Journal of Research and Advances in Mathematics Education

Volume 8, Issue 2, April 2023, pp. 123-139
DOI: 10.23917/jramathedu.v8i2.1625
p-ISSN: 2503-3697, e-ISSN: 2541-2590

JRAMathEdu

Classification of errors in determining trigonometry function values at standard positions

Yayan Eryk Setiawan[1]*, Sunardi[2]

[1]*University of Islam Malang, Indonesia*
[2]*Jember University, Indonesia*

Citation: Setiawan, Y. E. S., & Sunardi. (2023). Classification of errors in determining trigonometry function values at standard positions. *JRAMathEdu (Journal of Research and Advances in Mathematics Education),* 8(2), 123–139. https://doi.org/10.23917/jramathedu.v8i2.1625

ARTICLE HISTORY:
Received 15 February 2023
Revised 12 April 2023
Accepted 15 April 2023
Published 30 April 2023

KEYWORDS:
Trigonometry
Conceptual Error
Principle Error
Calculation Error
Factual Error

ABSTRACT

One of the important materials to study in a trigonometry course is the value of trigonometric functions in standard positions. However, the results of preliminary research show that there are still many first semester students who experience errors in determining the value of the trigonometric function in the standard position. These errors are important for further analysis. Therefore, this study aims to describe the mistakes of the first semester students in determining the value of the trigonometric function in the standard position. This type of research is a qualitative descriptive study with a case study approach towards students of the mathematics education study program at a university in Malang. The data collected in this study consisted of the subject's work and transcripts of interviews with the subject. Work data were analyzed descriptively to know the types of student errors. The transcript data from the interview results were analyzed by coding, to determine the factors that caused student error. The results showed that out of 34 students who made errors in determining.

INTRODUCTION

One of the important subjects to be taught to prospective teacher mathematics is Trigonometry courses. Trigonometry courses discuss triangles and trigonometry functions (Downing, 2009; Lial et al., 2016). The importance of the subject to be taught to prospective teacher is because prospective teacher will teach trigonometry material at the junior high school level. In addition, the trigonometry course is a prerequisite course for courses in calculus, vector analysis, and differential equations. Trigonometry material also has a lot to do with concepts and their applications in various disciplines (Nabie et al., 2018). So, in general the importance of trigonometry courses to be taught to prospective teacher is because prospective teacher must master trigonometry material before they teach trigonometry material at the high school level.

One of the trigonometry materials that students learn is trigonometry functions at standard positions. Trigonometric functions in standard positions are defined as comparisons of trigonometric functions in Cartesian coordinates (Lial et al., 2016). Learning trigonometry functions in this standard position starts with an introduction to trigonometry comparisons of triangles. This is because the application of trigonometry ratios of triangles at standard positions will produce trigonometry functions at standard positions. For example, the trigonometry ratio for the sine function in Indonesian is SINDEMI (Sin Depan Miring). If the trigonometry comparison for the sin

*Corresponding author: yayaneryksetiawan@unisma.ac.id

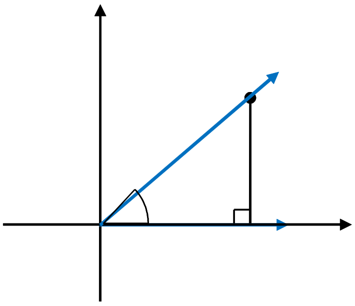

Figure 1. Position of standard angle θ in quadrant I

function is applied in the standard position, then the sin function in the standard position will be $\sin\theta = \frac{y}{r}$ (see Figure 1).

There are six trigonometric functions of angle θ that are studied, namely sine, cosine, tangent, cotangent, secant, and cosecant. These six trigonometric functions are abbreviated sin, cos, tan, cot, sec, and csc. If these six trigonometric functions are applied in standard positions (see Figure 1), we will obtain the definitions of the six trigonometric functions for angle θ in standard positions, namely $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}, x \neq 0$, $\cot\theta = \frac{x}{y}, y \neq 0$, $\sec\theta = \frac{r}{x}, x \neq 0$, and $\csc\theta = \frac{r}{y}, y \neq 0$. The use of the six definitions of trigonometric functions in standard positions is influenced by the location of the angle θ. For example, for the function $\sin\theta = \frac{y}{r}$ (for θ in quadrants I and II) and the function $\sin\theta = -\frac{y}{r}$ (for θ in quadrants III and IV). This means that in determining the value of the angle trigonometry function in standard positions, you must pay attention to the quadrants.

However, the results of the study showed that first-semester students made a mistake in determining the value of the trigonometry function in the standard position, not paying attention to the location of the quadrants (Jaelani, 2017). Furthermore, the results of the study also show that preservice teachers have difficulty solving trigonometry problems (Nabie et al., 2018). The results of the preliminary research also showed that of the 92 students in the first semester of the mathematics education study program who completed questions about trigonometry functions in standard positions, 58 students answered correctly, and 34 students answered incorrectly. This means that there are 37% of students answered incorrectly in determining the value of the trigonometry function in the standard position. These student errors are important for further analysis. This is because the analysis of student errors will provide insight to students or lecturers to find out student misunderstandings (Ball, 1993; Ball & Friel, 1991; Borasi, 1994; Webb & Mastergeorge, 2003). By knowing student misunderstandings in understanding material or in solving problems, students or lecturers can improve student understanding (Setiawan, 2021b, 2021c, 2021a). Research in cognitive failure or errors is an important area of research in mathematics education (Awofala & Odogwu, 2017). Errors in learning mathematics are of concern to researchers, scholars, and mathematics teachers (Kshetree et al., 2021). In addition, errors are considered by other researchers as a major component of the pedagogical content knowledge (PCK) required to teach mathematics (Barkai, 2021). Thus, it is important to research student errors in solving trigonometry problems because it will provide benefits to correct student misunderstandings.

The importance of research on student errors in solving trigonometry problems is of concern to several researchers. Research (Jaelani, 2017) analyzed student errors in solving trigonometry problems which showed that students experienced errors in determining the value of quadrant functions because they did not pay attention to quadrants. Research (Imelda, 2018) analyzed students' difficulties in solving trigonometry problems which showed students had difficulties in using trigonometry formulas to solve trigonometry problems. Research (Nabie et al., 2018) analyzed student teacher candidates' perceptions of trigonometry courses which showed that students said trigonometry was difficult, abstract, and boring. Previous research is analyzing errors in determining the distance between two points in the cartesian plane (Setiawan, 2022a), analyzing student errors in solving problems applying radian measurements (Setiawan & Surahmat, Find all the trigonometric functions for each of the following values.

(a) $tan\ \theta = -\frac{15}{8}$ and θ is in quadrant II

(b) $cot\ \theta = \frac{\sqrt{3}}{8}$ and θ is in quadrant I

(c) $sin\ \theta = \frac{3}{5}$ and θ is in quadrant II

(d) $cot\ \theta = \frac{\sqrt{3}}{8}$ and θ is in quadrant III

Figure 2. Research instruments

2021), analyzing errors in drawing a graph of the cosine function (Setiawan, 2022b). The research that has been done does not discuss student errors in determining trigonometric function values at standard positions. Therefore, research is still needed to analyze student errors in determining the value of the standard angle trigonometry function.

In contrast to previous research (Imelda, 2018; Jaelani, 2017; Nabie et al., 2018; Setiawan, 2021d, 2022b, 2022a; Setiawan & Surahmat, 2021), this study aims to describe the mistakes of first semester students in determining trigonometric function values at standard positions along with their causal factors. The results of this study have theoretical benefits and practical benefits. The theoretical benefit of this research is to develop a theory of errors in the matter of quadrant angle trigonometric function values. While the practical benefits of the results of this study are used by students to improve their understanding in understanding trigonometric function material in standard positions and used by teachers or lecturers in teaching material trigonometric function values in standard positions.

METHODS

This research method is qualitative research with a case study approach to 8 students of the first-semester mathematics education study program who were selected as research subjects. This research was carried out at one of the tertiary institutions in Malang City in the odd semester of the 2021/2022 Academic Year. The process of selecting subjects in this study consisted of three steps. The first step was to ask 92 students of the mathematics education study program to work on the Mid Semester Examination questions which consisted of 6 questions, one of which was a question about trigonometric function values in standard positions (see Figure 2). The second step is to correct student answers so that 46 students experience errors in determining the value of trigonometry functions in standard positions. The third step is to classify the work of 46 students based on the type of error. The results of the classification show that there are 7 different characteristics of errors made by students (see Table 2). Of the 7 characteristics of these errors, one subject was selected who could provide a detailed explanation of the results of his work (while for principal errors consisting of 12 students, two subjects were taken), so 8 subjects were obtained in this study.

The data collected in this study consisted of the subject's work results and interview transcripts. The procedure for collecting data on the work of the subject is carried out by the steps for selecting the subject. From the steps for selecting the subject, it was obtained that the subjects of this study were 8 students of the mathematics education study program in the first semester. Furthermore, the results of the work of these 8 subjects were used as data on the results of the subject's work which would be analyzed further. The data collection procedure in the form of interview transcripts with the subject was carried out in two steps. The first step is to conduct interviews with research subjects via WhatsApp. The second step is to transcribe the interview results word for word so that all interview results can be transcribed. Thus, the data was obtained in the form of transcripts of interviews with research subjects.

By the data collected in this study, the research instrument consisted of questions and interview guidelines. The two research instruments were developed by the researchers themselves. The research instrument in the form of this problem consists of 4 questions about trigonometric function values in standard positions which can be seen in Figure 2.

Table 1. Student error classification framework

No.	Error Type	Error Indicators
1	Conceptual Error	Errors in understanding the concepts contained in the matter of trigonometric function values in standard positions.
2	Principle Error	Errors in applying the formulas, definitions, and methods used to determine the value of the quadrant angle trigonometry function.
3	Calculation Error	Error in determining the result of arithmetic operations.
4	Factual Error	Errors in identifying information and errors in understanding the intent of the questions.

Source: (Muthukrishnan et al., 2019; Oktaviani, 2017; Setiawan, 2020)

Table 2. Result of classification error solving function value problem in standard position

Error Type	Error Indicators	Errors In Determining Trigonometry Function Values At Standard Positions	Many of Students
Conceptual Error	Errors in understanding the concepts contained in the matter of trigonometric function values in standard positions	Error in understanding the definition of trigonometric functions in standard positions (misunderstanding of inverse identity)	1
		Error drawing triangles on quadrants	3
Principle Error	Errors in applying the formulas, definitions, and methods used to determine the value of the quadrant angle trigonometry function	Error in applying the definition of trigonometric functions in standard positions without regard to quadrants	12
		The mistake of applying the method which is the Pythagorean identity	6
		Error applying the Pythagorean Theorem formula	1
Calculation Error	Error in determining the result of arithmetic operations	Errors in performing arithmetic operations	5
Factual Error	Errors in identifying information and errors in understanding the intent of the questions	Error in understanding the meaning of the question	6
Total			34

From Figure 2 it can be seen that the four questions involve various quadrants. Before the instrument was used, it was first tested the validity of experts consisting of two mathematics education lecturers at the University of Islam Malang. Validity test is done by using expert judgment. The results of the validity test show that the instrument used is valid for measuring student errors in determining trigonometric function values at standard positions. This is because if students do not pay attention to the location of the quadrants, they will be trapped into errors in determining the value of trigonometry functions. In addition, if students are wrong in using the opposite identity, they will be trapped in an error. If students misunderstand the intent of the problem and apply the wrong way of solving it, students will be trapped in an error.

Data analysis in the form of student work was carried out by classifying errors based on conceptual errors, principle errors, and factual errors which can be seen in Table 1. The reason researchers classify student errors in determining trigonometric function values at standard positions based on conceptual errors, principle errors, calculation errors, and factual errors is based on research materials and instruments. The concept contained in the trigonometric function value material at standard positions is the definition of trigonometric functions at standard positions and images of triangles at standard positions. If students are wrong in defining trigonometry functions and wrong in drawing triangles in standard positions, then students experience conceptual errors. This is in accordance with the opinion (Setiawan, 2020) which says that a conceptual error is an error in understanding a concept. Furthermore, this principle error is an error in applying rules, properties, formulas, or theorems in solving problems (Setiawan, 2020).

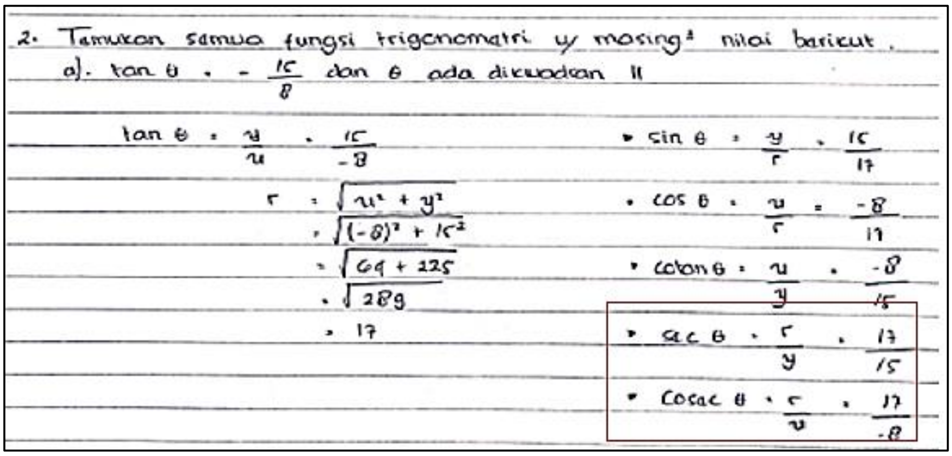

Figure 3. Errors in defining secant and cosecant

The principles in determining trigonometric function values at standard positions are the application of the Pythagorean theorem, the application of the definitions of trigonometric functions in various quadrants, and the application of the Pythagorean identity. While arithmetic errors are errors in determining the results of arithmetic operations. The application of the Pythagorean theorem in the matter of trigonometric functions at standard positions requires calculations. If students make mistakes in calculating, they will be trapped into calculation errors. Next, if the student is wrong in identifying the problem or misunderstood the meaning of the question, then the student will be trapped in a fact error. Where, fact errors are errors in identifying the information contained in the problem (Muthukrishnan et al., 2019; Oktaviani, 2017). So in general it can be said that student errors in determining trigonometric function values at standard positions can be categorized into conceptual errors, principle errors, calculation errors, and factual errors.

Validation of the results of these findings in the form of student errors in determining trigonometric function values at standard positions was carried out by asking the subjects again through interviews whether they agreed with these findings. and asked the subject to re-check the accuracy of the findings of this study (Creswell, 2012). The seven subjects said that the results of these findings were accurate and by the mistakes they made in determining trigonometric function values at standard positions.

FINDINGS

The results showed that of the 92 students who solved the function value problem in the standard position, 34 students answered incorrectly. Of the 34 students who answered incorrectly, an error classification was obtained which can be seen in Table 2. In Table 2 it can be seen that of the 34 students who experienced errors in determining the value of trigonometric functions in standard positions, the percentage of each of the many students who made mistakes obtained: 11% conceptual errors, 56% principle errors, 15% calculation errors, and 18% factual error. This means that first semester students experience more principle errors in determining trigonometric function values in standard positions. From the results of classifying student errors in determining trigonometric function values at standard positions, one subject will be taken from each type of error. There were 8 subjects in this study who would be analyzed further regarding the results of the subject's work and the factors that caused the subject's errors.

Conceptual errors

The first conceptual error is the error in defining trigonometric functions at standard positions. As a result of this error, students experience errors in determining trigonometric function values at standard positions. There was one student who experienced this error and was subsequently selected as the first subject (S1) in this study. The results of the first subject's work can be seen in Figure 3.

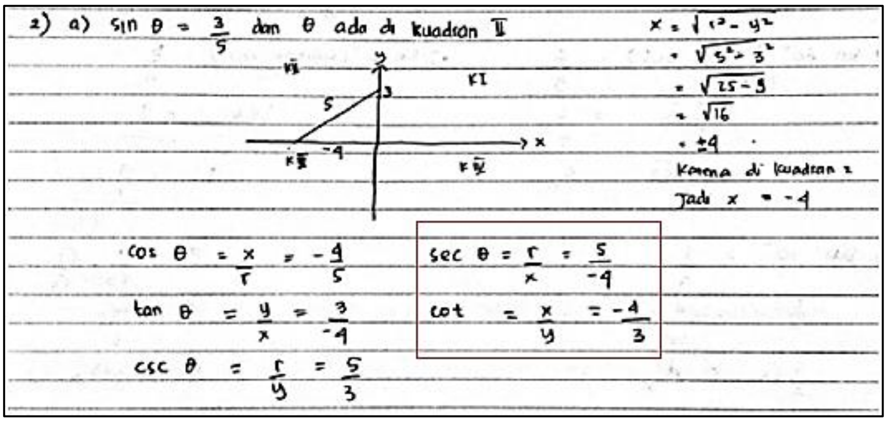

Figure 4. Error drawing triangles in standard position

From Figure 3 it can be seen that the subject was wrong in defining $\sec\theta$ and $\csc\theta$, namely $\sec\theta = \frac{r}{y}$ and $\csc\theta = \frac{r}{x}$. As a result of this error in defining, the subject is wrong in determining the comparison value for the $\sec\theta$ and $\csc\theta$ functions. Factors causing this error can be seen from the following interview excerpt.

R : *Please explain how do you answer this question?*

S1 : *I looked for r first, by **using the Pythagorean theorem**, so that r is equal to 17. **Because in quadrant II, the negative value is x,** so the x value is negative 8, the y value is 15 and the r value is equal to 17. Then I determine the value of each trigonometric function, ... for the secant function, it is r per y, **because the secant is equal to 1 per sin** and for the cosecant function it is r per x **because the cosecant is equal to 1 per cos**. ...*

From the interview transcript excerpts it can be seen that the subject has understood the problem, can apply the Pythagorean theorem, and also paid attention to the quadrants. However, there were two trigonometric functions that were misunderstood by the subject, namely the secant function and the cosecant function, where the subject's mistake was to think that $\sec\theta = \frac{1}{\sin\theta}$ and $\csc\theta = \frac{1}{\cos\theta}$. The correct inverse identities are $\sec\theta = \frac{1}{\cos\theta}$ and $\csc\theta = \frac{1}{\sin\theta}$. This means that the cause of the conceptual error made by the subject in defining the trigonometric functions in standard positions is that the subject uses the wrong inverse identity for the secant and cosecant functions.

The second conceptual error is the error in drawing a triangle in a standard position. 3 students made this mistake. Of these three students, 1 student was selected as the second subject (S2) of this study. The results of the work of this second subject can be seen in Figure 4. From Figure 4 it can be seen that the subject drawing a right triangle in quadrant II is not by the concept. The concept of a triangle in the quadrant position is that the corner point θ is located at the origin (0, 0), this is by the definition of the angle at the standard position, namely the result of the rotation of the initial side on the positive x-axis to the terminal side. However, the subject is correct in determining the value of the trigonometric function in quadrant II. The reason for the subject drawing the triangle in Figure 4 can be seen in the following excerpt of the interview transcript.

R : *Try to explain your answer?*

S2 : *There he found that **sin theta is 3 by 5**, **where sin theta is y per r**, so the y value is 3 and the r value is 5, so to do this, I used the Pythagorean theorem to find the x value. The x value meets plus minus 4, **because in quadrant II,** the x value is less than zero, **so the x value is negative four.** For the 5th one, sir, I'll try using the Pythagorean theorem too, sir. If x is negative 4, y is 3, find r plus minus 5, because r is in quadrant II more than zero. The result is 5. **So, I drew it just for illustration, sir. So, like in the triangle there will be y, x, and r, where y is 3, x is -4, and r is 5.** Like that sir.*

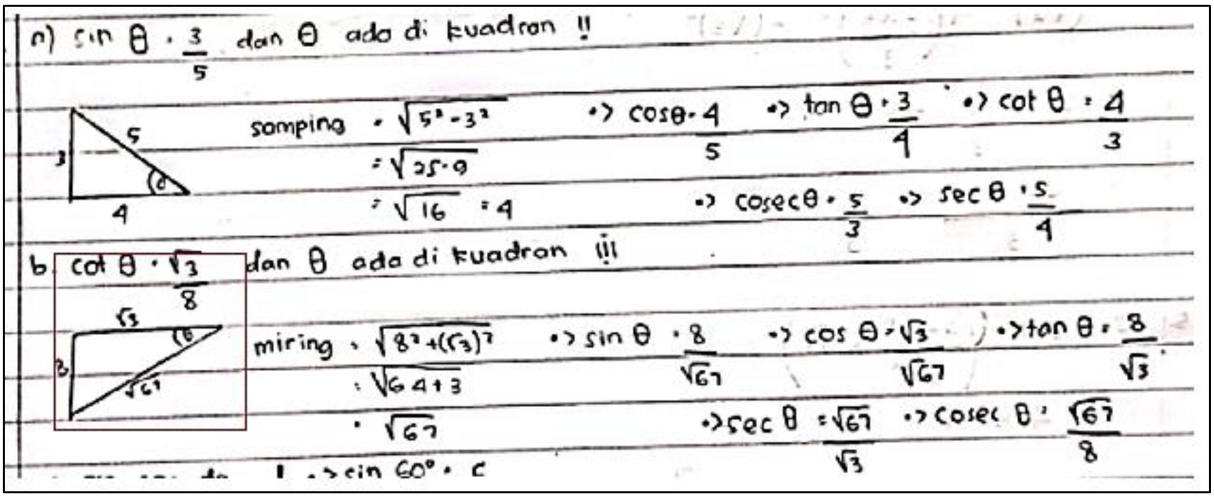

Figure 5. Error not paying attention to quadrants

From the interview transcript excerpts it can be seen that the subject has understood the definition of trigonometric functions in standard positions by saying that $\sin\theta = \frac{3}{5}$ which means $\frac{y}{r}$, so the values $y = 3$ and $r = 5$. The subject has also paid attention to the location of the quadrants and the subject can also apply the Pythagorean theorem so that the subject can find the value $x = -4$ (because it is in quadrant II). However, the subject did not understand the shape of the triangle in the correct standing position, where the subject only made an illustration of a triangle that has sizes $x = -4$, $y = 3$, and $r = 5$ in quadrant II, so the subject was wrong in drawing a triangle in the standard position. This means that the factors that cause misunderstandings in drawing triangles in this standard position are because the subject only makes illustrations of the measurements that have been calculated, without thinking about the correct location of a triangle in a standard position.

Principle errors

The first principle error is not paying attention to the quadrants in determining the values of trigonometric functions at standard positions. As a result of not paying attention to quadrants, students are wrong in determining the value of the quadrant angle trigonometry function. 12 students made this mistake. Of the 12 students, 2 students will be selected as the third subject (S3) and as the fourth subject (S4) in this study. The results of the third subject's work can be seen in Figure 5. From Figure 5 it can be seen that the subject made a right triangle with the correct side lengths, namely 3, 4, and 5. Because this triangle is in quadrant II, the length of the base side or x value should be -4. Because the subject does not pay attention to the triangle drawn which is located in quadrant II, the subject is wrong in determining the comparison involving the value of x. For example, the subject is wrong in determining the cos function ratio, namely $\cos\theta = \frac{4}{5}$, where the correct answer is $\cos\theta = -\frac{4}{5}$ (because it is in quadrant II). The factors causing the subject not to pay attention to the quadrants can be seen in the following excerpt of the interview transcript.

R : *Please explain how you solve question a?*

S3 : *There it is known that* ***sin theta is 3 per 5 and theta is in quadrant II****. I use* ***trigonometry ratios that exist in triangles.*** *The first, draws the triangle, the second enters the sin value. The sin value in trigonometry right, the front is slanted, the sides are unknown. So, I looked for the side by* ***using Pythagoras****. After all were met,* ***I entered the values into the trigonometry comparisons.*** *like cosine, cosine is the same side as the hypotenuse, so 4 per fifth*

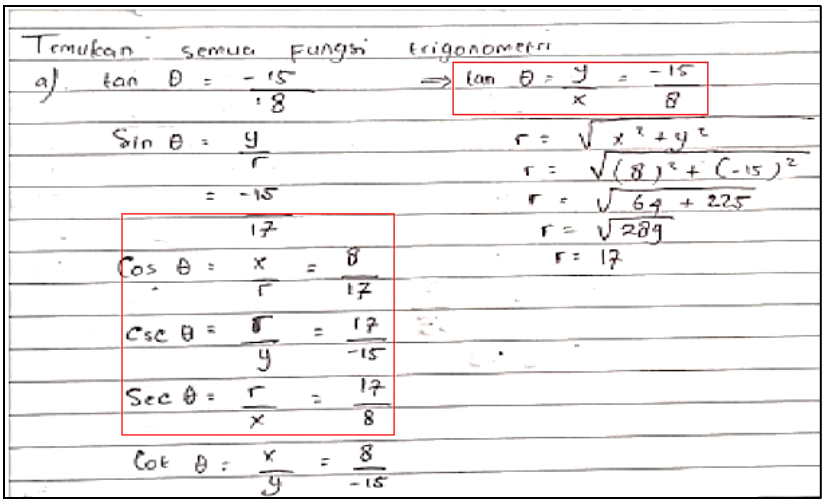

Figure 6. Error Not Paying Attention to Quadrants

From the excerpts of the interview transcript, it can be seen that the subject solved questions about trigonometry functions in standard positions using trigonometry comparisons. Then the subject was also able to use the Pythagorean theorem correctly, but the subject did not carry out further analysis or did not pay attention again to the problem which stated θ in quadrant II. As a result, the subject is wrong in determining trigonometric ratios in quadrant II. This means that the factor causing the subject not to pay attention to the quadrant is that the subject only does intuitive thinking, where the subject does not pay attention again that θ is in quadrant II. Therefore, it is important to look back for θ located in quadrants I, II, III, or IV.

While the results of the work of the fourth subject (S4) who also experienced the same error as the third subject can be seen in Figure 6. From Figure 6 it can be seen that the subject considers that the negative value of the ratio $\tan\theta = -\frac{15}{8}$ is -15. If you pay attention to the problem, the negative value of the ratio $\tan\theta = -\frac{15}{8}$ is -8, this is because θ is located in quadrant II. As a result of not paying attention to the location of this quadrant, the subject experiences errors in determining the values of other trigonometric functions. Factors causing this error can be seen in the following interview excerpts.

R : *Please explain how you solve question a?*

S4 : *What is already known in the problem is that* ***the theta tangent is negative 15 per 8****. First, I draw a* ***right triangle****, where* ***I determined the obverse, side, and hypotenuse sides****. ... For* ***cos theta the formula is the side divided by the hypotenuse, or x per r, x per r equals 8 per 17,*** *now to find the cosecant theta,* ***the cosecant theta is the opposite of sin theta****, so 1 per sin theta or r per y, so cosecant tetha r per y equals 17 per min 15. ...*

From the interview transcript excerpts it can be seen that the subject's attention is only focused on the value of the trigonometry comparison in the problem, where the subject does not pay attention that θ lies in quadrant II. The subject also understands concepts in the form of definitions of trigonometric functions at standard positions and the Pythagorean theorem. However, when the subject applies the definition of trigonometric functions in standard positions, they do not pay attention to the location of the angle θ in the quadrant. As a result, the subject was wrong in determining the value of the trigonometric function at the standard position. So, the factor causing the subject not paying attention to the quadrants is the subject not paying attention to the location of the angle θ which is known to be in quadrants I, II, III, or IV.

The second principle error is in applying the Pythagorean Identity method. 6 students experienced this error. Furthermore, one of the six students selected a subject as the fifth subject (S5) in this study. The results of the fifth subject's work can be seen in Figure 7.

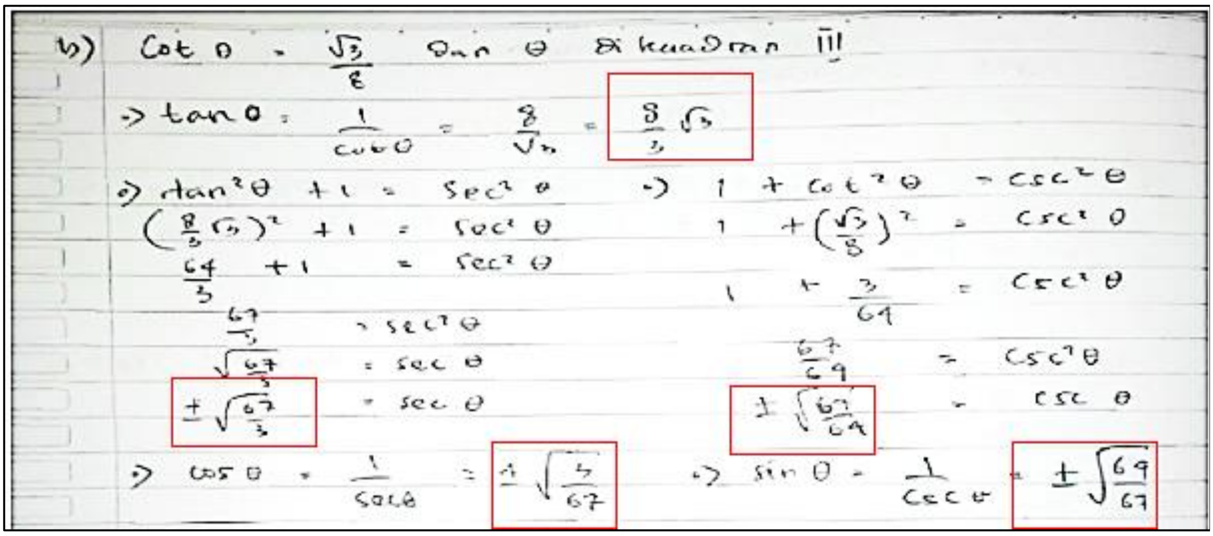

Figure 7. Mistakes in applying pythagorean identity

From Figure 7 it can be seen that the subject applied the Pythagorean identity to determine the value of the trigonometric function at standard positions. However, the subject did not determine the sign for the value of each trigonometric function at the standard position. As a result, the subject experienced an error in determining the value of the trigonometric function at the standard position. The cause of this subject error can be seen in the following excerpt from the interview transcript.

R	:	*Please explain how you solve question a?*
S5	:	*It is known that* ***cot theta is equal to the root 3 per 8****. The first thing I did was find tan theta because tan theta is equal to 1 per cot theta, so tan theta is equal to 8 per root 3.* ***Then I looked for the secant theta by applying the Pythagorean identity****, so that the* ***theta secant is equal to the plus-minus root of 67 per 3.*** *Then I look for the cos theta* ***because the theta cos is equal to 1 per the theta secant****, so the theta cos is equal to the plus-minus root of 3 per 67. ….*
R	:	*Why use this method?*
S5	:	*Because I think it's easier and at that time, I remembered that way.*

From the excerpts from the interview results it can be seen that the subject only pays attention to the value of the angle ratio, namely $\cot\theta = \frac{\sqrt{3}}{8}$ without paying attention to the location of θ in the quadrant. Then the subject applies the reverse identity and the Pythagorean identity. The subject is correct in applying the opposite identity. However, when applying the Pythagorean identity, the subject did pay attention to the location of θ in the quadrant which resulted in the subject not choosing a positive or negative sign for the trigonometric comparison value that was sought by applying the Pythagorean identity. As a result, the subject is wrong in determining the value of the trigonometric function at the standard position. The factor that causes the subject's error in applying the Pythagorean identity is that the subject does not choose the correct positive or negative sign for the value of the trigonometric comparison because the subject does not pay attention to the location of the angle θ in a certain quadrant.

The third principle error is the wrong application of the Pythagorean theorem. There is 1 student who experienced this error. then 1 student was chosen as the sixth subject (S6) in this study. The results of the work of the sixth subject (S6) who experienced an error in applying the Pythagorean theorem can be seen in Figure 8.

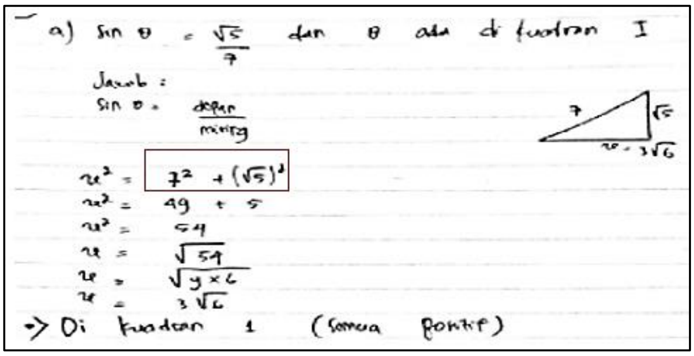

Figure 8. Errors in applying the pythagorean theorem

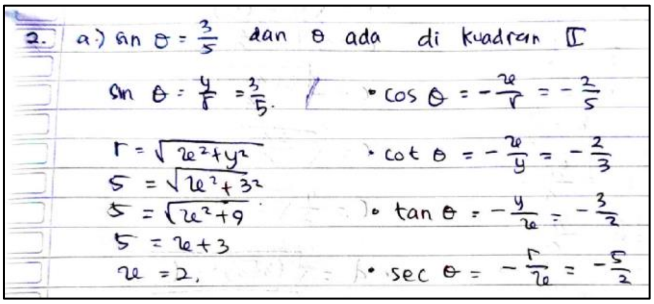

Figure 9. (a) Error calculating root results

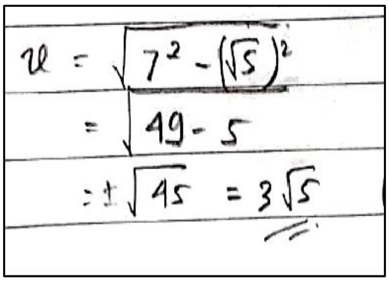

Figure 9. (b) Error calculating subtraction results

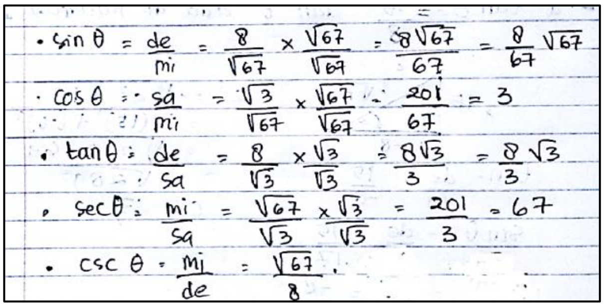

Figure 9. (c) Error calculating the multiplication of roots results

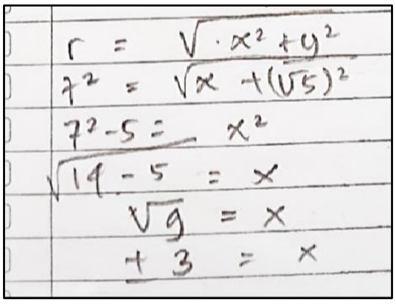

Figure 9. (d) Error calculating the result of the power of two

Figure 9. Errors in counting

From Figure 8 it can be seen that the subject experienced an error in determining the value of x, where the subject's error was $x^2 = 7^2 + \left(\sqrt{5}\right)^2$. While the correct answer according to the Pythagorean theorem is $x^2 = 7^2 - \left(\sqrt{5}\right)^2$. The causes of the subject's errors in applying the Pythagorean theorem can be seen from the following interview transcript excerpts.

R : *Please explain how you solve question a?*
S6 : *I'm sorry sir, my work is wrong. because* ***I'm in a hurry sir.*** *Supposed to find the value of x is reduced because x is a side.*

From the excerpts of the interview transcripts, it can be seen that the factors causing the subject's mistakes in applying the Pythagorean theorem were due to being in a hurry or not being thorough in doing the assignment.

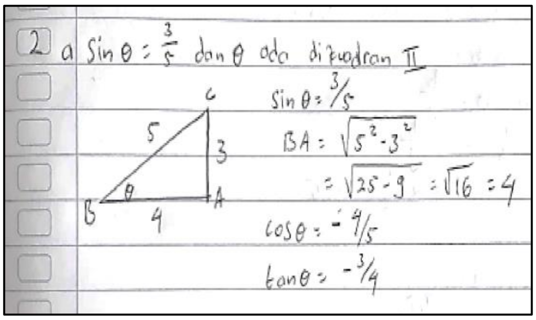

Figure 10 (a) Only Specifies Sin, Cos, & Tan

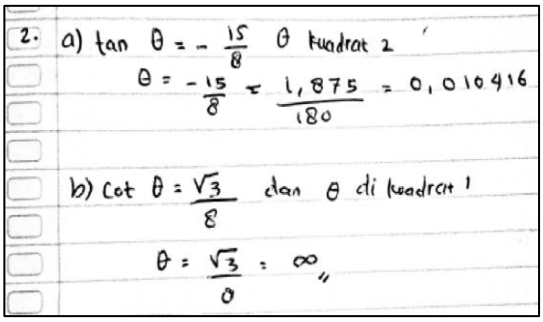

Figure 10 (b) Finding the Value of θ

Figure 10. Error in understanding the meaning of the problem.

Calculation errors

Any 5 students experienced a calculation error in determining the value of the trigonometry function at the quadrant angle. Of the five students, it was obtained: 2 students experienced an error in calculating the square root result of the addition of two exponential numbers (see Figure 9. a), 1 student experienced a subtraction error (see Figure 9. b), 1 student experienced an error in determining the multiplication of roots dissimilar (see Figure 9. c), and 1 student experienced an error in determining the cube of a number (see Figure 9. d).

From Figure 9 (a) it can be seen that the students were wrong in determining the root product, namely the result of $\sqrt{x^2 + 9} = x + 3$. From Figure 9 (b) it can also be seen that students are wrong in determining the result of $49 - 5$, namely answering 45. From Figure 9 (c) it can also be seen that students are wrong in multiplying the roots, namely $\sqrt{3} \times \sqrt{67} = 201$. Furthermore, from Figure 9 (d) it can also be seen that the students made a mistake in determining the result of 7^2, namely answering 14. Furthermore, the researchers conducted interviews to find out the cause of the calculation errors made by these five students. Transcript snippets showing the cause of the miscount of one of the students who became the seventh subject (S7) of this study are as follows.

R : *Please explain how you solve question a?*

S7 : *So here the problem is that **sin theta is equal to 3 per 5 and theta is in quadrant II**. So it is known that sin is y per r, y per r means that y is 3 and r is 5, so what is not yet known is x, so if we look for x using **the formula r equals the root of x squared plus y squared**, where r is 5 equals the root of x squared plus 3 squared. So, 5 is the same as the square root plus 9, **so 5 is the same as x plus 3.** So, the x meets 2. So, if everyone knows, the r is 5, the y is 3, and the x is 2, we can easily enter it like in cos but that's x per d, so it's 2 per 5. ....*

From the excerpts of the interview transcripts, it can be seen that the subject already understands the questions well. However, the subject experienced an error when calculating the square root results. The factor causing the subject's error in calculating the square root result is a misunderstanding of the concept of the root product of the sum of two square forms, where the subject thinks that the result of $\sqrt{a^2 + b^2} = a + b$. While the results of interviews from students who experienced subtraction errors and exponential results were caused by the subject not being careful in reducing numbers and determining the results of powers of two. Furthermore, for students who experience errors in determining the product of roots due to the subject's misunderstanding understanding the product of roots, where the misunderstanding of the subject is to think that the product of two numbers in the form of a root is to produce a multiplication of numbers whose roots are omitted. So in general the cause of the subject experiencing arithmetic error is a misunderstanding of the results of square roots and misunderstanding the results of multiplying two forms of roots and not being careful in determining the results of arithmetic operations.

Table 3. Description of errors in determining trigonometry function values at standard positions

Error Type	Description	Causative Factors
Conceptual Error	Conceptual errors in determining trigonometric function values at standard positions are errors in defining trigonometric functions at standard positions and misunderstandings in drawing right triangles in standard positions.	The causes of each error are errors in using the inverse identity (secant and cosecant functions) and errors in illustrating a triangle without taking into account the correct location of a right triangle in a standard position.
Principle Error	The principal errors in determining trigonometric function values at standard positions are: (1) not paying attention to quadrants when applying the definition of trigonometric functions at standard positions, (2) errors in applying the Pythagorean identity, and (3) errors in applying the Pythagorean theorem.	The causal factors for each of these errors are: (1) because they only used intuitive thinking and did not pay attention to the location of the quadrant of the angle θ, (2) the subject did not choose the correct positive or negative sign when applying the Pythagorean identity, and (3) because of being in a hurry or not being thorough in applying the Pythagorean theorem.
Calculation Error	Calculation errors in determining trigonometric function values at standard positions consist of: (1) errors in calculating the square root of the sum of two square numbers, (2) subtraction errors, (3) errors in determining the product of two roots that are not the same, and (4) errors in determining the second power.	The causal factors for each of these errors are (1) misunderstanding of the concept of the square root product of the sum of two square forms, (2) inaccuracy in subtracting, (3) misunderstanding of the product of two square roots, and (4) less careful in determining the result of a number raised to the power of two.
Factual Error	Errors in fact in determining the value of the trigonometry function is an error in understanding the meaning of the problem.	The factor that causes this error is a misunderstanding which thinks that if the problem contains sin, then what is sought is only cos and tan, similarly if in the problem what is known is sec, then what is sought is cot and csc.

Factual errors

Factual errors made by students in determining the value of trigonometry functions in standard positions, namely errors in understanding the meaning of the problem. Any 6 students experienced this error. Of the six students the misunderstandings in understanding the questions consisted of 1 student understanding the questions to determine x, y, and r values only (after being confirmed, it turned out that these students had not finished working on the questions), 4 students understood only determining 3 trigonometry comparisons (see Figure 10. a), and 1 student thought to find the theta value (see Figure 10. b)

From Figure 10(a) can be seen that students misunderstand the problem, where students think they only determine the values of $cos\ \theta$ and $tan\ \theta$ because what they know is $sin\ \theta$. From Figure 10(b) it can also be seen that students misunderstood the meaning of the questions, where these students understood the questions to determine the value of θ. Furthermore, to find out why students made this mistake, the researcher interviewed the eighth subject (S8) ie the student who wrote down the answer in Figure 10(a). An excerpt from the transcript of the interview results with the eighth subject is as follows.

R : *Please explain how you solve question a?*

S8 : *That's sin theta, I first made a right triangle with sin theta. Tetha is in corner b, sin right* ***sin for the slanted front****.* ***So, the front is 3 slanted 5****, so I'll look for* ***Pythagoras first to find the length of BA****, so how much, and find the Pythagorean formula, which is 4. And* ***because it's in quadrant II****, the x is negative, and the y is positive...*

R : *Why do you only specify cos equals tan?*
S8 : ***Because what Sin knows, sir, so in my opinion, just determine Cos and Tan, sir.*** *Likewise for question b, because what cot knows, so it's just a matter of looking for cosec and sec, sir.*

From the excerpts from the interview results it can be seen that the subject already understands the application of Pythagoras and the subject has also paid attention to the quadrants, and the subject has also understood the definition of trigonometric functions in standard positions. However, the subject has a wrong understanding of the problem. The cause of subject misunderstood the problem due to a misunderstanding of the information given in the problem, where the subject's misunderstanding is that if one knows one of sin, cos, or tan, then what is also sought is sin, cos, or tan, besides that what is known as one of cosec, sec, or cot, then what is sought is cosec, sec, or cot. Meanwhile, students who answered in Figure 10(b) said that they could not answer the questions, so they answered incorrectly.

Based on the results of the subject's job analysis and transcript analysis of interview results with the subject, a description of the error and the factors causing the subject's error in determining the value of the trigonometric function in the standard position are obtained which can be seen in Table 3.

DISCUSSION

In general, the results of this study contribute to the development of the theory of errors in solving mathematical problems in the matter of trigonometric function values at standard positions. The results showed that the mistakes made by students in determining the value of trigonometry functions at standard positions consisted of conceptual errors, principle errors, calculation errors, and factual errors. In this study, there were no procedural errors in determining trigonometric function values at standard positions. This is because the material and questions related to trigonometric function values in standard positions place more emphasis on concepts, principles of applying methods, and arithmetic operations rather than solving procedures. This means that the types of errors can appear according to the characteristics of the material and the questions worked on by students. The results of this study are by the results of previous studies which also show that first-semester students experience conceptual errors, principle errors, calculation errors, and fact errors (Abidin, 2012; Imelda, 2018; Jaelani, 2017; Nabie et al., 2018; Setiawan, 2021b, 2021d, 2021a, 2021c, 2022a, 2022b; Setiawan & Surahmat, 2021). However, the results of this study expand the results of previous studies by explaining these various errors in the matter of trigonometric function values at standard positions.

The first error in determining the value of a trigonometric function at standard positions is a conceptual error. The results of this study are the results of previous studies which show that first-semester students experience conceptual errors in solving trigonometry problems (Abidin, 2012). The results of other studies also show that 60% of math teachers are wrong in defining angles and 90% of math teachers are wrong in defining radians (Tuna, 2013). The results of the study also show that prospective teacher students have limited conceptual knowledge about the basic concepts of trigonometry (Nabie et al., 2018). The results of this study expand the results of previous studies by showing that the conceptual errors that occur in this study are caused by a misunderstanding of the use of a concept (i.e. an error in using the concept of inverse identities) to construct another concept (i.e. the concept of trigonometric functions at standard positions) and a misunderstanding of the representations used (i.e. error of drawing a triangle at standard position). Therefore, to build a concept can be started by understanding the correct basic concept and representation according to the concept.

The second error is an error of principle. The results of this study are the results of previous studies which show that first-semester students experience more principal errors in solving trigonometry problems (Abidin, 2012). The results of this study expand the results of previous research by showing that this principle error occurs due to errors in applying definitions (definitions of trigonometric functions at standard positions), the Pythagorean theorem, and identities

(Pythagorean identities) in solving trigonometric problems. The main reason for the failure of this principle is intuitive thinking, that is, thinking based solely on perception without further analysis. The results of previous research also show that the things that make trigonometry material difficult for prospective teachers to understand are that it requires analysis (Nabie et al., 2018). Therefore, applying definitions, theorems, and ways to solve trigonometry problems requires analytical skills.

The third error is a miscalculation. The results of this study are the results of previous studies which show that first-semester students also experience calculation errors in solving trigonometry problems (Gür, 2009; Hidayat & Aripin, 2020; Imelda, 2018). The results of this study expand the results of previous studies by showing that the cause of this calculation error is a misunderstanding of the concept of arithmetic operations and a lack of accuracy in performing calculations. This misunderstanding occurs when only using intuitive or hasty thinking in solving arithmetic operations involving square roots. Therefore, it is important to re-examine the results of the calculations that have been carried out, so that the results of they calculations that have been carried out are correct.

The fourth error is the error of fact. The results of this study are in accordance with the results of previous studies which show that first semester students also experience fact errors in solving trigonometry problems (Abidin, 2012; Imelda, 2018; Tuna, 2013). The results of this study expand the results of previous studies by showing that the main cause of this fact error is the wrong perception of the information contained in the problem. Perception is defined as the act of recognizing and interpreting sensory information to obtain an overview and understanding of the information provided (KBI, 2008). In simple terms, it can be said that the correct perception of the information contained in the problem can result in success in solving the problem. Conversely, if the perception is wrong, it will result in an error in solving the problem.

The results of this research also contribute to learning trigonometry to overcome student mistakes in studying trigonometry function values in standard positions. The first implication for reducing conceptual errors is carried out by understanding the concept of the definition of trigonometric functions at standard positions and the concept of a right triangle at standard positions. Various researchers also recommend emphasizing conceptual understanding in learning mathematics (Nabie et al., 2018). The concept of trigonometry functions in standard positions is built from trigonometry triangles, namely SINDEMI (Sin Depan Miring), COSAMI (Cos Samping Miring), and TANDESA (Tan Depan Samping). If this trigonometry triangle is applied in standard positions, then the definition of trigonometry functions in standard positions is obtained. The second is the concept of a right triangle in a standard position. The concept of a right triangle in the correct standard position is based on the corner point θ which is located at the origin (0, 0). For example, the angle θ is in quadrant I (see Figure 1), then the corner point θ is located at the origin (0, 0). Likewise, if the angle θ is in quadrants II, III, and IV, then the corner point θ must also be located at the origin (0, 0). In addition, the concept of a right triangle in a standard position is built by finding the reference angle. This reference angle is defined as an angle whose sides are the x-axis and the terminal side of the angle to which the reference is to be sought (Downing, 2009; Lial et al., 2016). For example, drawing a triangle for angle 120^0 is done by finding the reference angle from 120^0, which is $180^0 - 120^0 = 60^0$. So, the triangle drawing from angle 120^0 is the same as the triangle drawing from angle 60^0 in quadrant II with the sides being the *x*-axis and the terminal side from angle 120^0. By understanding that the corner point θ is located at the origin (0, 0) and the reference angle, it is expected that students can understand the concept of a right triangle in a standard position.

The second implication is learning to reduce principle errors by teaching how to apply concepts that are understood correctly and more thoroughly. Not only that, in applying the concept one must also be able to choose the most appropriate concept or strategy to solve the problem. The results of previous research show that someone who has high ability also has high flexibility, namely the ability to choose the most appropriate strategy in solving a problem (Rittle-Johnson & Star, 2007; Star & Rittle-Johnson, 2008). The results of this study expand the results of previous studies by showing that the most suitable strategy for determining the value of a trigonometric function is to apply the definition of a trigonometric function that takes into account the quadrants. By learning how to choose and implement an appropriate and careful strategy, principal errors can be overcome.

The third implication is learning to reduce calculation errors by understanding the concept of arithmetic operations and asking to be more careful in performing arithmetic operations. There are 2 misunderstandings about the concept of arithmetic operations that need to be corrected, namely: (1) a misunderstanding of the square root product of the sum of two square numbers and (2) a misunderstanding of the product of like square roots. The first misunderstanding can be overcome by explaining that if $\sqrt{a^2 + b^2} \neq a + b$. The second misunderstanding was caused by the subject making an analogy of the product $\sqrt{a} \times \sqrt{b}$ with the product of $\sqrt{a} \times \sqrt{a} = a$, so the subject wrote the wrong result, namely $\sqrt{a} \times \sqrt{b} = ab$. This misunderstanding can be overcome by giving an understanding that the product of similar roots produces a root number. Furthermore, if a number can be determined by its root product, then the result is written in the form of a non-root number.

The fourth implication is that reducing fact errors is done by asking students to understand questions or material not only based on perception but based on concepts that are already understood. The results of previous research indicate that errors in facts occur due to a misunderstanding of the intent of the questions (Setiawan, 2020). The research results also show that misunderstanding information can lead to wrong strategies used to solve problems. By asking students to understand the information contained in the problem properly based on the concept understood and not based on perception alone, it will be able to overcome fact errors when solving problems.

CONCLUSION

Student errors in determining trigonometric function values at standard positions consist of conceptual errors, principle errors, calculation errors, and factual errors. This conceptual error occurs due to an error in understanding the concepts contained in the trigonometry function value material, namely the definition of trigonometric functions at standard positions and images of right triangles at standard positions. This principle error occurs because of not paying attention to quadrants when applying the definition of trigonometric functions in standard positions, wrongly applying the Pythagorean identity and the Pythagorean theorem. This calculation error occurs due to a misunderstanding of the results of square roots and multiplication results of radical forms and lack of accuracy in determining the results of calculations. This factual error occurs because of a misunderstanding in understanding the meaning of the problem.

This research is limited only to the material values of trigonometry functions in standard positions, for this reason, the suggestion for further research is to identify errors made by students and students in trigonometry materials. Because the results of this study will contribute to the development of students' misunderstanding theory in understanding trigonometry materials. In addition, the recommendation to teachers and lecturers is to conduct learning that can reduce concept errors, principle errors, calculation errors, and factual errors by explicitly explaining the concepts, principles, and facts contained in the material trigonometric function values in standard positions and being careful in solving problems. trigonometry. Thus, it is hoped that teachers and lecturers can reduce student and student errors in learning material trigonometric function values in standard positions.

ACKNOWLEDGMENT

Thanks go to Allah SWT who has given me health and smoothness in completing research so that I can write this article. Thanks also go to the two people who have motivated the writer to always work. Thank you to the Rector of the Islamic University of Malang, Dean of FKIP, and Chair of the Institute for Research and Community Service of the Islamic University of Malang for supporting the publication of this scientific article.

AUTHOR'S DECLARATION

Authors' contributions	All authors contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.
Funding Statement	This research was funded by Institute for Research and Community Service of the Islamic University of Malang.
Availability of data and materials	All data are available from the authors.
Competing interests	The authors declare that the publishing of this paper does not involve any conflicts of interest. This work has never been published or offered for publication elsewhere, and it is completely original.

BIBLIOGRAPHY

Abidin, Z. (2012). Analisis Kesalahan Mahasiswa Prodi Pendidikan Matematika Fakultas Tarbiyah IAIN AR-Raniry dalam Mata Kuliah Trigonometri dan Kalkulus 1. *Jurnal Ilmiah DIDAKTIKA*, *13*(1), 183–196. https://doi.org/10.22373/jid.v13i1.472

Awofala, A. O. A., & Odogwu, H. N. (2017). Assessing Preservice Teachers' Mathematics Cognitive Failures as Related to Mathematics Anxiety and Performance in Undergraduate Calculus. *Acta Didactica Napocensia*, *10*(2), 81–98. https://doi.org/10.24193/adn.10.2.7

Ball, D. (1993). With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics. *The Elementary School Journal*, *93*(4), 373–397. https://doi.org/10.1086/461730

Ball, D. L., & Friel, S. N. (1991). Implementing the "professional standards for teaching mathematics": What's all this talk about "discourse"? *The Arithmetic Teacher*, *39*(3), 44–48. https://doi.org/10.5951/AT.39.3.0044

Barkai, R. (2021). Using cases as a means to discuss errors in mathematics teacher education. *Education Sciences*, *11*(10). https://doi.org/10.3390/educsci11100575

Borasi, R. (1994). Capitalizing on Errors as "Springboards for Inquiry": A Teaching Experiment. *Journal for Research in Mathematics Education*, *25*(2), 166. https://doi.org/10.2307/749507

Creswell, J. W. (2012). *Educational Research Planning, Conducting and Evaluating Quantitative and Qualitative Research* (Fourth). Pearson Education, Inc. http://repository.unmas.ac.id/medias/journal/EBK-00121.pdf

Downing, D. (2009). Dictionary of Mathematics Terms. In *Barron's Educational Series, Inc.* (Third Edit). Barron's Educational Series, Inc. https://doi.org/10.2307/3614426

Gür, H. (2009). Trigonometry Learning. *New Horizons in Education*, *57*(1), 67–80. https://eric.ed.gov/?id=EJ860819

Hidayat, W., & Aripin, U. (2020). Identifikasi Kesalahan Jawaban Mahasiswa Pada Mata Kuliah Trigonometri Berdasarkan Dimensi Pengetahuan Krathwohl. *Jurnal Nasional Pendidikan Matematika*, *4*(1), 142–153. https://doi.org/10.33603/jnpm.v4i1.3316

Imelda. (2018). Analisis Kesulitan Mahasiswa dalam Menyelesaikan Soal Pemecahan Masalah Pada Mata Kuliah Aljabar dan Trigonometri. *Journal of Mathematics Education and Science*, *4*(1), 49–56. https://doi.org/10.30743/mes.v4i1.868

Jaelani, A. (2017). Kesalahan jawaban tes trigonometri mahasiswa pendidikan matematika semester pertama. *Journal of Mathematics Education*, *3*(2), 1–13. https://jurnalnasional.ump.ac.id/index.php/alphamath/article/view/2750

KBI. (2008). *Kamus Bahasa Indonesia*. Pusat Bahasa Departemen Pendidikan Nasional. https://perpus.unimus.ac.id/wp-content/uploads/2012/05/Kamus-Besar-Bahasa-Indonesia.pdf

Kshetree, M. P., Acharya, B. R., Khanal, B., Panthi, R. K., & Belbase, S. (2021). Eighth Grade Students' Misconceptions and Errors in Mathematics Learning in Nepal. *European Journal of Educational Research*, *10*(3), 1101–1121. https://doi.org/https://doi.org/10.12973/eu-jer.10.3.1101

Lial, M. L., Hornsby, J., Schneider, D. I., & Daniels, C. J. (2016). *Trigonometry* (11th ed.). Pearson. https://www.pxu.org/cms/lib/AZ01001825/Centricity/Domain/1204/Trigonometry%20Lial%2011th%20Edition.pdf

Muthukrishnan, P., Kee, M. S., & Sidhu, G. K. (2019). Addition error patterns among the preschool children. *International Journal of Instruction*, *12*(2), GHBLOL,aQ. https://doi.org/10.29333/iji.2019.1228a

Nabie, M. J., Akayuure, P., Ibrahim-Bariham, U. A., & Sofo, S. (2018). Trigonometric concepts: Pre-service teachers' perceptions and knowledge. *Journal on Mathematics Education*, *9*(2), 169–182. https://doi.org/10.22342/jme.9.2.5261.169-182

Oktaviani, M. (2017). Analysis of students' error in doing mathematics problem on proportion. *Proceedings Ofthe 2nd Asian Education Symposium (AES 2017)*, *March*, GHBLOL,aQ. https://doi.org/10.5220/0007300601720177

Rittle-Johnson, B., & Star, J. R. (2007). Does Comparing Solution Methods Facilitate Conceptual and Procedural Knowledge? An Experimental Study on Learning to Solve Equations. *Journal of Educational Psychology*, *99*(3), 561–574. https://doi.org/10.1037/0022-0663.99.3.561

Setiawan, Y. E. (2020). Analisis Kesalahan Siswa dalam Menilai Kebenaran Suatu Pernyataan. *Jurnal Didaktik Matematika*, *7*(1), 13–31. https://doi.org/10.24815/jdm.v7i1.14495

Setiawan, Y. E. (2021a). Analisis Kesalahan Mahasiswa Semester Pertama dalam Menentukan Nilai Fungsi Trigonometri Sudut Istimewa. *SJME (Supremum Journal of Mathematics Education)*, *5*(1), 321–334. https://doi.org/10.35706/sjme.v5i1.4531

Setiawan, Y. E. (2021b). Identifikasi kesalahan mahasiswa dalam menyelesaikan masalah matematika (Studi kasus masalah segitiga pada mata kuliah trigonometri). *Jurnal Kependidikan: Jurnal Hasil Penelitian Dan Kajian Kepustakaan Di Bidang Pendidikan, Pengajaran Dan Pembelajaran*, *7*(3), 649–662. https://doi.org/https://doi.org/10.33394/jk.v7i3.3329

Setiawan, Y. E. (2021c). Kesalahan mahasiswa semester pertama dalam menyelesaikan masalah fungsi trigonometri sudut tidak lancip. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(2), 599–614. https://doi.org/http://dx.doi.org/10.24127/ajpm.v10i2.3458

Setiawan, Y. E. (2021d). Kesalahan mahasiswa semester pertama dalam menyelesaikan masalah kecepatan sudut pada mata kuliah trigonometri. *PYTHAGORAS Jurnal Pendidikan Matematika*, *16*(1), 19–32. https://doi.org/10.21831/pg.v16i1.38560

Setiawan, Y. E. (2022a). Analysis of Error in Determining the Distance Between Two Points in the Cartesian Plane in the First Semester Students. *Jurnal Pendidikan Matematika Dan IPA*, *13*(1), 25. https://doi.org/10.26418/jpmipa.v13i1.43259

Setiawan, Y. E. (2022b). Kesalahan Menggambar Grafik Fungsi Cosinus. *EDU-MAT: Jurnal Pendidikan Matematika*, *10*(1), 1–14. https://doi.org/10.20527/edumat.v10i1.10800

Setiawan, Y. E., & Surahmat. (2021). Error analysis of prospective mathematics teachers in solving of applying radian measurement problem in trigonometry courses. *Al-Jabar: Jurnal Pendidikan Matematika*, *12*(2), 343–357. https://doi.org/https://doi.org/10.24042/ajpm.v12i2.9874

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in Problem Solving: The Case of Equation Solving. *Learning and Instruction*, *18*(2008), 565–579. https://doi.org/10.1016/j.learninstruc.2007.09.018

Tuna, A. (2013). A Conceptual Analysis of the Knowledge of Prospective Mathematics Teachers about Degree and Radian. *Word Journal of Education*, *3*(4), 1–9. https://doi.org/10.5430/wje.v3n4p1

Webb, N. M., & Mastergeorge, A. M. (2003). The development of students ' helping behavior and learning in peer-directed small groups. *Cognition and Instruction*, *21*(4), 361–428. https://doi.org/10.1207/s1532690xci2104