Item M9: M041329 Which one is a right angle

Figure 4. Item pair on angle size

Regarding items related to reflection, 56% of the U.S. students were able to draw a reflection of the given right triangle (M5). Although this percentage is higher than the international average, tasks that require students to draw may pose additional challenges. Drawing can lead to more errors, particularly if the students' conceptual understanding of reflection is not strong..

Angle size

Table 7 shows that U.S. students perform well on the three angle-size-related items. Drawing an angle between 90 and 180 degrees (M4) is more challenging than solving multiple-choice tasks

(S3 and M9 in Figure 4). S3 and M9 share many similarities (understanding of angle property, angle size, and congruent angles, see Figure 4). Regarding U.S. students' performance on S3 and M9, 78% of U.S. students answered S3 correctly as they answered the ordering of four given angles by size. In comparison, only 68% were able to pick the right angle from the four different angles given (M9). S9 is less challenging for U.S. students as it requires students to visually order four angles positioned in horizontal orientation, whereas M9 asks students first to know what a right angle is and then to pick a right angle from given pictures but in different orientations. There was a 1% performance difference for the international average for S3 and M9; however, this difference was 10 % for U.S. students, indicating that identifying an "abnormal" right angle (based on a different orientation) is a particular challenge for U.S. students.

By examining U.S. students' most incorrect choices on S3 and M9, the data shows that option B in Item S3 and option D in M9 were prominent errors. 14.1% of U.S. students chose option B in S3, which means these students ordered the given four angles from the greatest to least, opposite from the proposed question. More likely, these students may have needed to read the item more carefully. Regarding M9, 24.2% of U.S. students selected option D as the right angle. The underlying rationale of their choice might be because they believe a "right" angle should be horizontally oriented.

2D-3D shape connections

On the topic of 2D-3D shape connections, Table 8 shows that U.S. students performed well on S7 and M1 but failed to do so for W3. W3 (see Figure 5) tests students' knowledge of composing 2-D shapes into common 3-D shapes. 34% of U.S. students identified the correct pattern (option D), while 38% chose option B as the correct option. Such common error suggests that a substantial proportion of U.S. students may know the 2-D components of a cylinder but fail to infer that the length of the rectangle should equal the circumference of the circle, not the rectangle's width.

2D Shapes

Identifying shapes (M6 and W1) is covered in kindergarten; however, it is challenging for all students when the task design is complicated (e.g., Item W1). This is also visible in the percentage of U.S. students and the international average (Table 9). Figure 6 shows Item W1. The core mathematical content of this task is to find the number of sides of given shapes and compare the side lengths. However, the item requires logical thinking as students need to understand the meanings "a shape has four sides, and all sides are not the same length," "a shape does not have four sides and are sides are the same length," and "a shape does not have four sides, and all sides are not the same length." Also, students need to coordinate the picture and the table to report their answers, which might have increased the complexity of the task.

Measurement

Table 10 shows that measurement items were the most difficult for U.S. students. U.S. students got the lowest ranking on W2 and W5 (Figure 7) across the 24 items. W2 asked students to measure the length of a string that is neither a straight segment nor starts at the zero point on the ruler. To solve this item correctly, students must understand the fundamental ideas underlying measurement, know the ruler reading strategies, and mentally unfurl the string. Only one-fifth of U.S. students correctly answered, 9% lower than the International Average. Many U.S. students (33.8%) selected option C, which can be obtained by reading the mark where the string visually ends.

W5 tested students' understanding of perimeter. Typically, students learn the formula or memorize the standard algorithm without grasping conceptual reasoning. To solve Item W5, students need to conceptualize – 'walk all the way around the edge of the square playground' means tracing the perimeter of a square, and 'the length Ruth walk' means the length of the perimeter. Around 43% of U.S. students reported the correct option C, which is 7% lower than the international average. Also, 37% of the U.S. students chose option A, indicating they may lack an understanding of the problem and a deep conceptual understanding of the perimeter or lack experience finding the perimeter of a real-life context.

Table 8
Information on 3D-2D Shape Connection Items

Item Code	Item Information	CD	VH	IA	US
S7	M031083 Pieces of cardboard to make shape	K	L1	69	83
M1	M041148 Statement about 3-D figures	K	L1	32	50
W3	M041265 Relating net with its 3-D figure	R	L1	37	34

Table 9
Information on the 2D Shape Items

Item Code	Item Information	CL	VH	IA	US
M6	M041143 Identify shapes in the picture	K	L1	53	63
W1	M041284 Identify the shapes	R	L1	15	13

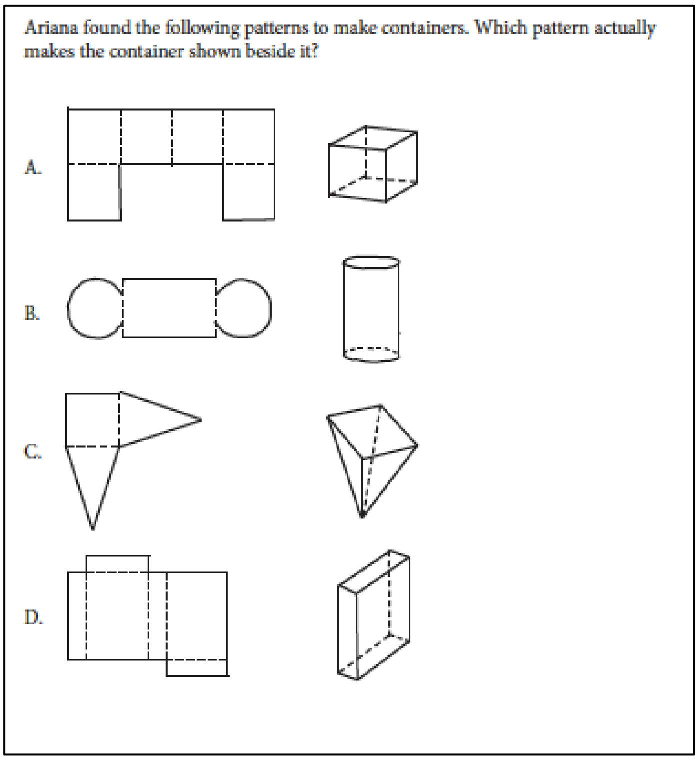


Figure 5. Item W3: M041265 relating net with its 3-D figure

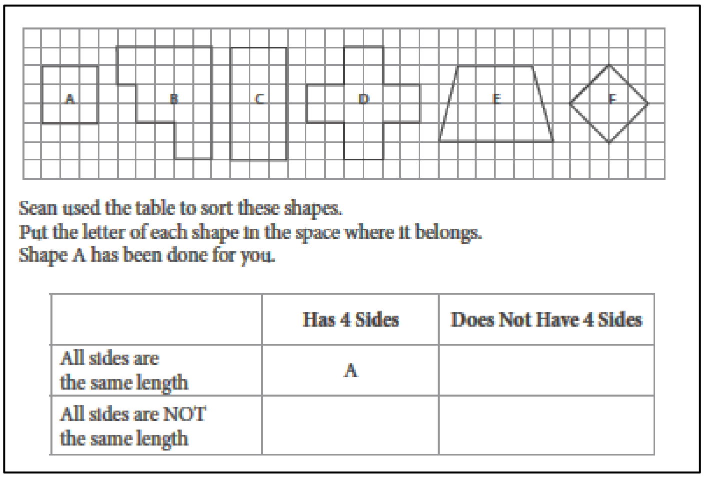
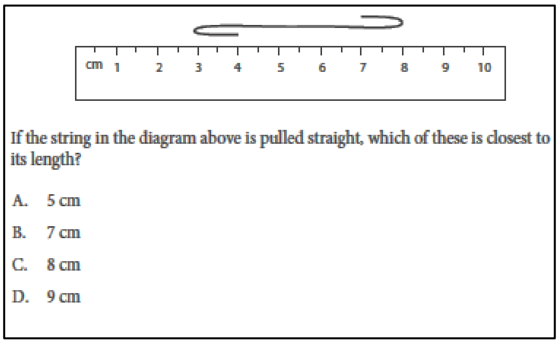


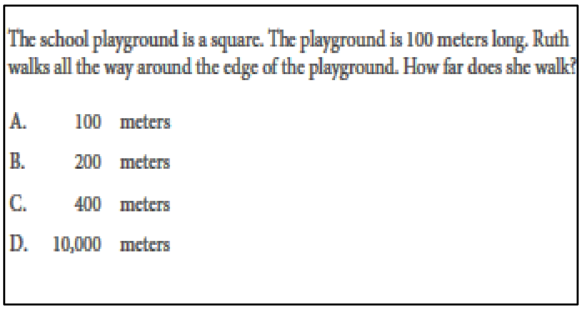
Figure 6. Item W1: M041284 identify the shapes

Table 10
Information on Measurement Items

Item Code	Item Information	CD	VH	IA	US
M10	M041158 number of boxes in the stack	A	L1	63	69
W5	M041155 How far does Ruth walk	A	L2	50	43
W4	M031297 Shaded area in square centimeters	A	L2	30	38
W2	M031004 Length of string pulled straight	A	L1	29	20



Item W2: M031004 Length of string pulled straight



Item W5: M041155 How far does Ruth walk

Figure 7. Two measurement-related items

DISCUSSION

This study is motivated to break the notion that large-scale assessments are often used to rank students' achievements (e.g., Chen et al., 2021; Mullis et al., 2012; Provasnik et al., 2012; Shapira-Lishchinsky & Zavelevsky, 2020). Guided by the idea of assessment *for* learning (Small, 2019), this study focused on analyzing the 24 publicly released geometric items in TIMSS 2011 to identify attributes of items that impacted students' performance. This study used content analysis and answered two central questions: What can be learned from the geometric items that U.S. students performed differentially? Moreover, what insights can be gained regarding assessment for learning from the variations in performance across these subtopics? By doing so, this study draws a detailed picture of U.S. students' geometry performance, which can inform *nationwide* mathematics teachers and curriculum developers with concrete evidence of what and how to make changes to improve students' geometry performance.

Building on students' strength for learning

Building on children's strengths in geometry can provide children with new learning opportunities (Sinclair & Bruce, 2015). Though there were items for which students' performance was weak, there was a range of items on which they performed well, especially on the items that focused on testing their knowledge about relative locations, lines of symmetry, and transformation. The U.S. students surpassed the international benchmark substantially for most of these items. These findings raise interesting questions, such as whether there are any underlying cultural factors (Izard & Spelke, 2009; Spelke et al., 2010) or curricular strengths (Chen et al., 2009) that support U.S. students' strong performance on some specific topics or types of items? If so, how could educators build on such strengths to empower U.S. students' learning of other subtopics of geometry?

In addition, U.S. fourth graders performed well on the items grouped under level 0 and level 1 of van Hiele Level but struggled with items grouped as level 2. Although having competence at level 0 and level 1 is also an achievement, the field needs to think about possible ways to use guidance from the existing theoretical frameworks to improve students' geometric thinking and performance (Sarama et al., 2021). Building on this observation, large-scale assessments, such as TIMSS, are recommended to include van Hiele Level as a domain of the designed items.

Explore effective approaches to address students' learning difficulties

Guiding teaching with students' learning trajectories

Our analysis results highlight that U.S. students struggle most with measurement-related items and require a deep conceptual understanding of geometric concepts or procedures. For example, students seemed to be confused between a line of symmetry and a line that divides a shape into two congruent figures (e.g., S5 and M8, suggesting that students did not conceptually understand the line of symmetry, which allowed them to untangle the confusion). Aligned with Clements and Battista's (1992) early observation that students often use measurement instruments or apply geometric formulas to solve tasks without understanding the conceptual underpinnings, similar situations from the national representative data were observed. For example, only 20% of U.S. students reported the measure of the length of a string that needs to be pulled straight correctly (7cm, W2), but 33.8% of U.S. students (33.8%) selected 8cm, which can be obtained by reading the mark where the string visually ends, suggesting most students lack an understanding of the fundamental ideas underlying measurement.

These observations underscore the importance of deepening students' conceptual understanding of measurement (Smith & Barrett, 2018). Building on previous literature, Kim et al. (2017) proposed a developmental sequence across the length, area, and volume measurements that reflect students' learning trajectories within the geometrical measurement. Scholars (Clements & Battista, 1986; Lee & Cross Francis, 2016) also have advocated a specific instructional sequence for effective measurement teaching, including "gross comparison of length, measurement with nonstandard units such as paper clips, measurement with manipulative standard units, and finally measurement with standard instruments such as rules" (Clements, 1999, p. 5).

Increasing variation in curriculum.

A noteworthy application of a summative assessment strategy within an assessment for learning context is identifying areas where students lack understanding, followed by establishing targets to address these gaps. This study highlights that U.S. students' understanding of geometric concepts needs more depth and flexibility. Despite the items discussed above, the U.S. students needed help with various items requiring them to apply learned knowledge flexibly. For example, about one-fourth of U.S. students recognized an obtuse angle with a horizontal base as a right angle (M9); they failed to recognize the given right angle, which may be because of its nonstandard orientation (Devichi & Munier, 2013; Fuys & Geddes, 1984; Izard & Spelke, 2009; Mitchelmore, 1998;). Although 92% of U.S. students marked the given position on the map correctly (S9), only 87% were able to report the position of the shop on the map (S8). Additionally, most students could correctly rotate a circular shape (S1), but fewer could correctly answer an item asking to turn a flag-like shape (M3).

These findings require further examination of the type of geometrical shapes presented in our curriculum and instruction. The shapes students encounter, either in school or out of school, are often too "perfect" (e.g., equilateral triangle, isosceles triangle, rectangles, and square with horizontal bases), which limits their notions of shapes (also see Aslan & Arnas, 2007; Bernabeu et al., 2021; Clements et al., 1999; Clements & Sarama, 2000; Guncaga et al., 2017; Walcott et al., 2009). Similarly, Monaghan (2000, p.192) also concluded that "What emerges is that students (over-)rely on standard representations of shapes as a means of identifying and discriminating between them. It was shown that curriculum materials tend to underpin such perceptions." Thus, examining the type of shapes and tasks used in the curriculum and taught in classrooms is essential. The curriculum and teachers should deliberately introduce shapes with more diverse appearances and orientations to expand students' notions of shapes (Clements & Sarama, 2000; Wang & McDougall, 2019). Promoting teachers' preparation in teaching geometry might be needed (Kaur Bharaj & Cross Francis, 2020; Steele, 2013; Žilková et al., 2015).

In general, our analysis shows U.S. students had good coverage of a range of geometric topics but struggled to solve more challenging tasks, which aligns with the early observation that the U.S. curriculum prefers to cover an extensive range of topics with limited time to go in-depth which was called "a mile wide and inch deep" (Schmidt & McKnight, 1998). The positive change is that the new Common Core State Standards for Mathematics (CCSSM)(Common Core State Standards Initiative, 2010) have made significant changes, including aligning the new standards with the standards from the top-performing countries and emphasizing the depth of topics and coherence across grade levels (Schmidt & Houang, 2012). These changes may lead U.S. students to achieve better on the geometric items in TIMSS. However, researchers (e.g., Zhou et al., 2022) also found that the development of geometry topics in the CCSSM needs to be more coherent and disturbs the learning progression. Thus, examining how the CCSSM responds to our findings of students' strengths and weaknesses in geometry learning and how students performed at TIMSS 2019 will be meaningful.

Informing teacher preparation and professional development

Instead of relying solely on scores from large-scale assessments like TIMSS as a summative evaluation of students' thinking, it is crucial for educational stakeholders to extract valuable insights from this dataset. For example, our analysis has uncovered intriguing aspects of students' geometrical thinking, and these findings should be integrated into teacher preparation programs and professional development settings. This ensures that pre-service and in-service teachers gain valuable insights into student thinking. The central concept behind assessment for learning is not merely to provide a final judgment but to narrow the gap between a learner's current understanding and their desired learning outcomes. To achieve this goal, K-12 teachers should incorporate the nuances of students' conceptions into their interactions as revealed within such assessments, allowing latent thinking patterns to be highlighted during instructional time.

Given that TIMSS data are derived from a diverse pool of students in the United States, it offers a comprehensive overview of students' thoughts that may not be readily apparent in individual states, schools, or students. Thus, similar analysis is recommended to be conducted at different levels. Teachers can adopt these items for specific classrooms as diagnostic items to understand their students' strengths and struggles. Building on this information, teachers can modify their instruction. Meanwhile, teachers can modify these items to develop an assessment of learning (Small, 2019).

Limitation

This study has several limitations. First, it analyzed only 24 released items, whereas TIMSS 2011 included a total of 54 geometry items. Incorporating more items in future analyses could strengthen the study's findings. Additionally, this study used data from TIMSS 2011 as a case to illustrate how large-scale assessments can be utilized for learning. Future research should consider using updated datasets from 2015 and 2019 to gain a more accurate understanding of current trends and developments in mathematics education. Notably, since 2011 marked the adoption of new curriculum standards in the U.S. (Common Core State Standards Initiative, 2010), using TIMSS 2011 data as a baseline to examine how U.S. students' performance evolved from 2011 to 2015 and 2019 to understand the impact of these new standards could be a valuable direction for further exploration. Second, this study has done extensive research and taken guidance from the literature to unpack the reasons for students' responses and hypothesized possible misconceptions. Still, the quality of this information can be strengthened by conducting interviews with students and probing into their thinking for the reasons for selecting specific answer choices. Furthermore, this study exemplified the comparison between the performance of U.S. students with the international average; future research could conduct cross-country comparisons. Such comparisons would help evaluate the strengths and weaknesses of students from different countries and provide insights into how curricula across cultures impact student learning.

CONCLUSIONS

Broadfoot et al. (2002) describe assessment *for* learning as the process of gathering and interpreting evidence to help learners and their teachers determine the learners' current progress, identify their learning goals, and decide the most effective strategies to achieve them. Similary, Black et al. (2004) argue that assessments, even those designed *of* learning, can be used *for* learning when they provide feedback that informs instructional adjustments to meet learning needs. Drawing on these ideas and continuing our work in studying geometry curriculum (Zhou et al., 2023), this study utilized data from U.S. fourth graders from the TIMSS 2011 assessment, analyzing 24 items qualitatively for learning. The analysis revealed that U.S. students performed well on items involving practical geometrical knowledge but struggled with questions requiring abstract geometrical thinking and reasoning. We also identified a range of potential misconception nationwide, highlighting the importance of enhancing students' conceptual understanding of geometry, particularly in measurement (e.g., length, angle), and their flexibility in applying geometrical knowledge. These findings provide valuable insights for designing tasks that develop students' geometrical thinking and support teachers and curriculum developers in creating targeted instructional tasks. In summary, our study underscores that large-scale assessments, which are often designed *of* learning, can be creatively utilized *for* learning, such as guiding professional development, diagnose teaching effectiveness, inform instructional planning, and more. Researchers and educators are also encouraged to replicate this analysis in other mathematical domains across different educational systems to foster meaningful educational improvements. Additionally, stakeholders are urged to make the most of the valuable yet costly data from large-scale assessments to enhance student success.

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AUTHOR'S DECLARATION

Authors' contributions	JL: Conceptualization, Methodology, Data Retrive, Data Analysis, Writing-Original Draft Preparation, Writing-Review and Editing; LZ: Conceptualization, Methodology, Data Analysis, Writing-Review&Editing; PKB: Conceptualization, Writing-Review and Editing, Data Analysis,; DZ: Conceptualization, Writing-Review and Editing; , JJL: Methodology, Writing-Review and Editing
Funding Statement	This study is not supported by any funding.
Availability of data and materials	All data are available from the authors.
Competing interests	The authors declare that the publishing of this paper does not involve any conflicts of interest. This work has never been published or offered for publication elsewhere, and it is completely original.

BIBLIOGRAPHY

Aslan, D., & Arnas, Y. A. (2007). Three- to six-year-old children's recognition of geometric shapes. *International Journal of Early Years Education*, *15*(1), 83–104. https://doi.org/10.1080/09669760601106 64

Barrett, J. E., Clements, D. H., & Sarama, J. (Eds.). (2017). Children's measurement: A longitudinal study of children's knowledge and learning of length, area, and volume. In *Journal for research in mathematics education monograph series* (Vol. 16). Reston, VA: National Council of Teachers of Mathematics.

Bernabeu, M., Moreno, M., & Llinares, S. (2021). Primary school students' understanding of polygons and the relationships between polygons. *Educational Studies in Mathematics*, *106*(2), 251-270. https://doi.org/10.1007/s10649-020-10012-1

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2004). Working inside the black box: assessment for learning in the classroom. *Phi Delta Kappan,* **86**(1), 8-21. https://doi.org/10.1177/003172170408600105

Bokhove, C., Miyazaki, M., Komatsu, K., Chino, K., Leung, A., & Mok, I. A. C. (2019). The Role of "Opportunity to Learn" in the Geometry Curriculum: A Multilevel Comparison of Six Countries. *Frontiers in Education, 4*. https://doi.org/10.3389/feduc.2019.00063

Broadfoot, P. M., Daugherty, R., Gardner, J., Harlen, W., James, M., & Stobart, G. (2002). *Assessment for learning: 10 principles*. Cambridge, UK: University of Cambridge School of Education.

Chen, J., Li, L., & Zhang, D. (2021). Students With Specific Difficulties in Geometry: Exploring the TIMSS 2011 Data With Plausible Values and Latent Profile Analysis. *Learning Disability Quarterly, 44*(1), 11–22. https://doi.org/10.1177/0731948720905099

Chen, J. C., Reys, B. J., & Reys, R. E. (2009). Analysis of the learning expectations related to grade 1–8 measurement in some countries. *International Journal of Science and Mathematics Education*, *7*(5), 1013-1031. https://doi.org/10.1007/s10763-008-9148-5

Cheng, Y.-L., & Mix, K. S. (2014). Spatial Training Improves Children's Mathematics Ability. *Journal of Cognition and Development, 15*(1), 2-11. https://doi.org/10.1080/15248372.2012.72518

Clements, D. C., & Battista, M. (1986). Geometry and geometric measurement. *The Arithmetic Teacher*, *33*(6), 29-32. https://doi.org/10.5951/at.33.6.0029

Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics*, *99*(1), 5-11. https://doi.org/10.1111/j.1949-8594.1999.tb17440.x

Clements, D. H. (2004). Geometric and spatial thinking in early childhood education. In D. H. Clements, J. Sarama, & A.-M. Di Biase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 267–298). Mahwah, NJ: Lawrence Earlbaum Associates Inc.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. *Handbook of research on mathematics teaching and learning*, 420-464.

Clements, D. H., & Sarama, J. (2000). Young children's ideas about geometric shapes. *Teaching children mathematics*, *6*(8), 482-482. https://doi.org/10.5951/tcm.6.8.0482

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of mathematics teacher education*, *14*(2), 133-148. https://doi.org/10.1007/s10857-011-9173-0

Clements, D. H., Swaminathan, S., Hannibal, M. A., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education*, *30*(2), 192–212. https://doi.org/10.2307/749610

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers.

Cullen, A. L., & Barrett, J. E. (2020). Area measurement: structuring with nonsquare units. *Mathematical Thinking and Learning*, *22*(2), 85-115. https://doi.org/10.1080/10986065.2019.1608619

Devichi, C., & Munier, V. (2013). About the concept of angle in elementary school: Misconceptions and teaching sequences. *The Journal of Mathematical Behavior, 32*(1), 1–19. https://doi.org/10.1016/j.jmathb.2012.10.001

Dingman, S., Teuscher, D., Newton, J., & Kasmer, L. (2013). Common mathematics standards in the United States: A comparison of K–8 state and Common Core Standards. *The Elementary School Journal*, *113*(4), 541-564. https://doi.org/10.1086/669939

Earl, L., & Katz, S. (2006). *Rethinking Classroom Assessment with a Purpose in Mind.* Retrieved from http://www.edu.gov.mb.ca/k12/assess/wncp/rethinking_assess_mb.pdf

Erlingsson, C., & Brysiewicz, P. (2017). A hands-on guide to doing content analysis. *African journal of emergency medicine : Revue africaine de la medecine d'urgence*, *7*(3), 93–99. https://doi.org/10.1016/j.afjem.2017.08.001

Fielker, D. S., Tahta, D. G., & Brookes, W. M. (1979). Strategies for teaching geometry to younger children. *Educational Studies in Mathematics*, 85-133. https://doi.org/10.1007/bf00311177

Foy, P., Arora, A., & Stanco, G. M. (2013). *TIMSS 2011 User Guide for the International Database*. International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, The Netherlands.

Furner, J. M., & Robison, S. (2004). Using TIMSS to Improve the Undergraduate Preparation of Mathematics Teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *4*.

Fuys, D., & Geddes, D. (1984). An Investigation of Van Hiele Levels of Thinking in Geometry among Sixth and Ninth Graders: Research Findings and Implications. https://doi.org/10.1177/001312458501700400

Guncaga, J., Tkacik, Š., & Žilková, K. (2017). Understanding of Selected Geometric Concepts by Pupils of Pre-Primary and Primary Level Education. *European Journal of Contemporary Education*, *6*(3), 497-515

Harlen, W. (2007). Criteria for evaluating systems for student assessment. *Studies in Educational Evaluation*, *33*(1), 15-28.

Hershkowitz, R. (1987). The acquisition of concepts and misconceptions in basic geometry - Or when “a little learning is a dangerous thing.” In: J. Novak (Ed.), *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, 3*, (pp 238–251).

Izard, V., & Spelke, E. S. (2009). Development of sensitivity to geometry in visual forms. *Human Evolution, 23*(3), 213–248.

Kamii, C., & Kysh, J. (2006). The difficulty of “length x width”: Is a square the unit of measurement? *Journal of Mathematical Behavior, 25*(2), 105–115. https://doi.org/10.1016/j.jmathb.2006.02.001

Kaur Bharaj, P., & Cross Francis, D. (2020). “A square is not long enough to be a rectangle”: exploring prospective elementary teachers’ conceptions of quadrilaterals. In A.I. Sacristán, J.C. Cortés-Zavala & P.M. Ruiz-Arias, (Eds.). *Mathematics Education Across Cultures: Proceedings of the 42nd Annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* (pp.727-728). Mazatlán, Mexico.

Kim, E. M., Haberstroh, J., Peters, S., Howell, H., & Oláh, L. N. (2017). *A learning progression for geometric* *measurement in one, two, and three dimensions (Research Report No. RR-17-55)*. Princeton, NJ: Educational Testing Service. https://doi.org/10.1002/ ets2.12189

Kloosterman, P., & Lester, F. K. (Eds.). (2007). *Results and interpretations of the 2003 mathematics assessment of the National Assessment of Educational Progress*. National Council of Teachers of English.

Lee, M. Y., & Francis, D. C. (2016). 5 Ways to Improve Children's Understanding of Length Measurement. *Teaching children mathematics*, *23*(4), 218-224. https://doi.org/10.5951/teacchilmath.23.4.0218

Lee, M. Y., & Lee, J. E. (2021). Spotlight on area models: Pre-service teachers' ability to link fractions and geometric measurement. *International Journal of Science and Mathematics Education, 19,* 1079–1102 https://doi.org/10.1007/s10763-020-10098-2

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 137–167). Mahwah: Erlbaum.

Liu, J. (2019). Identifying U.S. Elementary Students' Strength and Weakness in Geometry Learning from Analyzing TIMSS-2011 Released Items. In S. Otten, A. G. Candela, Z. de Araujo, C. Haines, & C. Munter (Eds.), *Proceedings of the 41st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, (pp. 435-436). The University of Missouri.

Liu, J., & Jacobson, E. (2022). Examining US elementary students' strategies for comparing fractions after the adoption of the common core state standards for mathematics. *The Journal of Mathematical Behavior, 67*, 100985. https://doi.org/10.1016/j.jmathb.2022.100985

Lowrie, T., & Logan, T. (2018). The interaction between spatial reasoning constructs and mathematics understandings in elementary classrooms. In K. S. Mix & M. T. Battista (Eds.), *Visualizing mathematics: The role of spatial reasoning in mathematical thought* (pp. 253-276). Switzerland: Springer Nature. https://doi.org/10.1007/978-3-319-98767-5_12

Mammarella, I. C., Giofrè, D., Ferrara, R., & Cornoldi, C. (2012). Intuitive geometry and visuospatial working memory in children showing symptoms of nonverbal learning disabilities. *Child Neuropsychology, 19*(3), 235–249. https://doi.org/10.1080/09297049.2011.640931

Mayberry, J. (1983). The van Hiele levels of geometric thought in undergraduate pre-service teachers. *Journal for research in mathematics education*, *14*(1), 58-69. https://doi.org/10.2307/748797

Mitchelmore, M. C. (1998). Young Students' Concepts of Turning and Angle. *Cognition and Instruction, 16(*3), 265–284. https://doi.org/10.1207/s1532690xci1603_2

Mix, K., & Cheng, Y. (2012). The relation between space and math: developmental and educational implications. In J. B. Benson (Ed), *Advances in child development and behaviour*, Vol 42 (pp. 197–243). Academic Press. https://doi.org/10.1016/b978-0-12-394388-0.00006-x

Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational studies in mathematics*, *42*(2), 179-196.

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College. https://doi.org/10.6017/lse.tpisc.tr2103.kb5342

Newton, J. (2011). An examination of K–8 geometry state standards through the lens of the van Hiele levels of geometric thinking. In J. P. Smith (Ed.), *Variability is the rule: A companion analysis of K–8 state mathematics standards* (pp. 71-94). Information Age.

Ohio Department of Education. (2018). *van Hiele Model of Geometric Thinking*. https://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics/Model-Curricula-in-Mathematics

Provasnik, S., Kastberg, D., Ferraro, D., Lemanski, N., Roey, S., & Jenkins, F. (2012). Highlights from TIMSS 2011: Mathematics and Science Achievement of US Fourth-and Eighth-Grade Students in an International Context. NCES 2013-009. *National Center for Education Statistics*.

Saldaña, J. (2021). *The coding manual for qualitative researchers*. sage. https://doi.org/10.29333/ajqr/12085

Sarama, J., Clements, D. H., Barrett, J. E., Cullen, C. J., Hudyma, A., & Vanegas, Y. (2021). Length measurement in the early years: teaching and learning with learning trajectories. *Mathematical Thinking and Learning*, 1-24. https://doi.org/10.1080/10986065.2020.1858245

Schmidt, W. H., & Houang, R. T. (2012). Curricular coherence and the common core state standards for mathematics. *Educational Researcher*, *41*(8), 294-308. https://doi.org/10.3102/0013189x12464517

Schmidt, W. H., & McKnight, C. C. (1998). What can we really learn from TIMSS? *Science*, *282*(5395), 1830-1831.

Schmidt, W. H., McKnight, C. C., Cogan, L. S., Jakwerth, P. M., & Houang, R. T. (2007). *Facing the consequences: Using TIMSS for a closer look at U.S. mathematics and science education*. Springer Science & Business Media. https://doi.org/10.1007/0-306-47216-3

Sevgi, S., & Orman, F. (2020). Eighth grade students' views about giving proof and their proof abilities in the geometry and measurement. *International Journal of Mathematical Education in Science and Technology*, 1-24.

Shapira-Lishchinsky, O., & Zavelevsky, E. (2020). Multiple appearances of parental interactions and math achievement on TIMSS international assessment. *International Journal of Science and Mathematics Education*, *18*(1), 145-161. https://doi.org/10.1007/s10763-018-09949-w

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM*, *47*(3), 319-329. https://doi.org/10.1007/s11858-015-0693-4

Small, M. (2019). *Math that matters: Targeted assessment and feedback for grades 3–8*. Teachers College Press.

Smith, J. P., III, & Barrett, J. E. (2018). Learning and teaching measurement: Coordinating number and quantity. In J. Cai (Ed.), *Compendium for research in mathematics education.* (pp. 355-385). Reston: VA: National Council of Teachers of Mathematics.

Spelke, E. S., Lee, S. A., & Izard, V. (2010). Beyond core knowledge: Natural geometry. *Cognitive Science, 34*(5), 863–884. https://doi.org/10.1111/j.1551-6709.2010.01110.

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. *Journal of Mathematics Teacher Education*, *16*(4), 245-268. https://doi.org/10.1007/s10857-012-9230-3

Tian, J., Ren, K., Newcombe, N. S., Weinraub, M., Vandell, D. L., & Gunderson, E. A. (2022). Tracing the origins of the STEM gender gap: The contribution of childhood spatial skills. *Developmental Science*, e13302. https://doi.org/10.1111/desc.13302

Uttal, D. H. (1996). Angles and distances: Children's and adults' reconstruction and scaling of spatial configurations. *Child Development, 67*(6), 2763–2779. https://doi.org/10.2307/1131751

van Hiele, P. M. (1984). A child's thought and geometry. In D. Fuys, D. Geddes, & R. Tischler (Eds.) (1959/1984), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele (pp. 243-252).* Brooklyn College.

van Hiele-Geldof, D. (1984). The didactics of geometry in the lowest class of secondary school (M. Verdonck, Trans.). In D. Fuys, D. Geddes, & R. Tischler (Eds.*), English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele (pp. 1-214).* Brooklyn College.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology, 101*(4), 817–835. https://doi.org/10.1037/a0016127

Walcott, C., Mohr, D., & Kastberg, S. E. (2009). Making sense of shape: An analysis of children's written responses. *The Journal of Mathematical Behavior*, *28*(1), 30-40. https://doi.org/10.1016/j.jmathb.2009.04.001

Wallrabenstein, H. (1973). Development and signification of a geometry test. *Educational Studies in Mathematics, 5*, 81–89. https://doi.org/10.1007/bf01425414

Wang, S., & Kinzel, M. (2014). How do they know it is a parallelogram? Analysing geometric discourse at van Hiele Level 3. *Research in Mathematics Education*, *16*(3), 288-305. https://doi.org/10.1080/14794802.2014.933711

Wang, Z., & McDougall, D. (2019). Curriculum matters: What we teach and what students gain. *International Journal of Science and Mathematics Education*, *17*(6), 1129-1149. https://doi.org/10.1007/s10763-018-9915-x

Yao, X. (2020). Characterizing Learners' Growth of Geometric Understanding in Dynamic Geometry Environments: a Perspective of the Pirie–Kieren Theory. *Digital Experiences in Mathematics Education*, *6*, 293-319. https://doi.org/10.1007/s40751-020-00069-1

Zhou, D., Liu, J., & Liu, J. (2021). Mathematical argumentation performance of sixth graders in a Chinese rural class. *International Journal of Education in Mathematics, Science, and Technology, 9*(2), 213-235. https://doi.org/10.46328/ijemst.1177

Zhou, L., Liu, J., & Lo, J. (2022). A comparison of U.S. and Chinese geometry standards through the lens of van Hiele. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 10*(1), 38-56. https://doi.org/10.46328/ijemst.1848

Zhou, L., Lo, J., & Liu, J. (2023). The journey continues: Mathematics curriculum analysis from the official curriculum to the intended curriculum. *Journal of Curriculum Studies Research*, *5*(3), 80-95. https://doi.org/10.46303/jcsr.2023.32

Žilková, K., Guncaga, J., & Kopácová, J. (2015). (Mis) Conceptions about Geometric Shapes in Pre-Service Primary Teachers. *Acta Didactica Napocensia*, *8*(1), 27-35.