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Developing pre-service mathematics teachers' curriculum knowledge for designing mathematics lesson: A professional development model

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ABSTRACT

This paper aims to describe an innovative Teacher Professional Development Program (TPDP) to develop pre-service mathematics teachers' curriculum knowledge. The participants of this study were eighteen pre-service mathematics teachers who have not learned about curriculum as a course in their teacher education program. Two different instruments to measure Pre-Service Teacher Curriculum Knowledge in the domain of statistics and algebra were delivered before and after the TPDP. The innovative TPDP for pre-service teachers in this study considered the current school mathematics curriculum as an entry object of learning. Furthermore, the triangle object such as the development of curriculum among countries, essential mathematics content, as well as designing lessons were elaborated in the model. The developed TPDP and pre-service teachers' lesson designs were described and analyzed qualitatively to see the pre-service teachers' curriculum knowledge. The result of the study shows that there is an improvement in Pre-Service Teachers' Curriculum Knowledge in almost all the components of Curriculum Knowledge, except basic skills and concepts. This implies that for designing a lesson plan, pre-service mathematics teachers need to carefully understand the school mathematics structure as a whole in a brief before the basic competence is reviewed.

INTRODUCTION

It is often claimed that teaching and learning quality is affected by teacher quality, closely related to teachers' competencies to develop the production of favorable educational outcomes (Cochran-Smith & Fries, 2005) including students' achievement. The competencies have a great impact on the teacher's beliefs, values, characters, skills, attitudes, and more importantly, they will affect their teaching practices and have an effect on the achievement of the students someday (Schmidt et al., 2011). If a teacher has fewer competencies, his or her teaching and learning quality will also be less powerful. In other words, teacher quality will be reflected in teaching and learning practice and vice versa. More specifically, in the Indonesian educational document, teacher competencies are pedagogical, personality, social, and professional (GoI, 2005). Pedagogical competencies are all related to knowledge of students, teaching planning and practices, and evaluation. Meanwhile, professional competencies are about the knowledge of the "mathematics"

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curriculum and concept. All competencies should be mastered by a teacher to hold a good teaching practice.

To have a good teaching practice, a curriculum designer should pay attention to pre-service teacher Mathematics content knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) in teaching (Ekawati et al., 2018). However, those two competencies have a special and parallel position. All four are needed to hold a good teaching practice. As for curriculum knowledge, its' existence could not be ignored since it is the core and guide of teaching. In the framework of mathematics knowledge for teaching (MKT) of Ball, (Ball et al., 2008), curriculum knowledge is included in MPCK. They stated that curriculum knowledge is part of the most influential in content knowledge. They define curriculum knowledge as the knowledge of a set of intervention programs designed for teaching certain topics at a given level, the variety of teaching materials available concerning those programs, as well as a set of characteristics that serve as both indications and contraindications for the use of a particular curriculum or program material in certain circumstances. In another way, in most scholars' conceptualization, curriculum knowledge is one of the four common components in PCK (Grieser & Hendricks, 2018), while Shulman (1987) categorizes it as a separate competency of pedagogic competence as well as mathematical content knowledge. Although curriculum knowledge is important, either as a part of pedagogical knowledge or mathematics content knowledge, there are insufficient studies and research on it (Basturk & Donmez, 2011; Lannin, et al., 2013; Sahin & Soylu, 2017). While numerous studies have researched the domain of pedagogic knowledge as well as mathematics content knowledge (Ekawati & Lin, 2014), the research does not cover the curriculum knowledge as well as an approach to develop it. The focus is on teaching knowledge, including developing the instruments, pedagogical knowledge, and learning innovation (Sothayapetch, 2013; Odumosu, 2018; Widodo, 2017). Furthermore, while mathematics knowledge for teaching has been studied by research on particular content in mathematics, e.g. concept of the function by Marbán and Sintema (2020) and tangent function by Malambo (2020), little research was carried out to study the curriculum knowledge for a particular area of content in mathematics.

Generally, the aspects of curriculum knowledge follow the framework of PCK (Ball et al, 2008). It is considered as a subcomponent of pedagogical content knowledge, including two categories, of knowledge aimed at the goals and targets of students to be achieved and the concepts and materials included in the curriculum, specifically for the subjects to be taught (Baştürk & Dönmez, 2011). Such two categories were used as the basis for creating an instrument to assess teachers' curriculum knowledge. For example, Sahin and Soylu (2017) implemented that basis to develop Curriculum Knowledge which specifically examines pre-service teachers' curriculum knowledge on School Algebra. Thus, it opens opportunities for constructing similar tests with different topics, such as geometry or statistics.

In professional education, the knowledge of pedagogy and mathematics content knowledge is given intensively. In the course about them, pre-service mathematics teacher has been taught about teaching planning (designing teaching and learning activity and process, as well as its evaluation). However, their learning designs indicate insensitivity to the curriculum. This is demonstrated by the prerequisite material selection of certain mathematical materials. For instance, most students interpret the prerequisite material as a previous material (taught at a previous meeting) regardless of the relationship or relation of the material to the material to be taught. In this condition, there is an implication on the growing interest to develop Teacher Education programs that support teachers' knowledge including curriculum knowledge. It can be explored through knowledge of the domain material based on a given indicator, the basic concept of curriculum, learning approach, basic skills, the role of teacher and students, the change in curriculum as well as pre-requisite knowledge (Sahin & Soylu, 2017).

TPD that aims to improve teacher PCK should pay attention to essential aspects of teacher professional practice, remove it providing opportunities to apply innovative instructional strategies and materials to facilitate teachers to also reflect, individually and collectively, on their own experience (Van Driel & Berry, 2012). These messages were captured by several innovative in-service Teacher Professional Development (TPD) done to improve mathematics teaching such as the

implementation of the Indonesian version of Realistic Mathematics Education (Ekawati & Lin, 2013). However, a limited study reported an Innovative TPD approach that specifically focuses on developing pre-service teachers' curriculum knowledge. For example, teachers experienced significant improvements when they shared sessions with other colleagues to make a collaboration in professional development which promote teachers' understanding of pedagogical content knowledge, including curriculum knowledge to create connections to the subject matter content across different grades (Sin, 2021). Furthermore, two characteristics can be included in TPD curriculum (Vinovska's, 2011), namely teachers engaged in relevant learning, and teachers are able to make classroom instruction assumptions to develop new learning. Ekawati and Lin (2014) propose Variation theory and develop models to project such as variations in the way students perceive learning objects, teacher understanding and pedagogical design guiding principles. Within the three variations, TPD's designer can propose a program with regard to the local current situation.

Therefore, in this paper, the writer elaborates on the implementation of a TPD approach considering those mentioned characteristics of TPD to develop pre-service teachers' curriculum knowledge as well as its impact on their lesson design. Lesson design is selected as the tool to analyze the emergence of teachers' PCK by several researchers (An et al., 2004; Yunianto et al., 2021). By investigating the feature of teachers' lesson design, the analysis of the integration of their subject matter knowledge and pedagogical content knowledge, including curriculum knowledge, can be obtained. In addition, the curriculum knowledge of pre-service teachers before and after the TPD approach was also described.

METHODS

This study aims to describe the implementation of the Pre-service Teacher Professional Development Program (TPDP) for developing curriculum knowledge as well as its impact on pre-service teachers' mathematics lesson design. There were 18 pre-service mathematics teachers involved in this study. We provide a Curriculum Knowledge Test (CKT) on statistics before the implementation of TPDP and a Curriculum Knowledge Test on algebra after TPDP. CKT was adapted from an instrument developed by Sahin & Soylu (2017). Before TPDP, the participants have not learned about the curriculum and its structure, but in previous courses they had learned about models, teaching strategies, designing learning and evaluation plans. Data collection was carried out in 2018 with direct observation. Each activity that took place in the classroom was recorded and analyzed qualitatively.

The built-in instrument to measure pre-service teachers' Curriculum Knowledge Test is based on the Secondary School Mathematics Curriculum and consists of a total of ten questions which is considered a valid instrument based on expert judgments. The aspects of curriculum knowledge tested to the participants are learning domain, basic concepts, learning approaches, basic skills, strategy/method/techniques, roles of teacher and students, prior knowledge, and acquisition statement. These aspects were used as guideline for creating the CK test for both pre-test (pre-curriculum knowledge) and post-test (post-curriculum knowledge). While the concerned topic in the pre-test is related to statistics, the concerned topic in the post-test is algebra. The topics for those two tests were different from each other, due to some of the topics discussed in the TPDP were about statistics, So it would be no longer relevant if the items examining participants' CK for post-test were released. However, all the aspects of CK in both pre-test and post-test were the same.

This CK instrument was mainly adapted from Sahin and Soylu (2017), where a number of CK factors are taken into account in the components of this CK instrument, some of which include the strategies specified in the curriculum, basic skills set in the curriculum, changes made in the curriculum (Gökkurt et al. al, 2015). In addition, other aspects such as the use of curriculum resource materials (Choppin, 2011), variety of instructional materials, teaching procedures, and learning objectives (Niemelä & Tirri, 2018) are also included in the CK tests. An example of a CK test for assessing the participants' pre-curriculum knowledge is shown in Figure 1.

APP. 1 CURRICULUM KNOWLEDGE TEST

Q.1. The acquisitions regarding the subject of algebra included in the mathematics curriculum are presented in the Table below. Write down which sub learning domain of the algebra learning domain these acquisitions belong to.

No	Acquisitions	Sub Learning Domain
1	They express the rule for an arithmetic sequence in letter; find the requested term of the sequence of which rule has been expressed in letter.	
2	They multiply the algebraic expressions.	
3	They recognize the coordinate system with its characteristics and show ordered pairs.	
4	They divide the algebraic expressions into multipliers.	
5	They establish equations with one unknown of the first order suitable for real life situations.	
6	They draw a graph of linear equations.	
7	They understand the principle of conservation of parity in equations.	
8	They solve inequalities with one unknown of the first order.	
9	They write a suitable algebraic expression for a situation given orally and write a suitable oral situation for an algebraic expression given.	
10	They solve the problems which require establishing an equation with one unknown of the first order.	

Q.2. Which **basic concepts** regarding statistics take place in the Mathematics Curriculum?

Q.3. On which learning approach is the Mathematics Curriculum based? What are the basic principles of this approach?

Q.4. The improvement of which **basic skills** in the Mathematics Curriculum does statistics ensure? Explain.

Q.5. Which strategies, methods, and techniques do you prefer in order to render teaching statistics more comprehensible for secondary school students within the basic philosophy of the Mathematics Curriculum? (Lecture method, expository teaching, exploratory teaching, question and answer method, discussion, brainstorming, case method, problem solving, demonstration, computer-assisted teaching, etc.) Explain your reasons?

Q.6. What are **the teacher and student** roles in a classroom environment according to the Mathematics Curriculum?

Q.7. How much do you know about **the changes made in the last 10 years** in the Mathematics Curriculum in our country? Explain.

Q.8. How can you reach the Mathematics Curriculum? Where and how often can you follow the changes regarding the curriculum?

Q.9. In your opinion, what are the prior knowledge and concepts students need to know before learning the statistics concepts given below?

Concepts	Prior Knowledge
Reading and interpretation of data or graphs Average, range and quartiles	
Histograms Scatter diagrams	
Correlation/linear regression	

Q.10. What are the characteristics of **an acquisition statement** regarding the Secondary School statistics subjects? **Q.11.** What are the characteristics and basic components of **a lesson plan**? How are the lesson plans procured in the current system?

Figure 1. Curriculum knowledge test

Figure 1 indicates the list of questions examined to the participants. Since curriculum knowledge is unique for particular content knowledge, the questions exam includes of two different topics for pre-test and post-test. Concerning teachers' pre-curriculum knowledge, the designed TPDP and teachers' lesson design were described and analyzed qualitatively.

RESULTS AND DISCUSSION

Pre-service mathematics teacher pre-curriculum knowledge

Curriculum Knowledge Test in the domain of Statistics was delivered to all participants. There are ten component questions in CKT on Statistics. For the component on "Learning Domain", there are participants who have problems in the sub-learning domain, namely in the differentiated learning domain and the sub-learning domain. In addition, participants also had problems distinguishing and classifying types of learning domains based on acquisition statements. The second question indicates that pre-service teachers lack knowledge of basic concepts in statistics. They have the same thinking between basic concepts and basic competencies in the curriculum. Some of them mention all the basic competencies in the mathematics curriculum and also previous knowledge in statistics.

For the fourth question about basic mathematics, skills were improved in statistics, most participants answered using basic knowledge and developed in statistical concepts. They have a problem differentiating between basic mathematics skills and prior knowledge in statistics. The description above shows that most students still have difficulties in differentiating between basic concepts, basic competence, basic skills in the mathematics curriculum, and prior knowledge in statistics. Furthermore, pre-service teachers have a problem distinguishing between approaches, methods, models, techniques, and strategies in a learning activity. They just answer the question using learning methods although the question is about the methods, models, techniques, and strategies in a learning activity.

In terms of theirs, 76% of participants could determine the role of the teacher with regard to the current curriculum. For example, the role of teacher which regards to the view of student-centered instead of teacher-centered were mostly found from the participants' responses. This is evidenced by the responses of teachers as facilitators of students' learning processes, who encourage students to gain a higher level of understanding the concept of statistics. Meanwhile, the remaining participants chose not to recommend student-centered teaching due to the nature of the statistic concept, which is to be directly demonstrated. Regarding the type of learning approaches, a few students answer with the type of approach based on centered in a learning activity, such as student-centered and teacher-centered, and there was a student who answers in classical learning. Furthermore, with regards to the change in curriculum, some of the participants did not pay attention to the question which ask about the changes during the last 10 years, so they answered all the changes made in the Indonesian mathematics curriculum. Then, most students answered it based on learning activity and or material. Furthermore, six participants answered based on the type of curriculum and four students answered based on curriculum system, learning activity, and material in mathematics. Thus, they have a limited understanding of the change in the curriculum.

For the question about prior knowledge, there are 38% of participants shared their prior knowledge of several concepts in statistics. Others mention the basic knowledge in statistics, but not the prior knowledge. Basic knowledge is the science of statistics to understand statistical concepts, which if students go through this learning, statistics will be more difficult to learn. When students do not understand such pieces of knowledge well, they will likely take some difficulties in learning statistics. Our findings indicate that their answers about the prior knowledge have a distance from the main knowledge in statistics. For example, several participants wrote that the prior knowledge of understanding reading and interpretation of data or graph, average, range, and quartiles, histograms, and scatter is the knowledge about data collecting, which is correct but incomplete. They did not write the knowledge of types of data (nominal, ordinal, scale, ratio) as the basis of collecting and analyzing data. Furthermore, the participants did not give correct responses on the prerequisite knowledge required to learn before trying to understand correlation/linear regression, which is regarded as an important topic of inferential statistics. Instead of providing the main knowledge of the requirements of examining a set of data using correlation or regression (e.g., normally distributed data, types of data for correlation test), the participants mentioned some statistic tests used to compare the means of two or more groups (t-test, Z-test). Such responses can be seen in Figure 2.

Q.9. In your opinion, what are the prior knowledge and concepts students need to know before learning the statistics concepts given below?

Concepts	Prior Knowledge
Reading and interpretation of data or graphs	Pengelompokan data.
Average, range and quartiles	Prosentase, decimals, fractions.
Histograms	Pengelompokan data,
Scatter diagrams	Pengelompokan data,
Correlation/linear regression	z table, t table, how to read table,

Figure 2. Example of participants' responses on the aspect of prior knowledge

In addition, regarding the component question about the lesson plan, students still have a misunderstanding about the last question. They could not mention the characteristics of the lesson plan. Only a few students complete their answers with the characteristics. For example, based on the result of the Statistics Curriculum Knowledge Test, there is still more opportunity to learn for pre-service teachers in the process of designing mathematics lessons. They need to improve their knowledge of the curriculum, especially in prior knowledge, basic Mathematics skill, and basic concept. So, they can design mathematics lessons well.

Pre-service teacher professional development program

Based on the results of pre-curriculum knowledge of Pre-service teachers, then developed a Pre-service Teacher Professional Development Model. It also considers the learning objects in the current school mathematics curriculum. This is in line with Ekawati and Lin (2014) that mathematics in schools is considered as an entry point to place pre-service teachers in active learning situations in community practices and authentic situations. With various experiences in secondary schools, pre-service teachers will be the object of the Competency-based Curriculum as well as the 2013 Indonesia Curriculum. They feel different opportunities to learn with two common curricula. In the mathematics curriculum and the 2013 Indonesian curriculum, they experienced more active learning in achieving mathematical concepts. After that, awareness of the important points of the mathematics curriculum such as problem solving, basic literacy skills, and technology integration must be possessed by all participants. Through the discussion, pre-service teachers who understand the three main points, will be able to mention about the stages of Polya problem solving and the implementation of mathematics teaching. In addition, several ideas emerged about mathematical literacy as well as the integration of technology in mathematics teaching.

The first stage occurs after two components of Pre-service Teacher Professional Development are carried out. Furthermore, as the second stage, we consider the triangle model activity that consists of The Development of mathematics curriculum among countries, the basic standard, and content on essential mathematics, and designing mathematics lessons. The dashed arrow is used in the model to facilitate the flexibility of the sequences. The overview above is presented within a model used for Pre-Service Mathematics Teacher Professional Development in Figure 3.

In the second stage of TPDP, we started with the activity of exploring the appearance and development of mathematics curriculum among countries. Five countries such as Singapore, Australia, UK, USA as well as South Korea Curriculum. The five different mathematics curricula were discussed and were also compared with the Indonesian mathematics curriculum. To be more specific, pre-service teachers' participants also considered the basic standard on each essential mathematics content such as algebra, statistics, numbers, and geometry. In addition, they were also allowed to share their finding on mathematics lesson design that can be found in the textbook performance and also some mathematics teaching videos across countries.

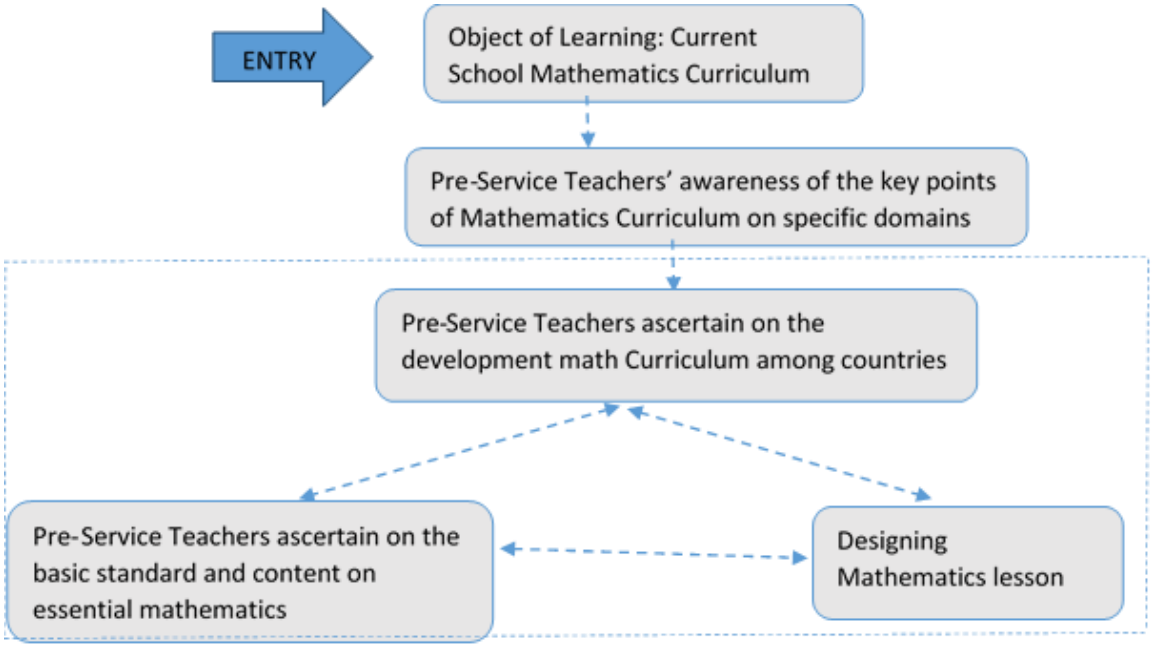

Figure 3. The used model for pre-service mathematics teacher professional development program

The third activity was designing a mathematics lesson. We consider adapting Mathematics teachers as instructional designers (Vinovska et al., 2011). All pre-service teacher participants planned the lesson by considering the curriculum program to achieve the lesson aims and its outcomes. They examined the mathematics textbook that includes the introduction of new content, solved examples, and exercises from several countries. After they evaluated the curriculum and textbook, they decided on the mathematics lesson design that fits the Indonesian situation. The TPDP model was implemented with 18 pre-service mathematics teachers' participants. As many as eighteen mathematics lesson plans were designed by all TPDP participants. Figure 4 shows the example of mathematics lesson design by pre-service teachers' participants.

Figure 4 shows the developed mathematics lesson plan by a pre-service mathematics teacher using their curriculum knowledge. The developed lesson plan shows that in terms of curriculum knowledge, students formulate teaching goals from the materials, yet there is no information related to the basic competence and indicators, in which they should be used as the foundation in formulating the teaching goals. However, the goals are still in synch with the materials.

In terms of the teaching model, Figure 4 tells that the pre-service mathematics teacher who designed that lesson plan chose to use active learning. This is evidenced by the use of contextual situations as starting point to learning algebraic expression, where students in groups are expected to be active in responding and collaborating with teachers and other students to find the solution to algebraic problems. The learning objectives have also been arranged hierarchically starting with paying attention to prior knowledge that is appropriate for learning algebra at the school level from constructing algebraic expressions to calculating algebraic products. This might be interpreted as one of the TPDP results which also encourage students to identify the key points of mathematics curriculum on specific domains. Within our TPDP, the pre-service teachers were also engaged in the activities of discussing some alternatives of learning sequences for given basic competencies from the Indonesian mathematics curriculum.

Material	Goal
EXPRESSIONS AND EVALUATION 1. Building expressions 2. Key words in algebra 3. Simplifying expressions 4. Algebraic products 5. Evaluating algebraic expressions	1. Student be able to building expression 2. Student know and use the key words in algebra properly 3. Student be able to simplifying expression in algebra 4. Student be able to calculate algebraic product, and improve their skill to calculate algebraic product 5. Student be able to improve their skill to evaluating algebraic expression

Lesson component (Based on Cambridge Book)	Teacher Activity	Times
Opening	Greeting, pray, check student attendance	10 minutes
Concept Development		50 minutes
1. Develop student thinking about building expression. (Student be able to building expression)	Teacher use contextual problem to improve the student skill about building expression ➢ Suppose we have some cups and some marbles . We can place a certain number of marbles into each cup, then write expressions for the total number of marbles present. Suppose we have cups with marbles in them plus marbles left over.  Find how many marbles are present if: Leni has put 7 marbles into each cup Teacher guide the student to think that 7 marbles on the cup and add with 2 marbles. Answer : If Leni has put 5 marbles into each cup, then there are 3 × 7+2 marbles.	
	➢ Like terms are terms with exactly the same variable form. ➢ The constant term of an expression is the term which does not contain a variable. The coefficient of any term is its numerical part, including its sign.	
3. Develop the student skill to simplifying expression (Student be able to simplifying expression in algebra) 4. Develop the student skill and thinking to calculate algebraic product. (Student be able to calculate algebraic product, and improve their skill to calculate algebraic product) 5. Develop the student skill to evaluating algebraic expression. (Student be able to improve their skill to evaluating algebraic expression)	➢ Teacher ask the student to make a group. every group contain 3 student. The student in every group is random. ➢ Teacher give the student's worksheet. Then ask the student to answer the question. ➢ After that ask the student to present their work in front of their friends	
Closing	➢ Teacher giving some problem and the student must looking for the solution with their group at home. ➢ Teacher and student conclude about they had learned.	20 minutes

Figure 4. The developed mathematics lesson plan on algebra by pre-service teachers

Learning plans on worksheets are used to facilitate students in developing simplification concepts and evaluating algebraic expressions in the learning process, teachers will accommodate and become facilitators in guiding students. Furthermore, the teacher does not directly explain the basic concept to the students, but makes them discuss the problem on the worksheet with their friends. This also indicates that the lesson plan is designed based on constructivism, indicated by facilitating the student to observe, construct their knowledge, discuss, and make a meaningful conclusion that the designed lesson plan supports the scientific approach. Overall, the implementation of curriculum knowledge components (using percentage) in designing mathematics lessons is shown in Figure 5.

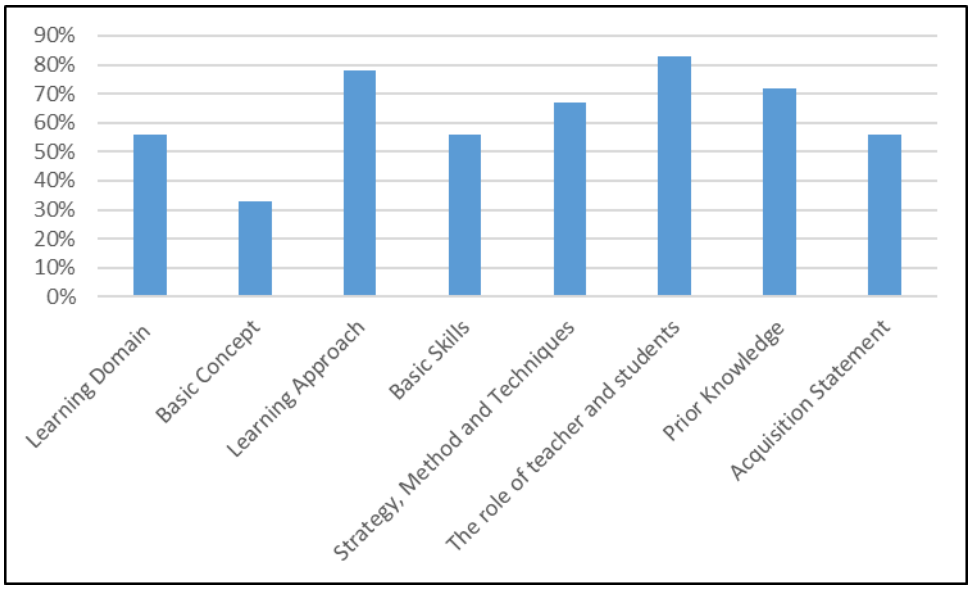

Figure 5. Percentage of pre-service teachers who implement curriculum knowledge as expected in the ongoing curriculum in designing a mathematics lesson plan

Figure 5 showed the use of teachers' curriculum knowledge when designing mathematics lessons. The percentage shows the number of pre-service teachers who provide eight aspects of curriculum knowledge as suggested in the ongoing curriculum document in Indonesia. It is clear that the most consideration is the role of teachers and students in the teaching and learning process that have been changed based on the change of curriculum. They tend to agree with the role of the teacher, as a facilitator. In addition, most learning approaches used in the mathematics lesson design were inquiry with constructivism. The mathematics lesson design is not considered to lead students as passive receivers, but rather as active learners within the developed activity. However, less than 50% answered correctly about basic skills in the learning design showing missing pieces of knowledge regarding the learning sequence that corresponds to the set learning objectives. Furthermore, the designed lesson plan in Figure 4 also does not contain the so-called apperception in which the teacher reminds the students of the prior knowledge and basic concepts that are important to learn Algebra.

Pre-service mathematics teacher post-curriculum knowledge

We delivered Pre-Service Mathematics Teacher Post-Curriculum Knowledge on the domain of algebra to all participants after a series of TPDP activities. Several changes appeared in every component of Curriculum knowledge. Table 1 shows the performance of Pre-Service Mathematics Teachers' curriculum knowledge after experiencing the TPDP. It also describes the score improvements of curriculum knowledge before and after the TPDP quantitatively.

It is obvious that there was a score improvement in the participants' curriculum knowledge before and after the TPDP. Overall, there was an increase of about 4% from 28.5 out of 44 to 29.65 out of 44 although some aspects were not indicated to show significant improvements, such as the aspects of basic concepts and basic skills. Our analysis is that those two aspects are highly related to the content of statistics (pre-test) and algebra (post-test), so it is incomparable for this issue. This finding indicates that the pre-service teachers experience difficulties in arranging learning pathways, in which basic knowledge and basic skills are simultaneously required in order to set learning goals in mathematics students' learning experience. In relation to learning trajectory, this is primarily related to learning goals and learning activities, in which knowledge of basic concepts and skills determine the uniqueness of learning sequences with the intent of supporting students' achievement of specific goals in particular mathematical domains (Clements & Sarama, 2012).

Table 1

Pre-service mathematics teacher curriculum knowledge before and after TPDP

Components on curriculum Knowledge	Results of post-curriculum knowledge	Pre-test		Post-test	
		Mean	SD	Mean	SD
Learning Domain	Pre-Service Teachers understand kinds of learning domains such as cognitive, affective, and psychomotor. They also describe the affective effects that they observe from learning in the real class	2.81	0.46	3.50	0.00
Basic Concepts	Pre-service teachers consider the basic concept as basic knowledge to learn algebra or rather related to prior knowledge.	1.3	0.92	1.14	0.64
Learning Approach	Pre-Service teachers describe the learning approach based on the Indonesian Curriculum as well as the Scientific approach. Only several of them try to relate potential approaches specific to mathematics learning such as Contextual Learning, Realistic Approach, and Problem Solving. It was not a problem because if we see in the Indonesian curriculum, especially in mathematics, theirs used in the learning activity.	2.85	0.67	2.85	0.63
Basic Skills	Basic skills in Mathematics used in Algebra are calculating, problem-solving, and applying.	2.65	0.93	2.05	0.90
Strategies, Methods, and Techniques	Pre-Service Teachers can choose a method for mathematics learning according to algebra. But they can differentiate between strategies, methods, and techniques.	2.6	0.60	2.90	0.29
Roles of Teacher and Student	Pre-Service Teachers can determine the role of teacher and student in mathematics learning. They describe the teacher as a facilitator in the learning activity.	3.85	0.49	3.52	0.91
The changes made in the last 10 years in the Mathematics Curriculum	All of the participants considered this part as the changes of the curriculum in Indonesia. They mentioned KTSP and K-13 with some information about the principal changes in the roles of	2.75	0.57	2.90	0.68

Components on curriculum Knowledge	Results of post-curriculum knowledge	Pre-test		Post-test	
		Mean	SD	Mean	SD
	teachers and students. The curriculum can't explain focus in mathematics, but just talked about the role of teacher and students, and the different learning approaches. Just a student who also answers about the indicator (IPK).				
Follow Math Curriculum	Almost all of the participants follow the mathematics curriculum using the internet, book, and lesson in college.	2.76	0.69	3.50	1.02
Prior Knowledge	Most pre-service teachers answer the prior knowledge using the prior knowledge connected with the material. However, some of the participants can answer correctly.	2.08	0.49	3.05	0.71
Acquisition Statement	The pre-service teacher performed some acquisition statements in statistic (for pre-test) or algebra (for post-test)	2.15	0.75	1.62	0.72
Lesson Plan	Pre-Service teachers know what a lesson plan is and what its components are, but some of the students can't mention all of the components of the lesson plan.	2.35	0.49	2.62	0.58
Total		28.5		29.65	

The improvements in the participants' curriculum knowledge can also be explained qualitatively from their written responses. For example, regarding the aspect of the learning approach, various types of learning approaches were mentioned such as realistic mathematics education, discovery learning, scientific approach, inquiry-based learning, problem-based learning, and project-based learning. Those approaches can be regarded as innovative learning approaches which have been implemented in many countries. Furthermore, project-based learning is considered a recommended approach in the current curriculum, e.g., Indonesia (MoE, 2020) and some recent research (Hossein-Mohand et al, 2021; Xiong, 2021). In the TPDP, the participants also got experience in comparing some innovative learning approaches used by other countries and discussing the potential of such approaches to be implemented in the Indonesian curriculum. However, the portion of using digital-based learning approaches as also encouraged by recent research is not explicitly stated in the participants' responses. Some digital-based learning approaches such as flipped learning (Wang, 2017), hybrid or e-learning (Shetu et al, 2021), which utilizes learning management system, gamifications (Ariffin et al, 2022; Hossein-Mohand et al, 2021). The approach regards to the

use of game-specific models, strategies, dynamics, mechanics and elements in contexts beyond these aiming at delivering messages or changing behavior, through a play experience that encourages motivation, engagement, and fun (Pérez-Ordás et al, 2021), digital technology as learning media for classroom teaching, are not found in such responses.

In this research, the ten aspects of curriculum knowledge are in line with other aspects chosen for the further analytical framework. According to Remillard & Kim, (2017) aspects of curriculum knowledge are basic mathematical ideas, representations and connections among these ideas, problem complexity, and mathematical learning pathways. Foundational mathematical ideas are related to basic skills and prior knowledge, while representation and connection are related to the acquisition statement. Meanwhile, mathematical learning pathways are related to basic skills, prior knowledge, lesson plan, the role of students and teachers, as well as strategies, methods, and techniques. These can be regarded as the aspects of curriculum knowledge related to learning trajectories (Clements & Sarama, 2012). The lesson plan designed by the participants of this study, however, was indicated to show only one alternative of learning pathways. In fact, it is possible to arrange more than one learning pathway for the same learning objectives. Thus, future research can consider this issue for exploring teachers' curriculum knowledge.

CONCLUSION

To conclude, we argue that the TPDP model for pre-service mathematics teachers can be chosen as an innovative model. The unique triangle models such as the development of curriculum among countries, basic standards and content on the essential concept of mathematics, and the design of mathematics lessons could be considered a promising model to lead the development of pre-service teacher curriculum knowledge with the active learning environment. The analysis of the curriculum Knowledge test shows improvements in pre-service teachers' curriculum knowledge. Furthermore, several components in the Curriculum Knowledge Test were implemented by several pre-service teachers in designing mathematics lessons.

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