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Mathematical Competencies in the Digital Era

Kaye Stacey

University of Melbourne, Australia

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ABSTRACT

The paper begins with a discussion of the characteristics of digitization that influence school mathematics, its goals, and practices: the digitization of everything, the invisibility of computation, the era of big data and information on everything, and changes in the social role of mathematics due to increased global competitiveness. This leads directly to Theme 1, where we discuss the diminished economic and personal value of the ability to carry out routine mathematical procedures from arithmetic to calculus. Two other themes are then discussed: the reorientation of school mathematics content to support students to solve real world problems, and the need for students to understand about how mathematics can provide insight into real world problems, even when they are not able to actually do this themselves. These themes are only three of the many aspects of mathematical competence in the digital era which might have been chosen for discussion.

INTRODUCTION

The digital era presents many challenges to mathematics education, but it also presents opportunities. One opportunity is to reconsider what mathematical competences are essential for the future and what aspects of the curriculum need to change. As teachers of mathematics, we want to give students a full appreciation of mathematics as an exciting discipline and an important part of cultural heritage, with opportunity for discovery, with elegant patterns and surprising theorems and with a unique way of thinking. My assessment is that what might be taught about this 'pure' side of mathematics is little changed by living in the digital era, and as in the past, only a few students are deeply attracted to this. I was. However, we mathematics teachers also have a responsibility to help all students see mathematics as relevant to their own lives, as individuals, citizens, and workers, and it is this second goal that my paper mainly addresses. It is where I see the greatest challenges for mathematics from our changing world. The paper therefore concentrates on this. It is also of current interest to me because Hugh Burkhardt, Daniel Pead and I have recently completed a book (Burkhardt et al., 2024) to be published by Routledge entitled "Learning and Teaching for Mathematical Literacy". Naturally, I have drawn on some of the thinking that has gone into the book. In what follows, I shall refer to this book as LTML. My interpretation of mathematical competencies in this paper is broad although with a focus on the use of mathematics in life and work – in other words the mathematical literacy part of mathematical competencies. Throughout the paper, I intend mathematics to include statistics.

METHODS

The paper begins with a discussion of the characteristics of digitization that influence school mathematics, its goals and practices. This leads directly to Theme 1, where we discuss the diminished economic and personal value of the ability to carry out routine mathematical procedures from arithmetic to calculus. Two other themes are then discussed: the reorientation of school mathematics content to support students to solve real world problems, and the need for students to understand

*Corresponding author: k.stacey@unimelb.edu.au

about how mathematics can provide insight into real world problems, even when they are not able to actually do this themselves. Burkhardt, Pead and I feel this has been underestimated in the past. These themes are only three of aspects of mathematical competence in the digital era which might have been chosen for discussion.

FINDINGS

Characteristics of the digital era

I am using the phrase the 'digital era' to refer to the present and the future, where digital technology is present almost everywhere in society, in professional and private lives, and in education. I appreciate that a major issue for education is that there is great variation in the technology that schools provide for their students, and what students have access to at home. Some schools in some countries have excellent technology, with strong technical and pedagogical support to use it. Other schools in most countries have inadequate technology and little support. However, I expect that the technology available in schools will gradually increase over time and I hope that the ideas in this paper are sufficiently broad to apply now to many levels of technology access and also to guide thinking about mathematics for the future.

The digital era has several characteristics relevant to school mathematics. First, almost everything can be digitized, in the sense that it is represented by numbers so transmitted and stored easily in multi-purpose devices and transformed mathematically. This applies not just to traditional quantities (prices, weights, speeds, light intensity etc.) but also to text, sound and images. In this way, digital technology can transform communication for education, between teachers and students, by presenting information (including textbooks with images and video), assignments, or tests and having students submit their work. It can also support collaborative work between students. This communication transformation has been the site of a great deal of important design work in mathematics education around technology (see for example, almost all of the papers in (Bates & Usiskin, 2016) at a major conference on digital technologies in mathematics education). However, it is not the main focus of this paper. Another educational spin off from the digitization of everything is the opportunity for students to solve mathematical problems using data about many different things. In addition to using prepared data, students can often now use probes (e.g., from the school physics department) or apps on their own phones to gather their own data on physical quantities (e.g., pH, sound, location) and analyze it to solve problems and develop concepts such as those of speed and acceleration. This is just one of the many ways in which technology can bring the real world into the classroom. Use of video and pictures are other examples.

Second, computation is increasingly invisible. It is embedded in machines and hidden from the user. At the supermarket, machines can weigh produce, look up the unit cost, calculate the price and add it to your bill. A navigation app tests many possible routes to your destination, finds current or predicted traffic speeds, calculates arrival times and shows you the best option. During a typical mathematics lesson, there are probably more calculations completed by the unused mobile phone in the teachers' pocket than in the minds of all the students in the room. The increasing invisibility of calculation and its outsourcing to machines presents what is possibly the greatest challenge to the mathematics curriculum, and I will discuss it in more detail below. Until recent decades (e.g., around 1970 in Australia) numerical skill was an important attribute for getting work. People were employed in banks and shops and elsewhere because of their speed and accuracy in calculation. All of this work, and much more, has now gone to machines. Whereas more advanced mathematical knowledge is very valuable in today's competitive economic environment, the ability to perform routine calculations is not required.

Third, there is information (both good and bad) everywhere. In the era of 'big data', the masses of data passing through technological systems can be analyzed to learn an unprecedented amount about the natural and human world. Within such data there is information of great importance to us all (such as millions of daily measurements used to predict weather), commercially important data such as the number of customers in a shop at various times of the day, and there is also 'trivial' information (such as how many likes a social media post receives) although information like that sometime has significant commercial value. It is now possible for students to learn much more about the world than ever before.

Finally, although it is not necessarily related to digitization, our era is one of globalization, global competition and national economies that prosper with technological advance. As a result, governments place a high value on having a mathematically proficient workforce, and consequently expect schools to give STEM subjects including mathematics a high priority. Further evidence of this high valuing in the support of governments for international assessments TIMSS and PISA in mathematics – especially, as with PISA, when there are direct links to real world problem solving. These characteristics set the scene for the discussion of mathematics curriculum and teaching that follows.

Theme 1. Decreasing VALUE of traditional mathematical skills

For at least 50 years, mathematics education around the world has been challenged by the advent of digital computation. The challenge is more pressing when technology becomes inexpensive so that all students can have access, and for this reason the pressure for change has been increasing over time. In many countries, it is now the case that most students can have reasonable access to technology such as four function and scientific calculators, spreadsheets and computer algebra systems and sophisticated statistics software either on the students' own device or on the internet. This technology can carry out all the routine procedures of school mathematics: arithmetic, trigonometry, probability and statistics, deal with algebraic expressions and equations both numerically and exactly including with parameters, graph relations and functions and find critical points, do calculus, and create charts of many types to visualize data. Computation in this sense goes beyond arithmetic to include routine mathematical procedures from any branch of school mathematics. With this computational power either already at the fingertips of students, or soon to become so, it is clear that practicing routine mathematical procedures cannot be the central component of mathematics lessons at any level. Instead of producing young people who can moderately quickly produce moderately reliable answers to routine computations, society no needs them to be able to use whatever mathematics they know to solve problems, drawing on technology in a sensible way to provide speed and high accuracy.

Forty years ago, when the four-function calculator was the only digital technology in schools, the U.K.'s Cockroft Report of 1982 considered that the main goal for teaching arithmetic should not be the traditional proficiency in carrying out the four operations with whole numbers, fractions and decimals, but to develop the ability to estimate answers and to carry out some calculations with simple numbers mentally. In problem solving, students should be able to identify what operations are needed and choose to use a calculator efficiently when the numbers are beyond the mental range. Brown's 2014 paper (Brown, 2014) explains the checkered history of this apparently straightforward proposal in the U.K.

Although some traditionalists see the proposed goal as 'dumbing-down' the curriculum, it is in fact harder to teach for understanding than it is to teach students to follow rules. The time that would be otherwise spent on practicing routine procedures to solve 'naked number' calculations has to be diverted to tasks where students can simultaneously develop problem solving skills, a feeling for number, and a strong understanding of place value and other key concepts. Typically, these tasks will require experimentation, reasoning, and explanations of students to each other and in writing. They may have multiple solutions, and through class discussion, students can come to see that some methods are better than others and begin to adopt them.

Beyond arithmetic, similar principles apply to learning algebra, statistics and calculus. Pierce and Stacey (Bates & Usiskin, 2016) for example, discuss 'algebraic insight' and its component of 'algebraic expectation' which is essentially a 'feeling for algebra' such as in required to use a computer algebra system. A simple example is that to enter algebraic expressions accurately into a machine or to read output, students need to be very aware of the importance of the order of operations and how order is indicated through algebraic notation. Using formulas in a spreadsheet requires a similar skill. Again, a very simple example is to know the differences between formulas

$$= A\$3 + 6 * B1 \text{ and } = (A\$3 + 6) * B1 \quad (1)$$

Although students need to carefully check their work, unless they are very aware of the demands and conventions of algebraic notation, they cannot use a machine efficiently. Selecting from the menus of routines available on mathematically able software also requires considerable understanding of the subject and its notation, without a high level of routine skill. For example, a spreadsheet is able to plot data in many different ways – users have to choose a good option that actually makes sense. It is surprising how often students make choices leading to meaningless graphs. There is also a broad menu of choices (e.g., linear, exponential) for a trend line - a line of best fit. Here the calculation is considerable, so automation is very valuable, but if the user makes choices without understanding, the results derived using the trend line can easily be nonsensical. Trend lines seem to be a concept where the general idea is easy to grasp, whilst the actual calculations and underlying least squares concepts are quite difficult.

In promoting these ideas, we need to be wary of living in the digital era. It is easy to create computer programs to drill and test students in routine calculations, with game features that make them attractive to students and hence to teachers and parents who view mathematics primarily just as calculation. It is harder to create digital materials that encourage students to think more deeply.

When students are permitted to outsource computation in any branch of mathematics to machines the goal is to develop "problem-solving strategies, concepts, and structures, rather than mechanical processes"(Stacey, 2013). Because this is such a major change in how time in mathematics lessons should be spent, there is substantial need for teachers and researchers to work together to design appropriate lessons and to decide where their country wants to draw a line between what students should be able to do with pen and paper, and what technology can be used for. Decisions will almost certainly need to change as technology changes. In doing this, whether for arithmetic, algebra or statistics, questions such as the following will be confronted.

1. How much by-hand skill in routine computation in (topic) is required for students to gain deep understanding?
2. `What types of new lessons best suit the national teaching traditions and values?
3. How can teachers learn to use the new lesson formats and features?
4. How can technology use be encouraged in a way that is as equitable as possible?
5. What should high-stakes examinations be like to promote and assess the new goals?

THEME 2. Mathematical competence for real life in all lessons

Context-focused mathematics framework

To organize the LTML book [1], Burkhardt, Pead and I created the "*Context-focused Mathematics Framework*" to group the broad competencies required for applying mathematics in life-related situations and most occupations. Goos, Geiger, Dole, Forgasz and Bennison [6] use a somewhat similar but more complex and more school-level framework. At the centre of the context-focused framework is 'taking the context seriously'. We have known for many years that students find it difficult to identify how mathematics can be used to illuminate real world problems, and also that many students feel the subject is irrelevant to their lives. Regularly taking the context of problems seriously is one step to remedying this. It implies using some problems in every-day mathematics lessons where a real-world context is intended to influence the solution, always including realistic data and always expecting answers that make real-world sense. Including some interdisciplinary work can take the role of the context to a higher level, so that students learn about how mathematics is really used in other subjects. A useful collection of interdisciplinary work that supports and requires mathematical understanding is in the Numeracy Across the Curriculum section of the Victorian Department of Education website.

In the Context-Focused Mathematics Framework, there are four components that accompany 'taking the context seriously'.

1. Students need a 'productive disposition' – the feeling that they can use mathematics successfully to gain information about real world concerns.
2. In taking the context seriously, a 'critical thinking approach' is essential: problem solvers have to carefully consider each aspect of their work, starting with how the problem is formulated mathematics, the appropriateness of the solution method, the interpretation of results and the adequacy of the real-world solution.
3. Turning to specifically mathematical content, it goes without saying that students need mathematical and statistical know how, but they also need to be aware of what makes quality data (from collection and analysis and interpretation) and know how to use technology. They need mathematical, data and technology know-how.

4. As we collected examples of how mathematics occurs in life-related contexts in preparation for the LTML book, Burkhardt, Pead and I were surprised to see that very often it seemed knowing about how mathematics is used is required, rather than knowing how to do it, This is most obviously true of the serious modelling of substantial problems such as climate change – all well-informed citizens and business leaders need to know about what is done and why in general terms, but they do not need to know how to do it. We felt this 'know about' was sufficiently important to elevate to the Context-focused Mathematics Framework. Theme 3 below elaborates on these ideas.

Orienting mathematical concepts and skills for mathematical literacy

This section moves down from the big picture in the Context-focused Mathematics Framework to the detailed picture of what aspects of mathematical knowledge may be especially important for the digital era, still with a focus on using mathematics. As we worked through the many mathematical literacy tasks that we considered in preparing LTML, Burkardt, Pead and I identified certain concepts, skills and understandings that deserved more emphasis in the normal school experience of all students. Different people with different interests, abilities and occupations need different amounts and types of knowledge for mathematical competence in their own lives, but the items listed below (adapted from LTML [1]) are sufficiently general to be part of compulsory education.

1. Number Calculations
    a. Using mathematical tools, especially calculators (on any device) and spreadsheets.
    b. Quick estimation of the expected results of calculation.
    c. Calculating with high reliability by the chosen method.
    d. Checking results are sensible.
    e. Very large and very small numbers – their size and how to write them.
    f. Understanding and calculating with positive and negative powers of ten.
2. Measurement Units and Quantities
    a. The size of units in relation to real world phenomena.
    b. Estimation of quantities and knowing the typical measurements of some common objects to use as benchmarks.
    c. Units for very large and very small quantities and their relationship to powers of 10 and 1000.
    d. Reporting results of calculation with sensible precision.
3. Proportional reasoning
    a. All aspects of proportional reasoning and linking them (percents, ratios, rates, etc.).
    b. Dealing with rates in less usual units (e.g., number of deaths per 100,000).
    c. Combining several rates (e.g., finding number of drops per minute to set an intravenous drip using volume and time required and number of drops per liter).
4. Statistics
    a. Four stages: reading the data, reading between the data, reading beyond the data and reading behind the data (Shaughnessy, 2007)
    b. Using technology (especially spreadsheet-like tools) to organize data sets, create data displays and do calculations.
    c. Increasing the range of data displays that students encounter in line with changing practice (e.g., animated graphs).
5. Probability
    a. Including discussion of risk to supplement work on chance.
    b. Linking mathematical language with common language and practices (e.g., betting odds).
    c. Visualizing probabilities (difficult because many of importance are very small).
6. Geometry, spatial reasoning and location
    a. Identifying geometry in the world around
    b. Appreciating that geometric features can make real objects work (e.g., keeping movement perpendicular to the ground).
    c. Using paper and digital maps and navigation tools.
    d. Visualizing, interpreting and making drawings and objects in three dimensions.
    e. Specific links to hobbies or vocational interests of students.

7. Algebra
    a. Reading and using formulas.
    b. Ideas of independent and dependent variables.
    c. Behavior of basic types of functions (linear, exponential, inverse proportion).
    d. That linear functions describe constant rate of change and exponential functions describe a constant percentage rate of change (constant addition or subtraction versus constant multiplication or division at each step).

Underlying these recommendations is an expectation that students learn to use digital tools appropriately and that many of the learning activities involve authentic contexts. Each of the dot points above could be elaborated. Just one is treated below.

Statistics: Expanding the range of data displays

. Whilst the school curriculum in many countries may still be focused on pictograms, bar charts and pie charts, new software and new capabilities of digital publishing has prompted the growth of new and more informative data displays. Even if a mathematics curriculum only requires that students know how to create traditional data displays, students dealing with data from other areas of interest will need to interpret new graph types.

Figure 1 presents one example of the new types of data displays that are now possible. It was created using the Gap minder website. which makes a huge and constantly updated database of statistics about people and countries available to all. The founder Hans Rosling intended to show that facts (data) are powerful, that things have been getting better over time for most people, and that people around the world with similar incomes have reasonably similar lifestyles no matter where they live (Rosling et al., 2016)

The Gap minder website has pioneered and popularized new ways of displaying data. Because the outcomes of concern to the Gap minder website are often linked to many variables, they have created ways of showing multiple variables simultaneously, using bubble charts with two axes for main variables, with bubble size indicating population perhaps, animation to show changing times, color to show region or religion, for example. Figure 1 is a simple example to enable printing (so for example, only three countries, time plus one other variable). The graph was created with ease, by selecting variables and other options from the graph site. It shows a best estimate of GDP per person over time, with ups and downs reflecting world events and in-country circumstances. The arrow below the graph shows that it can also be viewed as a movie. Gap minder offers vast potential for student investigations and lessons using real data, and enables explorations of history, social inequality, health, literacy and many other significant factors in human lives.

THEME 3. Knowing about mathematical modelling

To write LTML, Hugh Burkhardt, Daniel Pead and I considered how a mathematically literate person would use their mathematical knowledge as they encounter modern issues, such as considering what to buy, understanding pressing concerns such as climate change and social inequality, making financial decisions and dealing with technology. As we began the project, we agreed that mathematical modelling should be an important component of every student's mathematics education. Of course, we still advocate this.

However, we also were confronted with the many ways in which people need to know about mathematical modelling, rather than be able to do it themselves, and we noted many instances (some in newspapers, for example) where people's expectations were unrealistic. A recent example was the public reaction to the modelling for COVID-19. In my country, modelling was discussed by the general public in ways that it had never been discussed before. Gal and Geiger (2022) studied the newspaper reports of the time. The public discussion also revealed misconceptions about modelling. People expected models to be correct, rather than just useful. They did not understand that data and scientific information (e.g., about mode of transmission) was still emerging. They expected straightforward answers to quite small questions (e.g., exactly when to wear a mask).

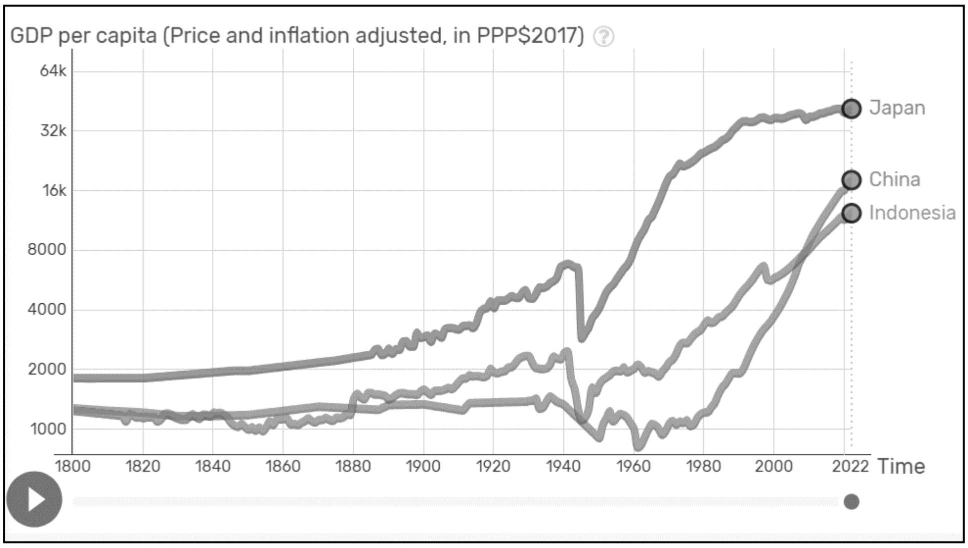

Figure 1. Simple graph of GDP per person for three countries from Gap minder website.

One of the themes of LTML therefore became the importance of teaching students about mathematical models commonly in use and what can and cannot be expected from them. A major example is how so many aspects of the social world are now being 'measured': social inequality, happiness, customer satisfaction etc. All of these measures require mathematical models.

As I noted above, in the digital era, almost everything can be given a number. There is, for example, a world index for happiness. For the last 6 years, Finland has scored the highest. Helliwell et al. (2023) explains what factors are considered.

There are multiple ways that products and services are ranked, perhaps an average score on a seller's website, or perhaps by careful testing. To take a real example, an article from Choice [https://www.choice.com.au/] describes how they rank washing machines. For each machine, scores are allocated to for each machine, scores are allocated to performance on 5 variables: dirt removal (d), rinse performance (r), gentleness (g), water efficiency (w) and spin efficiency (s). Each of these variables would have a separate scoring system. Then machines are ranked on their scores on a utility function:

$$U = 0.40d + 0.20r + 0.15g + 0.15w + 0.10s \qquad (2)$$

This utility function is a mathematical model of the 'goodness' of a washing machine. To make the mathematical model, decisions need to be taken about which variables to measure, and those not being measured (such as size which may be a critical factor for some buyers), the design of the scoring systems and choosing the weights to use. Even with carefully designed experiments, the decision of 'best washing machine' is not just a product of the data, but of all parts of the model. Consumers in the modern world need to understand this.

The system to decide the winners of the ten-event decathlon in track and field athletics is another complex real-world example of a utility function used as a mathematical model of skill in sports – see this article from NRICH [https://nrich.maths.org/8346]. Each athlete receives a score for performance (either time or distance) in each event. The scoring systems, defined and revised separately for each event, were originally set up so that a world record performance for that event would receive 1000 points. Events are equally weighted, so the scores are added to determine the winner. Other sports offer examples of mathematical models for deciding the best.

A worthwhile mathematical modelling activity is for students to consider how to find the "greatest of all time' for the sport or music style of their own choice or the best product in a category. Students with a common interest can decide on the contenders and the variables (factors) which make 'greatness', decide on how each factor should be scored and assign weights to create the utility function. They can then evaluate how well their mathematical model works to represent their own values. By experimenting with different weighting systems, they can see the importance of all aspects of the modelling. The main purpose of an activity such as this is that students understand more deeply what goes into the many ranking systems that we all see, what features are built into the model, and how they may or may not be useful for a particular purpose. This activity can be presented using the algebra of the utility function, but other students can tackle the same thing by thinking of the weights as percentages. The Choice washing machine website, for example, presents its system in this way: dirt removal has a weight of 40%, rinse performance 20% etc. The essence of mathematical modelling, as the interface between school mathematics and real life, can start well before students learn algebra.

CONCLUSIONS

This paper has selected just a few themes of mathematical competencies in the digital era. In particular, the focus has been on changes to what is taught, rather than how it is taught. There is much more that can be said and much more to be done, even to properly embed the simplest of digital technology (the four function calculator) properly within the school curriculum. Despite fears that the use of digital technology will destroy students' capacity to calculate, it seems likely to me that it is generally underused rather than overused. So, we have some distance to go before school curriculum catches up with society. And at the same time, digital technology races ahead. It seems that the conversation about mathematical competencies in the artificial intelligence era will be even more difficult!

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