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An analysis of mathematics textbook of Indonesian curriculum based on functional thinking framework

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ABSTRACT

Functional thinking (FT) is a cognitive ability needed by students to build an understanding related to functions. However, not many studies address the examination of FT problems within school mathematics textbooks, particularly in Indonesia. This study aims to describe the mathematics problems presented in junior high school mathematics textbooks of the Indonesian curriculum (Kurikulum Merdeka) based on the FT framework. The mathematics textbooks, Grade 7 and 8, published by The Indonesian Ministry of Education, were used to conduct the analysis. The data was collected by observing the textbook's contents in the algebra domain. The analysis was based on the framework of FT, namely recursive patterns, covariance, and correspondence. The results show that the FT tasks appear in the learning of algebraic terms, modelling with algebra, and linear functions. The tasks in algebraic terms satisfy all FT stages in hierarchical order, tasks in modelling with algebra focus on the correspondence stage, and tasks in linear functions include all FT stages but not in chronological order. In conclusion, this study finds that algebraic problems provided by Indonesian textbooks already support students' FT. Thus, designing more FT tasks and connecting each task to different algebra topics would help students nurture their FT.

INTRODUCTION

The issue of addressing early algebra within school mathematics has become notorious among researchers since traditional arithmetic-algebra learning raises a problem for students to develop their thinking from concrete to abstract (Bråting, 2023; Carraher & Schliemann, 2007; Kieran et al., 2016). According to Kaput (2017), the essential aspect of early algebra is generalisation, namely identifying patterns from specific instances, and symbolisation, which is how to express the generalisation with a symbolic system. Derived from Kaput's idea, Blanton et al. (2018) proposed mathematics content strands of early algebra in three main topics, i.e., generalised arithmetic; equivalences, expressions, equations, and inequalities; and functional thinking. The recent study will focus on functional thinking.

Functional thinking (FT) did not mean teaching formal functions we presently understand. It focuses on supporting students in generalising relationships between quantities, representing those relationships, and reasoning with these representations to predict function behaviour (Krüger, 2019; Stephens et al., 2017). Moreover, FT is essential since many mathematics problems faced by students require this ability to solve. For instance, students start recognizing numerical patterns in elementary school, progress to generalizing geometric patterns and learning of functions in secondary school, which is followed by geometric patterns and functions in secondary school, which serve as essential foundations for comprehending calculus in higher education (Blanton & Kaput, 2005; National Council of Teachers of Mathematics [NCTM], 2000). Taking these topics appropriately, students must

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cultivate an understanding of the functional relation between co-varying quantities and attain proficiency in FT, as highlighted by Lichti and Roth (2018).

Students' FT can be traced by how they identify the relations between variables, which are categorised into three stages (Confrey & Smith, 1994; Doorman et al., 2012; Smith, 2017). The lowest stage is *recursive patterns*. At this stage of thinking, students considered the variables' relation as the calculation process, where a specific input can produce a particular output through an operation. Following the process, students may identify that the change of values in independent variables influenced the change of values of the dependent variable. This stage of thinking is called *covariation*. Finally, students can recognise the relation between variables in general; that is, the relation can be applied to any value of both variables. This process is the highest stage of functional thinking, namely *correspondence*. In addition, growing pattern problems, which are central to FT, can be depicted either figuratively or geometrically (Hunter & Miller, 2022).

Although the importance of FT ability is frequently discussed in the literature, different studies have reported students' difficulties in coping with this ability in mathematics learning (Pinto & Cañadas, 2021; Ramírez et al., 2022). The most common issue found in studies is students' inability to recognise the pattern within a sequence of values; instead, they choose to count the values in consecutive numerical steps (El Mouhayar, 2018; Rivera, 2018; Wijns et al., 2019; Wilkie & Clarke, 2016). More advanced students frequently use natural language to express the relation between values because they have an insufficient understanding of variables (Ayala-Altamirano & Molina, 2020; Lucariello et al., 2014; Wilkie, 2016). In FT, which leads to the function's concept, students are required to understand variables to represent co-varying quantities, as mathematicians first used the term (Doorman et al., 2012; Kleiner, 1989). However, students' restricted image of variables is often influenced by previous learning that states the variable as the unknown (as a single quantity) (Jupri et al., 2014; Kaput, 2017).

One particular research that examines how Indonesian students solve word problems that require FT ability is the Utami et al. (2023) study. They emphasized their study on identifying ninth-grade students' solution methods in generating patterns. The findings revealed that many students only reach the lowest FT level in identifying patterns. They commonly attended to the value changes in one variable without considering that values in both variables change simultaneously. Additionally, their failure to write the algebraic representation was primarily caused by their lack of understanding of the variable concept (Beeh et al., 2018; Tekin-Sitrava, 2017). Students considered that a variable is only denoted by the word x and y; thus, they found difficulties when the variable is denoted with n in this study. Considering the students' challenges in acquiring FT ability, especially for Indonesian students, there is an urgency to investigate further the source of their difficulties. In this case, a curriculum analysis can be the starting point to assess how students' FT ability is developed at schools.

One of the essential parts that reflects the learning objective within the curriculum document is the textbook used by students and teachers (Fan et al., 2013; Rezat et al., 2021). A survey conducted by the TIMSS group reported that textbooks are used as the primary teaching resources for more than seventy percent of teachers in the world; meanwhile, in Indonesia, the use of textbooks is prevalent among ninety-three percent of teachers (Mullis et al., 2012; Word Bank, 2010). Textbooks also provide opportunities for students to learn mathematics since the detailed contents and problems presented in textbooks form the way for them to acquire knowledge and mathematical abilities. As suggested by van Zanten and van den Heuvel-Panhuizen (2021), mathematics is not ready-made knowledge, but it is essentially seen as a set of activities of the learners. Therefore, conducting research in mathematics textbooks is noteworthy, which strives to improve the teaching and learning of mathematics.

The government of Indonesia, through The Ministry of Education, Culture, Research, and Technology (*Kemdikbudristek*), has made several curriculum changes to provide solutions to global challenges. The most recent curriculum established by *Kemdikbudristek* is *Kurikulum Merdeka* in 2022. According to the National Education Ministry's regulation (*Permendiknas*) No.11/2005, textbooks are strategically positioned to improve education quality, especially for primary and secondary education. Thus, the quality of textbooks contributes to the student's learning quality.

A number of studies have analysed Indonesian mathematics textbooks using various approaches. Wijaya et al. (2015) reported that in Indonesian textbooks, only ten percent of the tasks presented are context-based. Their study also resonates with Hidayah and Forgasz's (2020) study that Indonesian textbooks offer more routine problems than Australian textbooks. Using the context of PISA, a study showed that mathematical tasks within Indonesian textbooks were in accordance with PISA's frameworks (Murdaningsih & Murtiyaya, 2016). Different types of tasks presented in Singaporean and Indonesian textbooks, where Singaporeans provide more high-level cognitive demand tasks, become the reason for different PISA mathematical abilities between students of these two nations (Yang & Sianturi, 2017). Moreover, some analyses were done using the praxeological framework. For instance, the Indonesian textbook analysis on the proportion topic by Wijayanti & Winslow (2017) and the spatial figures topic (Yunianta et al., 2023). Specifically, in the context of functions, Wijayanti (2018) found that linear function learning in Indonesian textbooks mostly begins with a naïve set theory without creating opportunities for students to build a connection of proportion to function.

Although these textbook investigations make essential contributions to textbook research, no study focuses on the analysis of problems provided by Indonesian textbooks based on the functional thinking framework. Remarkably, not many studies address the examination of the contents with *Kurikulum Merdeka* since the curriculum is relatively recently implemented in Indonesia.

In this study, the researchers focused on the analysis of student textbooks for seventh and eighth-grade *Kurikulum Merdeka* with the subject of mathematics. Therefore, this study aims to analyse and describe the contents and contexts in seventh and eighth-grade mathematics textbooks based on the framework of FT. The focus on analysis lies on (1) the use of FT tasks in learning algebra and (2) the appropriateness of the tasks with FT framework.

METHODS

The study was qualitative research with a documentary method. Documentary research method refers to the analysis of documents that contain information about the phenomenon that is wished to be studied (Bailey, 2008). The method is used to investigate and categorize physical sources, most commonly written documents, whether in the private or public domain (Payne & Payne, 2004). In this study, the documentary research method was used to analyse the public Indonesian mathematics textbooks, where the analysis was conducted to investigate task sequences in the textbooks based on the functional thinking framework.

Object of study

The object of study was mathematics textbooks for seventh and eighth-grade junior high school students under the Kurikulum Merdeka. The textbook resources utilized in this study were the grade seven mathematics textbook authored by the Indonesian team, while the Japanese team, Gakko Tosho, authors the grade eight mathematics textbook. The selection of these textbooks was motivated by the Indonesian Ministry of Education's initiative to adapt Japanese mathematics learning within the Indonesian educational context through the Kurikulum Merdeka. Therefore, we would like to consider both perspectives. The textbooks' title is "Matematika untuk Sekolah Menengah Pertama Kelas VII" and "Matematika untuk Sekolah Menengah Pertama Kelas VIII". The textbooks are released and authorised by the Indonesian government and are available online at https://buku.kemdikbud.go.id/katalog/buku-kurikulum-merdeka.

Data collection

The data collection technique was done by examining task sequences in each selected textbook. Specifically, the data were collected from tasks within the algebraic domain, that is, *Chapter 4: Algebra Form* in the seven-grade textbook and *Chapter 3: Linear Function* in the eight-grade textbook. This study also focused on selecting tasks in the introductory and exercises sections of each chapter, which satisfied the FT framework.

Data analysis

Data analysis was done by data reduction, data presentation, and conclusion. The phases of data analysis were as follows: first, data of introductory tasks and exercises in each chapter were scrutinised and classified based on the indicators of FT, namely recursive patterns, covariation, and correspondence. Second, the analysis result was presented in tables (describing the FT stages and problem context presented in each subchapter) and figures (linkage between tasks sequence with the order of FT stages). Moreover, the analysis was described in detail. Third, a conclusion was made from the analysis results.

To ensure the validity of this study, we refer to the four principles that must be satisfied in handling documentary research, namely authenticity, credibility, representativeness, and meaning (Ahmed, 2010). We uphold the authenticity principle by scrutinizing tasks directly from the official mathematics textbooks issued by the Indonesian government. We refrained from modifying these tasks, preserving their originality to maintain the credibility principle. As the textbooks are government-published, the task sequences they contain faithfully represent the typical mathematics learning in most Indonesian schools, thereby satisfying the representativeness principle. Finally, fulfilling the meaning principle, we conducted focus group discussions with experts, ensuring the analysis results clarity and alignment with the true meaning of the phenomenon being studied.

FINDINGS

Based on the analysis of the tasks in the grade-seven textbook, this study found that tasks about identifying patterns are presented in several sections of the algebraic form chapter. First, it appears as the initial context to introduce algebraic terms. This task is in the *Ayo Bereksplorasi* section, which we considered an introductory task. The introductory task provided students with a step-by-step solution method; sometimes, some steps are left blank to be filled in by students, and the end of the solution is intended to build students understanding of particular mathematical objects. Second, identifying pattern tasks also appears in exercises. From eight algebraic exercises, two tasks are given about generalising and symbolising geometric patterns.

Moreover, symbolising patterns are also presented as the first context about the following subchapter: modelling with algebraic form. Unlike the geometric patterns in the algebraic term subchapter, the textbook presents daily-life contexts in modelling with algebraic form tasks. Symbolising patterns tasks appeared in the *Ayo Berekplorasi* and *Ayo Mencoba* sections. The task is also presented in exercises, occupied one out of five questions in the section. The detailed analysis of each task is discussed as follows.

Grade 7 textbook, Chapter 4: Algebraic Forms, Sub Chapter: Algebraic Terms

According to the textbook, identifying patterns tasks are presented to introduce students to algebraic terms. We chose the task since FT's characteristics can be categorised by identifying patterns activity. In the textbook, the context *Pola Korek Api* (match's pattern) is used to encourage students to find the pattern: *how many matches are needed to shape several squares?* Figure 1 illustrates the first stage of the student's activity to find the pattern in the textbook.

In Figure 1, the textbook provides examples of mathematical expressions to find the number of matches to shape 1, 2, 3, and 4 squares. Notice that the textbook also writes the number of matches in each term following a specific pattern. For instance, instead of writing 7, the textbook expands it with $1 + (2 \times 3)$. Additionally, the textbook also labels each number in the expression with A, B, and C. We analysed that the three letters are expected to help students determine which numbers are constants and which vary. At this stage of activity, students are asked to pay attention to Figure 1. The stage requires students to calculate the output (the number of matches) from a given input (the number of squares). Unfortunately, finding the values of a sequence is not included in students' activity since it is delivered exclusively by the textbook.

Following the context given in Figure 1, the book encourages students to focus on mathematical expression, which is labelled with A, B, and C. Students' activity in this task is to name each letter based on their observation in Figure 1 and to determine whether or not the value of each letter varies, as shown in Figure 2.

In Figure 2, students are asked to name each letter based on the figural patterns observed in Figure 1. For example, A is 1, B means the number of squares, and C is 3. Based on students' observation of the mathematical expression in Figure 1, students can decide which labels whose values change or not. In Figure 2, students can write that the values of A and C remain the same while

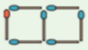
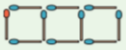
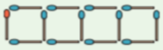
The pattern	The mathematical expression (the number of matches used)	A	B	C
	$1 + (1 \times 3)$	1	1	3
	$1 + (2 \times 3)$	1	2	3
	$1 + (3 \times 3)$	1	3	3
	$1 + (4 \times 3)$	1	4	3
⋮	⋮	⋮	⋮	⋮

Figure 1. The first student's activity to generalise pattern (translated from Susanto et al., 2022, p. 127)

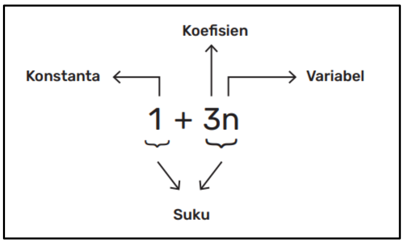
Label	Label according to the square pattern characteristic	Does the value remain the same or vary?
A	...	...
B	...	...
C	...	...

Figure 2. Students' activity to determine variation in the pattern (translated from Susanto et al., 2022, p. 128)

$$1 + (1 \times 3)$$
$$1 + (2 \times 3)$$
$$1 + (3 \times 3)$$
$$1 + (4 \times 3)$$
$$\vdots$$
$$\downarrow$$
$$1 + (\underline{\hspace{6cm}} \times 3)$$
Isi dengan judul kolom yang sesuai

(a)

$$1 + (\_\_ \times 3) = 1 + 3\_\_$$



(b)

Figure 3. Students' activity to generalise the pattern: (a) fill the blank space with natural language, (b) fill the blank space with n (translated from Susanto et al., 2022, p. 128-129)

the values of B vary. Students are encouraged to realise that the output's value depends on the input's value.

At the end of the task, the textbook presented a way to generalise the number of matches to make n squares. Before using the letter n to label the number of squares, the book bridges students to use their answer in Figure 2 to fill the blank space in Figure 3. In Figure 3a, students can write *the number of squares* or similar phrases and obtain the generalise number of matches needed, that is, $1 + (\textit{the number of squares} \times 3)$. At the end of this task, students are expected to understand the meaning of variables, constants, coefficients, and terms, which are unified within an algebraic form (Figure 3b).

Furthermore, the task consists of generalising patterns also given in exercises. Table 1 shows the two tasks of identifying patterns included in exercises, where the task context still focuses on geometrical patterns.

Table 1

Exercises in identifying patterns (translated from Susanto et al., 2022, p. 134-135)

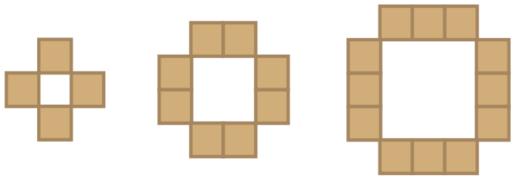
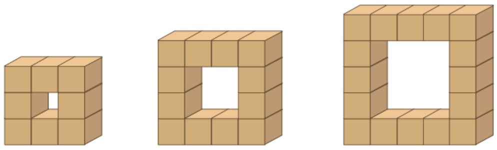
 Ukuran 1 Ukuran 2 Ukuran 3	Task 1: a. Describe in a sentence the number of tiles used to shape the square area in each term. b. Represent the description in (a) in algebraic form. Use a letter to describe a variable and give meaning to the variable used. c. How many tiles are used to shape the area of a square with 10 sizes?
 Pola 1 Pola 2 Pola 3	Task 2: a. Write down the algebraic form representing the number of cubes to shape the patterns in each term. Use a letter to represent a variable and give meaning to the variable used. b. Explain how you know if the algebraic form you generated is correct.

Table 2

Example of the introductory task from modelling with algebraic forms topic (translated from Susanto et al., 2022, p. 147)

AYO BEREKSPLORASI TASK The distance between Wisnu's house and his school is 5000 m. Wisnu goes to school with his father, riding a motorbike with a speed of about 15 m per second. Determine: a. The algebraic form of the distance travelled by Wisnu after t second. b. The algebraic form of the remaining distance that Wisnu needs to take to arrive at the school. c. The distance travelled by Wisnu after 2 minutes. d. The remaining distance that Wisnu needs to take to arrive at school after 2 minutes driving from home. e. The distance travelled by Wisnu after 3 minutes.

To answer Task 1 part a, in Table 1, it requires students to count the number of tiles needed in each term. Following that activity, students may start to identify the pattern, such as *if the size of the square's side is 1, then the number of tiles needed is 4*. Finally, they can answer the a part with *the number of tiles used is four times the side's size of the square*. Moreover, Task 1 part b requires students to symbolise generalisation, which can be answered if the student correctly answers the a part. Task 1 part c part can be answered using the answer in b. Assessing these types of problems, we can see that Task 2 part a also possesses a similar characteristic to Task 1 part b. An additional task is needed, that is Task 2 part b, to encourage students to justify their answers in a.

Grade 7 textbook, Chapter 4: Algebraic Forms, Sub Chapter: Modelling with Algebraic Forms

The following topic learned by students is modelling with algebraic forms to solve daily-life problems. Moreover, identifying the relation between two different values is presented as this topic's introductory task. The tasks appear in the *Ayo Bereksplorasi* and *Ayo Mencoba Sections*, as described in Table 2.

According to our analysis, the tasks in Table 2 also support students in developing their FT. These tasks focus on modelling the relation between values given in algebraic form. For instance, the a part in *Ayo Bereksplorasi* task can be answered with $15t$ and the b part can be filled with $5000 - 15t$. The subsequent questions can be answered using the algebraic form written in the a and b parts.

Table 3

Exercises in modelling the relation between different values (translated from Susanto et al., 2022, p. 152)

FINANCIAL LITERACY

Bayu often uses online transportation. He constantly compares the cost from several online platforms to find the cheapest price. Write down the algebraic form of the driving cost from several online platforms in a) until c).

a. Gogo Company decides the constant administration fee of Rp5.000,00 and the rate of Rp1.500,00 per km.
b. Gaga Company does not have an administration fee, but the company costs Rp2,000.00 per km.
c. Gugu Company decides the constant administration fee of Rp3.000,00 and the rate of Rp1.800,00 per km.
d. If Bayu uses the cheapest online transportation platform to go to school from his home at 5 km distance, which platform will Bayu use?
e. If Bayu uses the cheapest online transportation platform to go to his cousin's house at a 20 km distance, which platform will Bayu use?

Table 4

The task's characteristics based on the functional thinking framework in algebraic form chapter

Sub Chapter	The Task's Characteristics	The Functional Thinking Level Involved	The Context
Algebraic terms	Focuses on identifying the pattern to symbolising the pattern with algebraic form	Recursive patterns, covariation, and correspondence	Object configuration
Modelling with algebraic forms	Focuses on symbolising the relation between two or more values with algebraic form	Correspondence	Daily life (financial & motion)

Furthermore, the task consists of modelling the relation between two or more different values is also given in exercises. Moreover, the textbook labels this task as a part of financial literacy problems (Table 3).

Based on our analysis, the task presented in Table 3 requires students to symbolising the relation between the total cost, the administration fee, and the cost per km. For instance, the algebraic form of task a is $5.000 + 1.500d$, with d as the distance in kilometres. After representing the driving cost with algebraic form in a until c part, the students can decide which platform offers the cheapest cost to answer the d and e part.

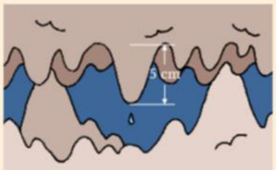
To conclude our analysis of each task that supports students' FT in learning the algebraic form chapter, Table 4 highlights the characteristic of each FT task in both algebraic terms and modelling with algebraic forms subchapter. Table 4 summarises that each subchapter has different task presentations, from the context, questions, and level of functional thinking at the end of the solution.

Grade 8 textbook, Chapter 3: Linear Functions

Developing students' FT through solving tasks in the algebra domain aims to prepare students to understand the concept of a function. In this study, the textbook analysis reveals that from the beginning of algebra learning in grade 7, some tasks provided by the textbook already equip students to nurture their FT.

Despite a good introduction to functional thinking, a missing link exists to connect the FT tasks in grade 7 (a part of the algebraic form chapter) with the tasks presented in grade 8 (linear function). In grade 7, one of the learning objectives is to make an algebraic expression from a given problem. Meanwhile, in grade 8, learning functions requires students to make a function equation. An expression does not support students to "see" how two different variables relate to each other. As an illustration, the sentence "the driving cost from Gaga company is Rp2.000 per km" will be denoted

1 When we measure the length of a stalactite in a cave, it is known that the length is 5 cm. if the length of stalactite increases 1 cm every 30 years, how many years needed for the stalactite to reach the length of 15 cm?



2 To solve the problem in (1), Dewi lets the length of stalactite after x year is y cm, makes the following table, and find how many years needed to reach the length of 15 cm. Complete the table and find how many years needed to reach the length of 15 cm.

x (year)	0	30	60	90	120	150	180	210	240	270	300
y (cm)	5	6	7								

3 In question (2), can we represent that y is a function of x? Can we describe that they have a direct or inverse relation, which we have learned in Grade 7?

Figure 4. Example of tasks to introduce the linear function (translated from Gakko Tosho, 2021, p. 61)

Q In the linear function $y = 2x + 3$, complete the following table and investigate the magnitude of the increase in the y value when the value of x increases by 1. If the value of x becomes 2 times, 3 times, …., does the value of y become 2 times, 3 times, …..?

x	...	-3	-2	-1	0	1	2	3	...
y	...	-3	-1						...

Figure 5. Example of the task to examine the degree of change (translated from Gakko Tosho, 2021, p. 64)

by 2.000d in grade 7 (expression), whereas it will be written as $y = 2.000x$ in grade 8 (function). The absence of the dependent variable's notation in grade 7 might raise an issue for students.

Furthermore, before beginning with linear functions, the textbook provides a recall explanation about the proportion topic: direct and inverse proportions. However, we did not obtain any preliminary tasks about proportions, leading to the concept of a linear function. As depicted in Figure 4, the introduction of linear function goes straight to the use of x and y.

As seen in Figure 4, Task 1 presents the problem context that is introduced to the students. Here, in Task 1, the book already informed the relationship between variables' quantities; that is, *the length of stalactite increases by 1 cm every 30 years.* To solve Task 1, the book provides Task 2, which asks students to complete the values of y in the table until they find the intended value of y. finally, Task 3 requires students to make a symbolic generalisation, that is, the function equation from the pattern identified in Task 2.

Since the problems presented in Figure 4 do not emphasize students' ability to see how each variable's quantity co-vary, for instance, *if x_1 then y_1, if x_2 then y_2, and so on*; we further analysed the subsequent tasks within the linear functions chapter, which supports students in analysing the variability between quantities. We found these types of problems of expressing generality presented as a "degree of change", as shown in Figure 5.

An example of a task in Figure 5 encourages students to understand how variables relate to each other. To identify the changes, the book lists some sequence values of x within the integer set and asks the student to complete the values of y. Moreover, the textbook hints that each value of x

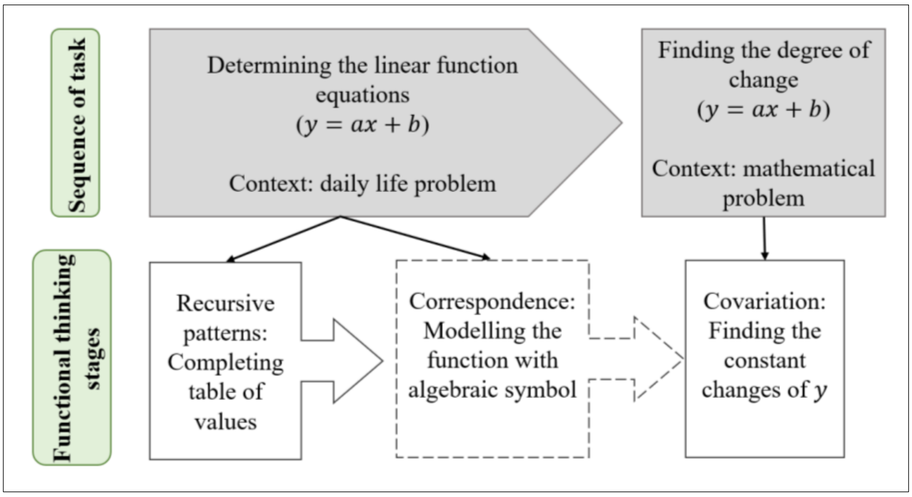


Figure 6. The linkage between tasks in the textbook with functional thinking stages

increases by 1, and the students need to determine the changes of y values. At the end of the task, the book explains that when the value of x increases by 1, the value of y increases by 2.

Even though the tasks within the linear function topic include all thinking stages in FT, our analysis examines that the type of task presented does not align with the order of the FT of tasks of linear functions. The figure shows that the correspondence level appears after the recursive patterns. At the same time, the ideal order of developing students' FT requires them to pass the covariation stage before going to the correspondence.

DISCUSSION

This study focuses on analysing the task sequences in Indonesian lower-secondary mathematics textbooks, which can support students in developing their FT. The tasks analysed are in the *Algebraic Forms* chapter, which is situated in the grade-seven textbook, and *Linear Functions*, which is provided in the grade-eight textbook.

Based on the findings on task sequences in the *Algebraic Forms* chapter, the book begins by introducing the algebraic terms, namely variables, coefficients, and constants. The context used to introduce these terms is a growing pattern. This study also analyse that the problem presented satisfies all levels of FT. The first activity, which is finding *how many matches are used* in a numerical sequence, supports students' recursive pattern ability, the initial stage of FT (Smith, 2017; Wilkie, 2014). The next activity asks students to analyse how the change of value on the input alters the value of the output while they both change simultaneously. At this stage, the activity involves the higher stage of FT, namely covariation (Smith, 2017; Wilkie, 2014). In the last activity to introduce the algebraic terms, students are tasked to determine the relation between the input and the output and use the generalisation they have made to find the number of matches in n squares without recursively counting one by one. This activity, according to Smith (2017) and Wilkie (2014), supports the highest level of FT, i.e., correspondence. Moreover, problems related to geometrical patterns or object configurations are commonly used by other research to assess, analyse, or develop students' FT (Lichti & Roth, 2018; Pinto & Cañadas, 2021; Ramírez et al., 2022). Therefore, the decision to select object configuration activities in this book is appropriate to support students' FT.

Furthermore, this study also found that geometrical patterns or object configurations are the initial context applied to help students generate algebraic expressions. Besides the *matches pattern* activity, the book also equips students with exercises that present different patterns to let them practice determining the expression in the n-term. Each exercise requires students to count the number of *tiles* needed in each term, to identify simultaneous changes in both quantities (*tiles* and *term*), and to symbolise the relationships between *the number of tiles* and *the term*. These three activities satisfy all stages in FT, i.e., recursive patterns, covariations, and correspondence, respectively (Smith, 2017; Wilkie, 2014). Likewise, tasks presented in the following subchapter, that is, modelling with algebraic forms, emphasize only the correspondence level of FT (making an algebraic expression of the problem) and how students can use the expression to solve the other problems.

While the tasks in algebraic terms are presented in object configuration problems, the tasks in modelling with algebraic forms are presented in word problems. This finding indicates that the seven-grade textbook provides FT problems with both figures and narratives. This finding elaborates that nurturing students' generalisation ability is not only by figural growing pattern problems, as stated by Hunter and Miller (2022), but also by narrative or word problems. The textbook prefers to begin learning algebra using geometrical patterns because those patterns are considered tangible tasks to be solved by students. The findings are supported by previous studies which gave evidence that performing generalisation with geometrical shapes is more accessible for students since they are able to concretely see the changing part of the object in each sequence (Pinto & Cañadas, 2021; Radford & Peirce, 2006; Ramírez et al., 2022; Wilkie & Clarke, 2016). Supporting students to model a certain situation with algebraic form may help them to bridge mathematics with the real world. However, the context and language structure of the word problem given are essential aspects to be considered by the textbook. Based on recent studies, identifying patterns within word problems can be a challenge for students to solve (Utami et al., 2023; Wilkie & Ayalon, 2018) because the problem requires students to translate the context into mathematical expressions (Jupri & Drijvers, 2016).

Moreover, the FT problems in the eighth-grade textbook are identified in the *Linear Functions* chapter. As discussed above, the tasks in the seven-grade textbook support students in understanding the notion of variables as things that vary, which prepares them to understand the role of variables in functions. This result resonates with Küchemann (1978), who highlights six levels of letter in algebra that can be used, where the highest level denotes a variable. However, the seventh-grade book does not directly explain using common letters such as x and y. It provides n or any letters to denote the variable. In the linear functions topic covered in the eighth-grade book, all variables are changed formally to x and y.

The first introduction to linear functions is presented by a real-life situation, namely *the increase of stalactite length in a cave every year*. Using the tables of values, students are tasked to find a certain value of the x (the year) from a given value of the y (the stalactite length); thus, supports the recursive patterns stage of FT (Smith, 2017; Wilkie, 2014). According to Doorman et al. (2012), the recursive patterns stage entails a function as "a request" to calculate. Known as the input-output assignment, the task develops students' ability to assign the element of x (domain) with one element of y (codomain). However, the covariation stage of FT is not mentioned here since it is already stated in the problem. The final task requires students to determine y as the function of x from the situation given, which evidences for supporting the highest stage of FT, namely correspondence.

Since the initial task on linear functions only satisfies the recursive pattern and correspondence level of FT, this study found that the covariation stage appears in the subsequent tasks presented as a "degree of change" in a function. Although this task emphasizes students' ability to examine the simultaneous changes between variables, i.e., the alteration of x values happens simultaneously with the changes of y values (Smith, 2017), the tasks in linear functions do not follow the sequential order of FT's stages. Instead of developing students' covariation ability before the correspondence one, the book provides it vice versa. Hence, the order of tasks is not aligned with the development of students' generalization proposed by Radford and Peirce (2006) since the covariation (factual generalization) emerges before students reach the correspondence (contextual and symbolic generalisation).

To conclude, this study provides another perspective on analysing mathematical tasks in textbooks, especially on algebra. An existing study by Wijayanti (2018) of how linear functions are treated in the Indonesian textbook emphasizes the theoretical perspective of the function concept. Her study revealed that the concept of function in Indonesian textbooks usually begins with naïve set theory and then proceeds to function representations such as symbols, graphs, and tables. With the reform of the Indonesian curriculum, a recent study shows that introducing function with its set definition is not implemented again in Kurikulum Merdeka. Instead, the textbook opts for patterns' tasks and determining change and relationship to bridge the students to the concept of linear functions. Additionally, this study also finds evidence for Wijayanti's (2018) study that the topic of proportion is still dominated in the arithmetic domain with less contribution to the linear function introduction.

CONCLUSIONS

In Indonesian textbooks, the tasks in which their solutions require FT ability are commonly found within three mathematics topics: introducing the algebraic form, modelling with algebraic form, and introducing the linear function. Generally, the type of task presented in algebraic form is about generalising patterns in object configurations, while the task's type in modelling with algebraic form and linear functions is about word problems. Moreover, this study also reveals another difference in how "generalising patterns" are arranged in each topic. The object configuration tasks in algebraic form begin by identifying the pattern in a given sequence to express the generality. In contrast, tasks in two other topics focus on expressing the generality (the patterns are given).

Based on the FT framework, this study also found that tasks in each topic develop students' FT at different stages. The tasks presented in the algebraic form topic satisfy all FT's stages in hierarchical order: recursive patterns, covariation, and correspondence. The tasks in modelling with algebraic form emphasize the correspondence stage. The tasks in linear functions also include all stages of FT. However, the linear function's tasks do not satisfy the ideal order of ability's stages in FT: recursive patterns, correspondence, and then covariation.

Although existing research in mathematics textbooks significantly contributes to mathematics learning at schools, this study offers a new reference to analysis tasks in textbooks using the FT framework. Focusing on tasks in algebra, the findings of this study imply that designing more FT tasks and connecting each task to different algebra topics might help students develop their FT and, thus, be more proficient in understanding the concept of function. Moreover, the textbook research in this study limits the analysis of a potentially implemented curriculum. Further research on how these existing tasks were used in the classroom might be required to complete the current study as the implemented curriculum.

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