

Defragmentation of construction holes in students with high mathematical ability: Addressing “skipping steps” errors using scaffolding

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ABSTRACT

High-ability students generally excel in mathematics but frequently make “skipping steps” errors during complex problem-solving. These errors occur due to cognitive construction holes, where specific schemas are unconsciously bypassed, leading to incomplete final solutions. This study aims to identify and defragment these construction holes in high-mathematical-ability students using Level-3 scaffolding to restore complete cognitive processing. A qualitative multiple-case study was conducted involving three high-ability eighth-grade students from three different middle schools. Data were comprehensively collected through written tests, semi-structured interviews, and observations. The analysis utilized cognitive mapping based on Polya’s problem-solving stages to pinpoint specific cognitive gaps. Each student exhibited a “Skipping Steps” construction hole, omitting one or two essential schemas in the final stages. The application of Level-3 scaffolding successfully guided students to recover these missing schemas. It enabled them to self-detect errors, reconstruct their mathematical reasoning, and achieve accurate solutions while fully maintaining their procedural independence. Targeted conceptual scaffolding effectively remediates light construction holes in high-ability learners, ensuring problem-solving completeness without compromising their learning autonomy.

INTRODUCTION

Geometry learning plays a foundational role in mathematics education, supporting the development of measurement, spatial reasoning, proportional thinking, and problem-solving skills that are essential across scientific and technical disciplines (Flavin et al., 2025; Wu et al., 2024; Xu et al., 2025; Yang et al., 2025). Despite its importance, numerous studies report that students’ understanding of geometry remains fragmented, even after prolonged exposure to formal instruction (Hidayat et al., 2023; Moosapoor, 2023). Students may recall formulas or procedures yet fail to integrate underlying concepts coherently, resulting in incomplete or unstable solution structures. This condition indicates the presence of conceptual gaps, commonly referred to as construction holes, where essential schemas are absent or insufficiently constructed within students’ cognitive structures, leading to pseudo-complete or unjustified solutions.

One major source of this difficulty is not merely a lack of knowledge, but the presence of fragmentation in students’ thinking structures, where conceptual schemas are partially formed or disconnected (Wibawa et al., 2020). In such cases, students may understand the problem situation but are unable to construct a complete solution pathway because essential conceptual links are

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missing (Isnania et al., 2021). This phenomenon has been described in mathematics education as a construction hole, namely a gap in the cognitive schema that emerges during the process of concept construction (Putri & Indrawatiningsih, 2023; Rosimanidar et al., 2024). Moreover, this phenomenon becomes even more critical when considering students with high mathematical ability, as their conceptual difficulties are often concealed behind procedural fluency and thus remain underdiagnosed.

Importantly, fragmentation and construction holes are not limited to students with low mathematical ability. Recent studies indicate that students with high mathematical ability may also experience conceptual breakdowns, particularly in non-routine or geometrically complex tasks (Dorner et al., 2025; Prast et al., 2025). However, errors made by high mathematical ability students tend to be subtle, pseudo, or hidden, like they often appear fluent procedurally, yet omit or skip a crucial step, especially at the final stage of reasoning that leading to incorrect or unjustified conclusions (Ayhanöz & Altun, 2024). Such errors are therefore difficult to detect through conventional assessment focused on final answers alone (Zhu et al., 2024). These characteristics suggest that high mathematical ability should not be defined solely by performance outcomes, but also examined through the underlying cognitive, representational, and metacognitive processes that shape students' reasoning.

Students with high mathematical ability are generally characterized not only by high test scores, but also by their capacity to flexibly coordinate multiple mathematical processes. From a cognitive perspective, high mathematical ability students demonstrate strong conceptual understanding, efficient retrieval of prior knowledge, and the ability to connect representations, like visual, symbolic, and verbal when solving problems (Bley et al., 2024; Hadi & Csíkos, 2025; Patmawati & Unaenah, 2025; Rahmawati et al., 2025). They tend to show advanced reasoning, effective problem-solving strategies, and greater metacognitive awareness, such as monitoring solution progress and evaluating the plausibility of results. From an instructional perspective, these students often display procedural fluency and speed, which can mask underlying conceptual weaknesses (Aladwan et al., 2023; Gao et al., 2025; May, 2024; National Council on Teacher Quality, 2025). Consequently, while high-ability students may arrive at correct answers in routine tasks, their errors in non-routine or geometrically complex problems are more likely to appear as subtle omissions, skipping steps, or incomplete justification rather than overt misconceptions. This dual profile, strong surface performance alongside potential hidden conceptual gaps, makes high-ability students a critical group for investigating light or pseudo construction holes that are difficult to detect through conventional assessments alone.

In addition to achievement scores, mathematical ability, particularly at a high level, is commonly reflected in the quality of students' problem-solving processes. One widely used framework for examining problem solving in mathematics is Polya's four stages: understanding the problem, devising a plan, carrying out the plan, and looking back (Ermilia & Sutarni, 2024; Pathuddin et al., 2024). Previous studies suggest that students with high mathematical ability tend to perform consistently across these stages, demonstrating not only correct solutions but also coherent planning, execution, and reflection (Pathuddin et al., 2024). For this reason, Polya-based problem-solving performance is frequently employed as a complementary indicator to achievement scores when identifying students' mathematical ability levels, especially in qualitative investigations focusing on cognitive processes rather than outcomes alone.

Previous research on students' fragmented thinking has largely emphasized representational shifts, such as transitions between visual and symbolic forms. For example, studies have shown that students may correctly interpret geometric visuals but fail to translate them into formal mathematical procedures, resulting in incomplete solutions (Sholihah & Maryono, 2020; Triyani et al., 2023). While these findings are relevant to fragmentation in general, they do not explicitly address construction holes as a specific type of conceptual error, nor do they examine how missing schemas can be systematically identified and repaired within a structured intervention framework.

Within the Indonesian mathematics education literature, construction holes are conceptualized as errors arising from incomplete or imperfect schema construction, where essential conceptual components are absent from the student's thinking structure (Rosimanidar et al., 2024). When such schemas are missing, students may produce answers that appear correct superficially,

Table 1
Defragmentation steps (Putri & Indrawatiningsih, 2023)

Defragmentation Steps	Explanation
Scanning	<ul style="list-style-type: none"> • Researcher make the structure of students' thinking. • Students present their work.
Check some error	<ul style="list-style-type: none"> • Researcher checked the wrong parts.
Repairing	<ul style="list-style-type: none"> • Researcher made improvements and structuring the thinking structure of students who experienced fragmentation by using scaffolding building blocks.
Give a chance to re-work	<ul style="list-style-type: none"> • Students are allowed to rework the problems given previously.
Verify the result	<ul style="list-style-type: none"> • Researcher checks and re-assures that the answers given are correct. • Researcher also asked again what the students did and understood regarding the given problem.

yet the underlying reasoning process remains conceptually flawed or incomplete. To address this issue, defragmentation of thinking structures has been proposed as a targeted intervention aimed at reorganizing and repairing fragmented cognitive schemas (Mardhiyatirrahmah & Abdussakir, 2021; Wibawa et al., 2020).

Defragmentation is commonly operationalized through a sequence of analytical and instructional steps, including scanning students' initial thinking structures, identifying conceptual errors, repairing missing schemas through targeted support, allowing students to rework the problem, and confirming the reconstructed understanding (Isnania et al., 2021). Prior studies suggest that this process can effectively generate new, more coherent schemas, a process referred to as schema generation, that enable students to solve problems meaningfully rather than procedurally (Putri & Indrawatiningsih, 2023; Wibawa et al., 2020). However, the effectiveness of defragmentation largely depends on the nature of instructional support provided during these steps, particularly the type and level of guidance used to trigger missing schemas without overtly directing students' solutions, shown in Table 1.

Among various instructional supports, scaffolding plays a central role in the defragmentation process. According to Anghileri's framework, scaffolding can be organized into three levels: environmental support, interactive restructuring, and the development of conceptual thinking (Anghileri, 2006). Level-3 scaffolding, which focuses on prompting conceptual connections and reflective reasoning, is particularly relevant for students with high mathematical ability. Because of their construction holes tend to be "light" and occur at advanced stages of reasoning, direct procedural guidance may be unnecessary or even counterproductive. Instead, conceptual prompts that encourage students to re-examine assumptions, relationships, and justifications are more likely to activate missing schemas and restore conceptual coherence (Anghileri, 2006; Huda & Marzal, 2023).

Despite growing interest in defragmentation and scaffolding, several research gaps remain evident. First, empirical studies that explicitly investigate construction holes among high-ability students are still limited, especially in the context of geometry. Second, few studies examine defragmentation processes involving geometric problems that require identifying interacting or touching surfaces (e.g., flat-sided solids), which impose high mathematical ability on spatial visualization and conceptual integration. Third, the use of Level-3 scaffolding as a deliberate intervention strategy for repairing construction holes in high-ability students has not been sufficiently explored or documented.

To address these gaps, the present study focuses on defragmenting construction holes in high-ability eighth-grade students as they solve geometry problems involving flat-sided solids. Using a qualitative case-study approach, this research aims to (1) identify patterns of construction holes that emerge in high-ability students' geometric reasoning, (2) examine how Level-3 scaffolding supports schema reconstruction during the defragmentation process, and (3) describe changes in students' thinking structures before and after intervention. The findings are expected to contribute theoretically by refining the characteristics of light construction holes in high mathematical ability

Table 2
Polya-based problem-solving performance of selected high-ability students

Subject Code	Problem-Solving Ability				Problem-Solving Ability (Planning and Implementation)	Overall Problem-Solving Ability
	Understanding the Problem	Planning the Solution	Carrying Out the Plan	Looking Back		
S1	Very Good	Very Good	Good	Very Good	Very Good	Very Good
S2	Very Good	Good	Good	Very Good	Good	Very Good
S3	Very Good	Very Good	Very Good	Very Good	Very Good	Very Good

from students and practically by providing teachers with diagnostic insights and intervention strategies for addressing subtle conceptual errors in geometry learning.

METHODS

This study used a qualitative multiple-case study design to examine in depth the process of defragmenting construction holes in the thinking structures of high-ability eighth-grade students while solving geometry problems. Research permission was obtained from school administrators, and informed consent from parents/guardians as well as student assent were secured prior to data collection. Participant confidentiality was ensured through anonymous coding (S1–S3), with all audio files and transcripts stored on a password-protected drive accessible only to the research team. Each case (S1, S2, S3) is treated as a single unit of analysis with within-case and cross-case comparisons reported. The multiple-case approach was chosen to allow detailed, context-sensitive description and to identify patterns that may generalize across similar contexts.

The subjects in this study were three eighth grade students from the Insan Cendekia Islamic Integrated Middle School in Malang. The initial sample consisted of 116 eighth-grade students (73 males, 43 females) from three schools. The sampling process followed these steps:

1. Screening test administration: All 116 students completed a geometry test designed to elicit possible construction holes.
2. Preliminary analysis and shortlisting: Responses were analyzed to detect evidence of construction holes. Thirteen students whose responses exhibited substantive fragmentation (i.e., missing or weakly connected schemas) were shortlisted for follow-up.
3. Ability classification and final case selection: Mathematical ability was determined using students' mathematics achievement scores and their problem-solving performance. Achievement scores were categorized as high (85–100), moderate (75–84), and low (≤ 74) following established benchmarks (Jaenudin et al., 2024). Problem-solving ability was analyzed using Polya's four stages and rated on a five-point Likert scale like from very poor to very good (Pathuddin et al., 2024). Students who demonstrated very good problem-solving performance were classified as having high mathematical ability. From the 13 shortlisted students, three students who showed clear construction holes and met the high ability criteria were purposively selected as the cases (S1–S3) shown in Table 2.
4. Justification for three cases: Three in-depth cases allow rich within-case description and cross-case pattern identification while remaining feasible for detailed cognitive mapping, repeated interviews (scaffolding), and rigorous triangulation.

The results of the test were selected based on the presence or absence of construction holes, the number of construction holes formed, and students who had high mathematical abilities. Specifically, construction holes were identified when students provided correct answers but demonstrated inappropriate conceptual construction, or when the relevant concepts were only partially formed. The Figure 1 following is an illustration of the stage of determining the subject carried out.

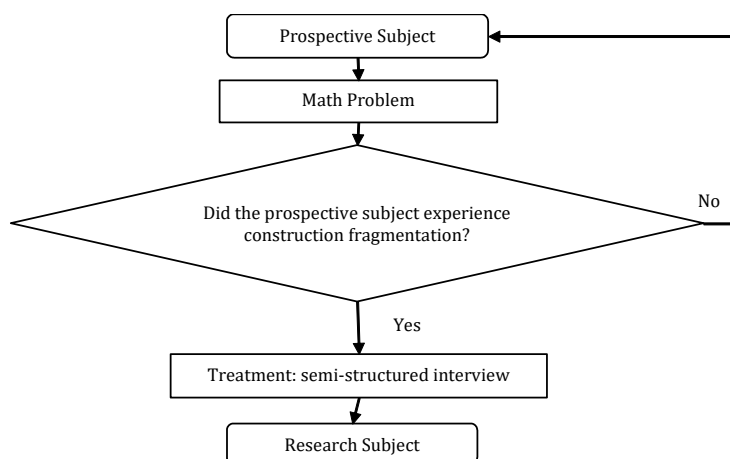


Figure 1. Determination of Research Subjects

A monument in the form of a stack of three cubes will be painted blue as shown in the picture below. The first cube is the smallest of the three cubes that make up the monument. The first cube has a volume 8 times smaller than the second cube. The second cube has a volume of 8 m^3 . The third cube has edges that are 3 times longer than the edges of the first cube. Determine the surface area of the monument to be painted!



Figure 2. The Question of the Test

Sources of data used in this study consisted of student answer sheets, interviews, and observations. The questions used are in the form of descriptive questions that will be used to analyze the structure of students' thinking in solving geometrical problems of flat-sided geometry. The test questions are validated by three doctors of mathematics before being tested on prospective research subjects. The test was piloted with 23 comparable students from a nearby middle school. Items were revised for clarity based on pilot results through four rounds of revision, including expert review by three doctors of mathematics and field testing, before being finalized. The following is a math problem with geometry problem that will be given, shown in [Figure 2](#).

Conclusions in this study were carried out with the following steps.

1. Identifying construction holes in solving geometry problems:
 - a. Describing the thinking structure of each subject in solving geometrical problems, then analyzed based on the Polya stage problem solving indicators
 - b. Choose the subject according to the structural error of thinking in the construction hole based on a predetermined error indicator.
2. Analyzing the defragmentation process of students' thinking structures:
 - a. Comparing the defragmentation processes across cases by examining differences in scaffolding implementation and types of construction-hole fragmentation. Cognitive maps were used to represent changes in students' thinking structures before and after scaffolding. The following is an image of a cognitive map of solving geometric problems from the questions that will be given to the subject, show in [Figure 3](#).

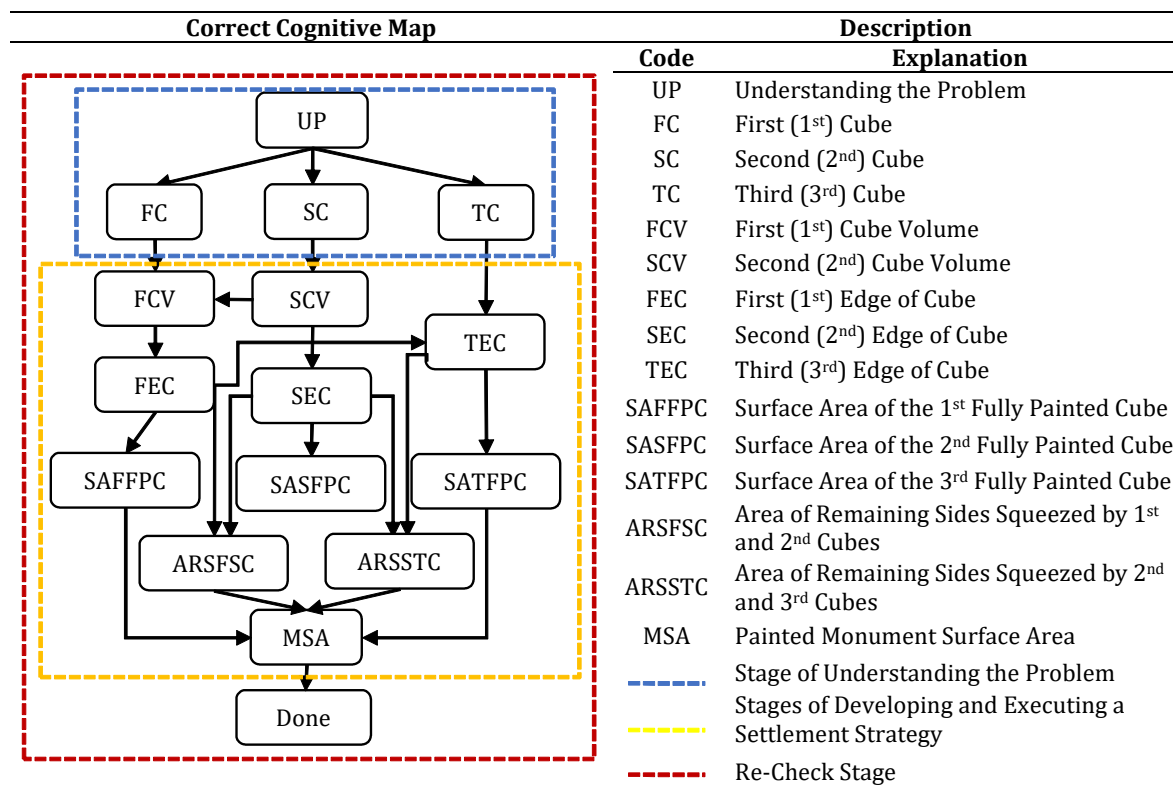


Figure 3. Correct Cognitive Map on Geometry Problem Solving

Table 3
Mapping of interview guidelines, level-3 sScaffolding, and construction-hole indicators

Problem-Solving Stage	Interview Focus / Guideline	Level-3 Scaffolding Prompt Type	Example Scaffolding Prompt	Construction-Hole Indicator
Understanding the Problem	Identifying knowns and unknowns; explaining problem meaning	Clarification	“Can you explain what information is given and what is being asked in this problem?”	Student provides correct interpretation but cannot explain the conceptual relationship between known and unknown elements
Devising a plan	Selecting concepts, models, and strategies	Justification	“Why did you choose this concept or strategy to solve the problem?”	Student selects an appropriate strategy but shows incomplete or inappropriate conceptual justification
Carrying out the plan	Applying concepts and strategies consistently	Clarification / Justification	“Can you explain how this step follows from the concept you selected?”	Student produces correct intermediate results while skipping essential conceptual steps
Checking Solution	Verifying steps and detecting errors	Reflection	“Are you confident that each step is correct? Why?”	Student accepts the result without validating the reasoning or conceptual consistency
Looking back	Confirming final answer and drawing conclusions	Reflection	“Does your solution fully answer the question? How can you be sure?”	Student reaches a correct final answer despite incomplete conceptual construction

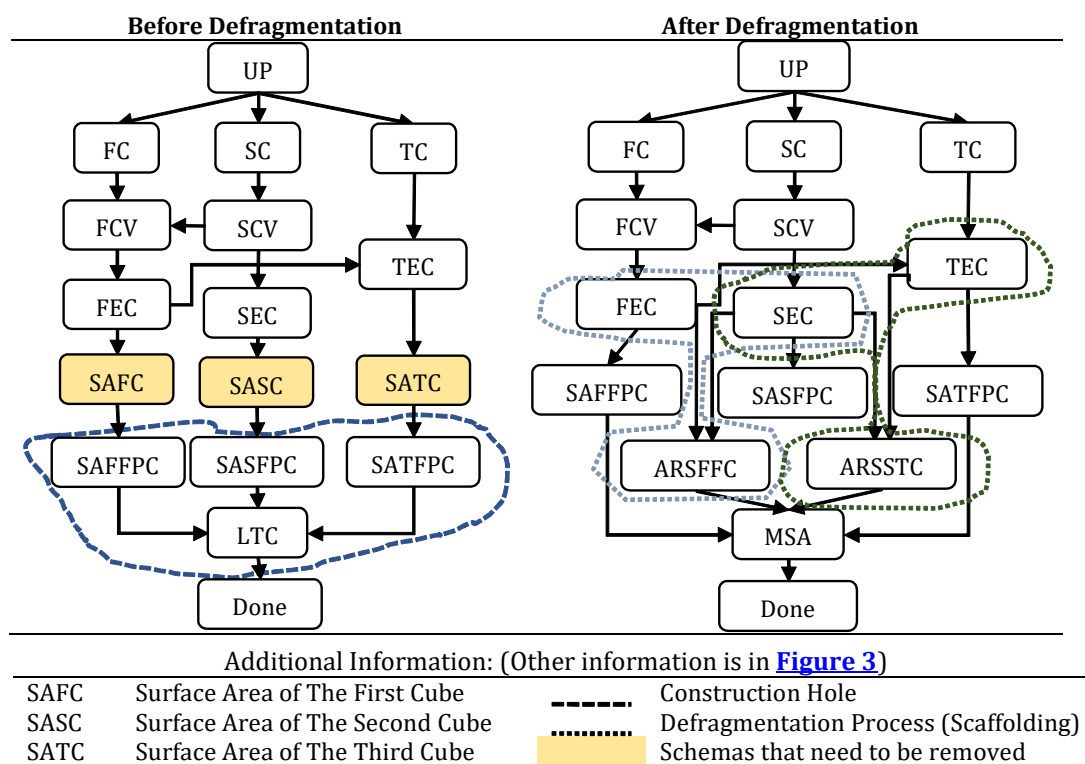


Figure 4. Cognitive Map of S1 Before and After Defragmentation

- b. Quantifying defragmentation outcomes: The degree of defragmentation was calculated using a defragmentation percentage, defined as the proportion of previously missing cognitive nodes that were recovered after scaffolding relative to the number of missing nodes identified at baseline. This percentage was used to describe the extent of schema recovery for each case.

$$\text{Defrag. \%} = \frac{\text{Number of previously-missing nodes that are Recovered after scaffolding}}{\text{Number of missing nodes observed at cognitive map}} \times 100$$

For example, cognitive map shows 9 total expected nodes; 3 are Missing ($M = 3$). After scaffolding the student recovers 2 of the 3 missing nodes ($R = 2$). Defragmentation $\% = (2/3) \times 100 = 66.7\%$. This percentage quantifies the degree of schema recovery for each case; thresholds (e.g., $< 15\%$ as “light”) are reported descriptively and justified in the Results Discussion.

3. Integrating interview data, scaffolding, and construction-hole indicators: To ensure analytical coherence, the interview guidelines in [Table 3](#) were systematically mapped to Level-3 scaffolding prompts and predefined construction-hole indicators. This mapping clarifies how specific scaffolding actions during interviews correspond to students’ cognitive responses and the identification of construction holes in each problem-solving stage.

FINDINGS

The process of defragmenting the structure of students’ thinking involves construction holes that have been identified and their handling with scaffolding in the form of guidance without changing the method of completion that has been done before the repair. The following describes the defragmentation process data for each subject.

First subject (S1)

The S1 answer was first converted into a cognitive map and then each step was coded to analyze before and after the defragmentation process. Based on the S1 cognitive map, the researcher found out that there were differences such as some missing schemas formed or appeared after the defragmentation process, either through guiding scaffolding or building blocks. Figure 4 shows a cognitive map of S1 before and after defragmentation, show in [Figure 4](#).

$54 - (4) = 50 \rightarrow \text{Balok ke-3}$
 $20 - 1 = 19 \rightarrow \text{Balok ke-2}$
 $1 \times 5 = 5 \rightarrow \text{Balok ke-1}$
 $\underline{74 \text{ m}^2}$

Figure 5. Construction Hole of the First Subject in S1 Answer Sheet

Based on the cognitive map, the subject is known to be able to understand the problem as a whole and identify information that is known from the problem. Even the subject can formulate a problem-solving strategy but is constrained in carrying out the plan due to the construction holes that are formed. There is only one construction hole that is formed and there are no other thinking structure errors when solving problems. The construction hole, which is the area of the remaining sides of the three cubes, is not calculated due to misunderstanding the concept of painted sides. The following is the result of S1's work which stated that there were construction holes identified (Figure 5).

The handling for this construction hole is only with level 3 scaffolding, namely developing conceptual thinking. The subject was only given a few questions related to the answer, he immediately knew where his mistake was. Scaffolding at this level only aims to develop conceptual knowledge, meaning that the researcher does not lead to changing all the answers according to the researcher's alternative answers. This can be seen when S1 corrects the answer without changing the answer from the start and interview which confirms the defragmentation.

- P_{3,6} : Why $5\frac{1}{2}$?
 S_{1,3,6} : Because if you look at it from the side... oh yes...
 P_{3,6} : Why?
 S_{1,3,6} : I think this, the bottom is also painted. It should be $4\frac{1}{2}$ anyway.
 P_{2,3} : Does that mean the bottom should be coloured or not?
 S_{1,2,3} : It shouldn't be because this is a monument, not 3D modelling.

Several schemes can be omitted to shorten the resolution of the problem. These schemes are related to the surface area of the three cubes that make up the monument. The subject can just solve it without knowing the surface area first, but he chose another alternative solution that he understands. It does this naturally when solving problems. In addition, the researcher realized that the mistakes made were not because they did not understand the concept, but the schema was missed or when completing it accidentally left the section. The researcher concluded that this subject was again given level 3 scaffolding, namely developing the concept of thinking to find alternative solutions. This is shown based on the following interview transcript.

- P_{3,7} : This means, then what is the correct answer?
 S_{1,3,7} : Means minus the area of the bottom, the surface area of the bottom side.
 P_{3,7} : So how much will the result be?
 S_{1,3,7} : Hold on... it must be 56..
 P_{4,3} : Period 56? How much is the bottom one? How much was the 74 deducted earlier?
 S_{1,4,3} : 182

Based on the analysis of the results of the subject's work both before and after the defragmentation process, the researcher concluded that the subject was able to understand the problem as a whole and identify information that was known from the problem. Even the subject can formulate a problem-solving strategy and is not constrained by anything even though an error is found. Subjects are also able to identify parts of the monument that are painted and can develop the basic formula for the surface area used. Not only that, the subject can identify mistakes made independently and fix them. The defragmentation process carried out to bring up the missing schema

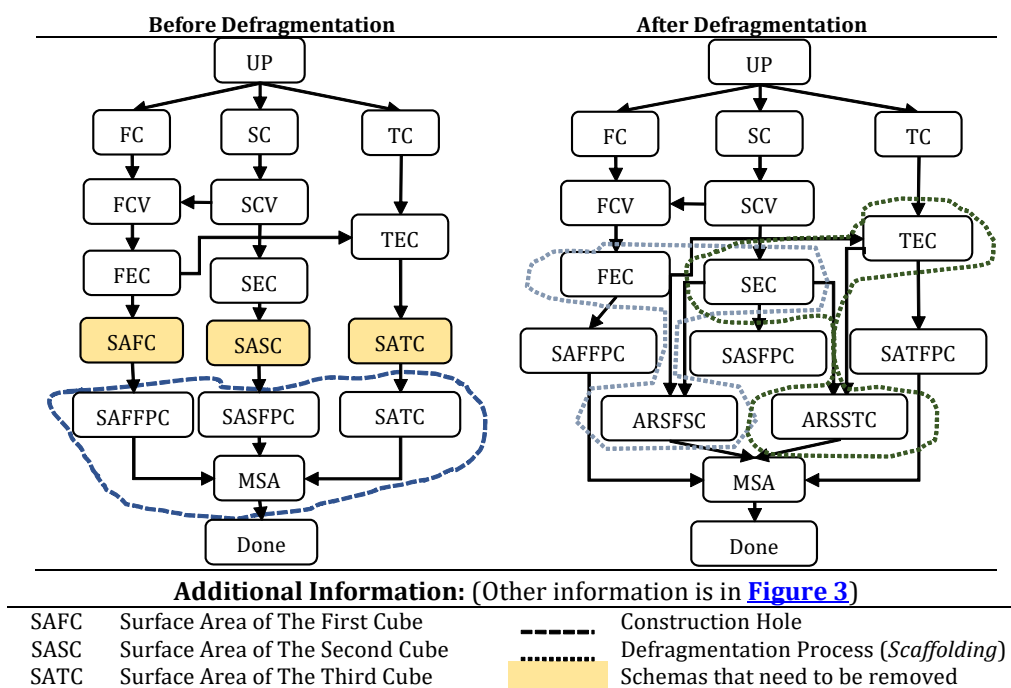


Figure 6. Cognitive Map of S2 Before and After Defragmentation

$$\begin{aligned}
 & \text{luas permukaan yg harus dicat} \\
 & (L \cdot pk_1 - 1) + (L \cdot pk_2 - 2 - 1) + (L \cdot pk_2 - 3 - 2) \\
 & = (6 - 1) + (24 - 3) + (54 - 5) \\
 & = (5) + (21) + (49) = 75 \text{ m}^2
 \end{aligned}$$

Figure 7. Construction Hole of the Second Subject in her Answer Sheet

can be done with the help of questions or statements aimed at developing their thinking concepts or using level 3 scaffolding. Therefore, the subject only focuses on developing the concepts he has chosen in solving geometric problems. The percentage of defragmentation of construction pits in all completion steps carried out to completion reaches 11%.

Second subject (S2)

The S2 answers were first converted into a cognitive map and then each step was coded to analyze before and after the defragmentation process. Based on the master's cognitive map, the researcher found out that there were differences such as some missing schemas formed or appeared after the defragmentation process through scaffolding. Figure 6 shows a cognitive map of S2 before and after defragmentation, show in [Figure 6](#).

Based on the cognitive map, the subject is known to be able to understand the problem as a whole and identify information that is known from the problem. Even the subject can formulate a problem-solving strategy and there is no difficulty in carrying it out. There is only one construction hole that is formed and there are no other thinking structure errors when solving problems. The construction hole, which is the area of the third side of the cube, is not calculated due to misunderstanding the concept of painted sides. The following is the result of the S2 work which stated that there were construction holes identified, show in [Figure 7](#).

The handling for this construction hole is only with level 3 scaffolding, namely developing conceptual thinking. The subject was only given a few questions related to the answer, he immediately knew where his mistake was. Scaffolding at this level only aims to develop conceptual knowledge, meaning that the researcher does not lead to changing all the answers according to the

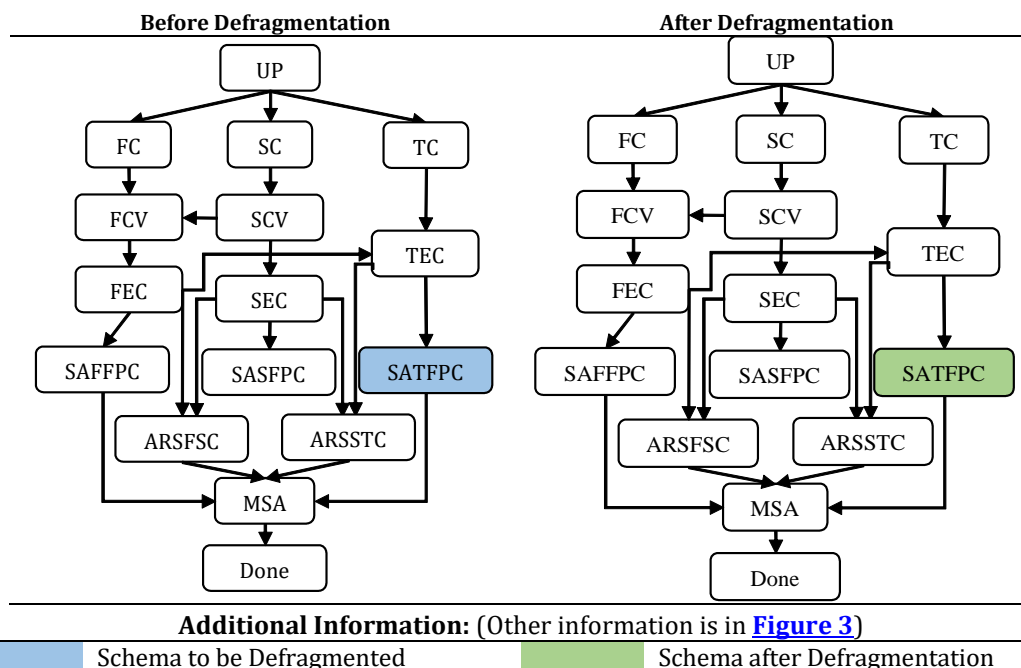


Figure 8. Cognitive Map of S3 Before and After Defragmentation

researcher's alternative answers. This can be seen when S2 corrects the answer without changing the answer from the start. The following is a transcript of the S2 interview which confirms the defragmentation.

- P_{3,4} : *This is if the first cube has the front, back, left, right, and topsides that are coloured. So, how many sides are coloured?*
- S_{2,3,4} : *There are 5.*
- P_{3,5} : *How about the second cube?*
- S_{2,3,5} : *There are 5, sides, top, and bottom.*
- P_{2,6} : *The bottom one is coloured too, right? Isn't the bottom of the monument closed?*
- S_{2,2,6} : *Nope.*

Several schemes can be omitted to shorten the resolution of the problem. These schemes are related to the surface area of the three cubes that make up the monument. The subject can just solve it without knowing the surface area first, but he chooses another alternative solution that he understands. It does this naturally when solving problems. The researcher concludes that this subject can be given level 3 scaffolding, namely developing thinking concepts to find alternative solutions. This is shown based on the following interview transcript.

- P_{2,4} : *Let's read the problem first. Yesterday it was one of the answers, only minus one, right? How many painted sides does this first cube have?*
- S_{2,2,4} : *There are 3.*
- P_{4,5} : *Why 3? The one on the back isn't coloured?*
- S_{2,4,5} : *The sides are coloured too. So, 5 means.*
- P_{2,5} : *Well, some of the answers have been correct. The first cube is a total of 6 minus 1. So 5. The second cube is 24. Why 24 - 3?*

Based on the analysis of the results of the subject's work both before and after the defragmentation process, the researcher concluded that the subject was able to understand the problem as a whole and identify information that was known from the problem. Even the subject can formulate a problem-solving strategy and is not constrained by anything even though an error is found. Subjects are also able to identify parts of the monument that are painted and can develop the basic formula for the surface area used. Not only that, the subject can identify mistakes made independently and fix them.

The defragmentation process carried out to bring up the missing schema can be done with the help of questions or statements aimed at developing their thinking concepts or using level 3

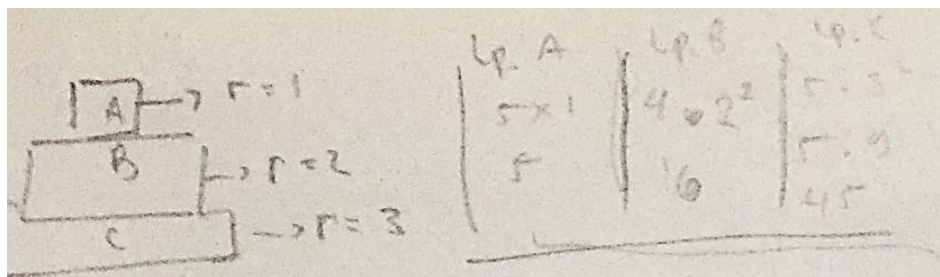


Figure 9. Construction Holes of the Third Subject in her Answer Sheet

scaffolding. Therefore, the subject only focuses on developing the concepts he has chosen in solving geometric problems. The percentage of defragmentation of construction pits in all completion steps carried out to completion reaches 11%.

Third subject (S3)

The S3 answers were first converted into a cognitive map and then each step was coded to analyze before and after the defragmentation process. Based on the S3 cognitive map, the researcher found out that there were differences such as some missing schemas formed or appeared after the defragmentation process through scaffolding. Figure 8 shows a cognitive map of S3 before and after defragmentation.

Based on the cognitive map, the subject is known to be able to understand the problem as a whole and identify information that is known from the problem. Even the subject is able to formulate a problem-solving strategy and there is no difficulty in carrying it out. There is only one construction hole that is formed and there are no other thinking structure errors when solving problems. The construction hole is the area of the fully unpainted side of the third cube. The following is the result of the S3 work which stated that there were construction holes identified (Figure 9).

The handling for this construction hole is only with level 3 scaffolding, namely developing conceptual thinking. The subject was only given a few questions related to the answer, he immediately knew where his mistake was. Scaffolding at this level only aims to develop conceptual knowledge, meaning that the researcher does not lead to changing all the answers according to the researcher's alternative answers. This can be seen when S3 corrects the answer without changing the answer from the start. The following is a transcript of the S3 interview that confirms the defragmentation.

- P_{3,5} : *What about B (the second cube)?*
 S_{3,3,5} : *4 fully coloured sides, 1 incomplete side.*
 P_{3,6} : *which C (the third cube)?*
 S_{3,3,6} : *5 sides are full, then 1 side is not full.*
 P_{2,4} : *It means that the C (third) below it is coloured too?*
 S_{3,2,4} : *Yes, uh wait...*
 P_{2,4} : *Is it coloured or not?*
 S_{3,2,4} : *Yes, may be coloured.*
 P_{2,4} : *That was the question, if you still remember, it's a monument. Is the bottom of the monument coloured too? What's not?*
 S_{3,2,4} : *No? haha...*

This subject does not suffer from other errors that make many schemas lost. The schematic identified as having construction holes does not need much improvement. This is proven when the subject can solve it directly without knowing the surface area first, but he chooses another alternative solution that he understands. It does this naturally when solving problems. The researcher concludes that this subject can be given level 3 scaffolding, namely developing the concept of thinking without changing the problem solving that has been done previously. This is shown based on the following interview transcript.

- S_{3,4,2} : *Previously, sure. However, I didn't know that (sambal demonstrating the monument) was on the ground. In my point of view, which is squared as seen from all directions, continues to be counted below.*

- P_{3,7} : *So, that was because it was a monument, so it was connected to the ground, right? So, it means that the bottom is not coloured. So, the actual result is 74. 74 what is this? What's below is not included. So how much?*
- S_{3,3,7} : *65, right?*
- P_{4,2} : *How? Are you sure or not?*
- S_{3,4,2} : *This is wrong, only one side is below. That means $3 \times 3 = 9$, $74 - 9$. Yes, 65.*

Based on the analysis of the results of the subject's work both before and after the defragmentation process, the researcher concluded that the subject was able to understand the problem as a whole and identify information that was known from the problem. Even the subject can formulate a problem-solving strategy and is not constrained by anything even though an error is found. Subjects are also able to identify parts of the monument that are painted and can develop the basic formula for the surface area used. Not only that, the subject can identify mistakes made independently and fix them.

The defragmentation process carried out to bring up the missing schema can be done by scaffolding in the form of questions or statements. This assistance does not need to be provided by the relevant expert or teacher. That is, the subject can identify errors only with questions from peers. The percentage of defragmentation of construction pits in all completion steps carried out to completion reaches 6%.

DISCUSSION

This construction hole is known as a light construction hole because it occurs in students with high mathematical abilities. This ability helps students to identify mistakes made independently so that they can correct them (Putri & Indrawatiningsih, 2023). Not only that, students can understand the problem and plan a solution strategy. This can be seen when students can identify parts of the painted monument and can develop the basic formula for the surface area used. However, students were constrained in carrying out the plan which led to the emergence of construction holes. When this construction hole appears, the damage can be handled by the third level scaffolding, namely developing conceptual thinking or level-3 scaffolding (Anghileri, 2006). Thus, the application of Level-3 scaffolding described here is specifically tailored for high-ability students, while students with heavier construction holes may require different forms of support.

Practically, teachers can calibrate Level-3 scaffolding prompts to encourage self-monitoring and conceptual reconstruction without providing the solution (Vo et al., 2025). For example, after a student executes a plan and skipping step, a teacher might ask:

1. "Can you check if each part of your solution aligns with the problem question?"
2. "Which concepts did you use here, and could anything be missing?"
3. "Could you explain this step to someone else, what would you say?"

These prompts guide students to recover omitted schemas, reinforcing metacognitive regulation and conceptual thinking while preserving their procedural independence (Vo et al., 2024; Yuriev et al., 2017).

Notably, the distinction in scaffolding levels is not solely determined by general mathematical ability. In our initial pool of 13 students, several students with low mathematical ability based on Polya problem-solving stages, exhibited severe construction holes, rather than light ones. These students required different interventions, including concrete manipulatives, rather than Level-3 scaffolding. Therefore, the assignment of Level-3 scaffolding in the current study was specific to high-ability students with light construction holes, consistent with the treatment described in prior thesis (Isnania et al., 2021).

Initially, students are given questions or statements that make students doubt so that they can find mistakes in solving the problems they are doing. Students with this construction hole do not need a lot of questions even when they find their mistakes, students can immediately fix them without being helped. In addition, students with this level of construction holes usually solve problems in different ways. Problem-solving that is done is an alternative answer and this is included in the third level scaffolding. This is due to the ability to solve the problem so that the supervisor only needs to direct without changing the original answer (Vo et al., 2024; Yuriev et al., 2017). Then, the handling for this construction hole is only with level 3 scaffolding, namely developing conceptual thinking. Students are only given a few questions related to the answer, their immediately knows

where his mistake is. Scaffolding at this level only aims to develop conceptual knowledge, meaning that the supervisor does not lead to changing all the answers according to the supervisor's alternative answers (Ajayan et al., 2025; Pakpahan et al., 2025; Yuriev et al., 2017).

Light construction holes referred to as first-category construction holes can be defragmented or repaired by schema appearance. This category can be named as "*Skipping Steps*". It means that the student has the construction hole because they skipped steps accidentally. Then, the mention is like that because the errors made are less than 15% and are not influenced by other types of thinking structural errors. Students only lose 1 or 2 basic schemas that can be re-emerged through level 3 scaffolding. The missing schemas contain important concepts that at least must exist in problem-solving. However, these schemas are lost due to students skipping or forgetting the schematics at the final stage when solving problems. Therefore, scaffolding is given to stimulate students to find or come up with these schemas (Mardhiyatirrahmah & Abdussakir, 2021).

High-ability students' tendency to skip steps during the execution phase can be understood through the lens of Cognitive Load Theory (CLT). Even learners with strong prior knowledge face working-memory limits when a task requires simultaneous management of multiple spatial and procedural schemas, so some final-stage schemas may be omitted under load (Gupta & Zheng, 2020; Paas & van Merriënboer, 2020). In this view, skipped steps reflect a transient overload of processing resources rather than a lack of conceptual knowledge, appropriately timed conceptual prompts therefore serve to regulate information flow and offload working memory, enabling retrieval or reconstruction of missing schemas without supplying answers. Empirical and review work on expert scaffolding and CLT supports the idea that well-calibrated scaffolding reduces extraneous load and helps learners reorganize complex visual or multi-step problems (Appiah-Twumasi, 2024; Faber et al., 2024; Kranz et al., 2026; van Nooijen et al., 2024).

Complementing this cognitive explanation, findings align with research on metacognition: high-ability students frequently exhibit stronger monitoring and evaluation skills, like they can detect inconsistencies and reflect on solution plausibility, which explains why light construction holes are often recoverable with minimal, concept-focused prompts. Level-3 scaffolding functions partly as metacognitive support, prompting students to check, justify, and reconnect elements of their solution (Anghileri, 2006; Huda & Marzal, 2023; Vo et al., 2024). So, it leverages learners' existing self-monitoring to trigger schema generation rather than imposing procedural fixes. Prior studies show that metacognitive scaffolding improves students' ability to plan, monitor, and review mathematical solutions, and that gifted learners commonly possess sophisticated metacognitive strategies though with variability, making them particularly responsive to conceptual, non-directive prompts (Shahzad et al., 2025; Vo et al., 2025).

Scaffolding can help students overcome difficulties in solving problems with various assistance provided (Jiang & Wang, 2025). The assistance provided can be adjusted to the difficulties experienced by students (Huda & Marzal, 2023). Each level or level of the construction pit has its handling. The light construction pit or the first level is handled with the third level of scaffolding.

CONCLUSIONS

This study aimed to explore how construction holes in high-ability students' thinking structures can be defragmented using scaffolding. The first category of construction holes, termed "*Skipping Steps*", arises when students omit or forget one or two final-stage schemas while solving geometry problems, without other structural thinking errors. This study demonstrates how students' thinking structures changed from pre- to post-defragmentation, indicating patterns of missing schemas and their recovery through scaffolding. These construction holes occurred in high-ability students who were able to understand problems, plan and execute solution strategies, and identify mistakes independently, but occasionally skipped essential schemas at the final step.

Defragmentation of these light construction holes was effectively achieved through Level-3 scaffolding, focusing on developing conceptual thinking without directing students to change their original solutions. Students were guided with conceptual prompts that helped them recover omitted schemas, reinforcing metacognitive regulation while preserving procedural independence. These prompts enabled students to check alignment with the problem, reflect on used concepts, and

articulate reasoning, allowing missing schemas to re-emerge naturally. Students with construction holes at this level could understand the problem as a whole, identify known information, plan and execute problem-solving strategies, and detect and correct their own mistakes. The cause of these construction holes stems from skipping or forgetting schematics at the final stage of problem-solving. Applying Level-3 scaffolding facilitated schema reconstruction while preserving procedural independence.

These findings are indicative for this specific context and should be interpreted with caution due to the small sample size (N=3). Practically, teachers can employ Level-3 conceptual prompts to diagnose and remediate skipped end-stage schemas in high-ability learners. Theoretically, the study refines the notion of light construction holes as omissions of final-step schemas rather than broader structural errors. Overall, Level-3 scaffolding is sufficient to restore skipped end-stage schemas in high-ability students, highlighting its value for targeted cognitive support. Future research could extend this work to students of different ability levels, diverse problem types, and include multiple coders for cognitive map analysis to improve reliability and generalizability. Not only that, Further research could expand to different ability levels, task types, and employ double-coding of cognitive maps to validate and generalize these patterns.

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AUTHOR'S DECLARATIONS

Authors' contributions

LM and DR contributed to the study's conception and design. Material preparation, data collection, and data analysis were performed by both authors. The first draft of the manuscript was written by LM, and DR critically reviewed and edited the manuscript. Both authors read and approved the final manuscript.

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Availability of data and materials

The datasets generated and/or analyzed during the current study are not publicly available due to the privacy and confidentiality of the student participants, but are available from the corresponding author on reasonable request.

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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