Journal of Research and Advances in Mathematics Education

Volume 8, Issue 2, April 2023, pp. 87– 99
DOI: 10.23917/jramathedu.v8i2.1402
p-ISSN: 2503-3697, e-ISSN: 2541-2590

Examining pre-service elementary mathematics teachers' views regarding critical thinking and doing mathematical proof

Elif Akşan Kiliçaslan*, Tuba Aydoğdu İskenderoğlu

Trabzon University, Turkiye

Citation: Kiliçaslan, E. A., & İskenderoğlu, T. A. (2023). Examining pre-service elementary mathematics teachers' views regarding critical thinking and doing mathematical proof. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 8(2), 87–99. https://doi.org/10.23917/jramathedu.v8i2.1402

ARTICLE HISTORY:
Received 22 December 2022
Revised 8 February 2023
Accepted 21 March 2023
Published 30 April 2023

KEYWORDS:
Critical thinking
Doing mathematical proof
Pre-service teachers

ABSTRACT

Like in proving, teachers have an important role in critical thinking because critical thinking is not an innately given skill, rather, it is acquired later in life. However, at this point, educators have a lot of responsibilities because if the individual can transfer critical thinking to his life at school age, he can live his life more meaningfully, be inquisitive and find solutions to problems after school. For this reason, the opinions of teachers and hence prospective teachers concerning both critical thinking and doing proof are important. In this study, survey and correlational research methods were used together since it was intended to explore the views of pre-service elementary mathematics teachers regarding critical thinking and doing mathematic proof and to understand whether these two correlates. The study was carried out on 158 pre-service elementary mathematics teachers enrolled in a state university in the Eastern Black Sea Region of Turkey. According to the results, the pre-service teachers in this study have a low level of critical thinking disposition. service teachers depending on the grade level at which they were studying. When the participating pre-service teachers' opinions on mathematical proof were examined, it was noticed that mental process was the highest while self-confidence was the lowest.

INTRODUCTION

Thinking is a fundamental feature that distinguishes humans from all other living things. Although it has different meanings, one of its meanings in the dictionary of the Turkish Language Association (TDK) is "to examine, compare and contrast units of information in order to reach a conclusion; and to produce thoughts, to form mental abilities, to do reasoning by means of the relations among them". As can be seen understood from the definition, thinking includes different skills. There are different thinking skills depending on these skills, and critical thinking is one of them.

Critical thinking is defined as making judgments and making decisions in line with a purpose by explaining evidence, concepts, methods, criteria and contexts as well as interpretation, analysis, evaluation and inferences (Özdemir, 2005). It can also be described as a process in which an individual makes logical inferences independently (Külahçı, 1995). A critical thinker organizes thoughts, uses proven information with expertise and authenticity; is successful in developing regular plans for the solution of complex problems and open-minded; can easily express their feelings and thoughts thanks to these abilities, postpones their interpretation of an issue in the absence of enough evidence to support a decision, makes observations on the subject in an objective and detailed manner, insists on collecting data and evidence, can make original decisions on any

*Corresponding author: aksanelif@gmail.com

subject without the need for the opinion of another individual, always questions the reliability of the sources from which he receives help, can initiate discussions about an event or issue and maintain these within a scientific framework, is willing to resolve an issue and learn more about it no matter how complex it is, has a ground for all their conclusions, and they can ask their questions about a subject clearly and unequivocally (Özdemir, 2005). All these show that critical thinking contains many skills in itself. These skills are about spotting the difference between proven facts and asserted claims, testing the reliability of the sources of information, discarding irrelevant information from the evidence, being aware of bias and cognitive errors, being aware of inconsistent judgments, doing proof, asking effective questions, effectively using the spoken and written language, and having metacognition, in which the individual becomes aware of his own thoughts (Kökdemir, 2003). As can be seen, one of the skills that critical thinking includes is the ability of proof.

Proof provides the correctness or falsity of every situation in mathematics (Tall & Mejia-Ramos, 2006). However, it shows not only whether a situation is right or wrong, but also justifies it (Hanna, 2000). Doing mathematical proof means communicating mathematically and recording mathematics (Schoenfeld, 1994). Creating proof is quite similar to mathematical problem solving in that the right idea comes to the mind of the individual at the right time (Selden & Selden, 2003). At the same time, proving is defined as a mental act used to eliminate the doubts of an individual or community about the accuracy of a claim (Harel, 2008; Harel & Sowder, 1998; Harel & Sowder, 2007)Therefore, proof is a very important tool in learning mathematics (Knuth, 2002). The development of proof, in turn, depends on individuals' gaining different ways of logical thinking (Almeida, 2003). In Turkey, the meaning and importance of proof in mathematics and mathematics education is rapidly increasing. When the curricula are examined, it can be seen that proof skill is now given a larger part compared to the past. For this reason, it can be said that the level of proof and the perceptions and opinions of mathematics teachers and prospective teachers matter as they are going to train students who can become mathematicians in the future (Moralı, Uğurel, Türnüklü, & Yeşildere, 2006). However, when the literature is examined, it is seen that the number of proof-making studies carried out with teachers and teacher candidates is limited (Flores, 2002; Jones, 2000; Özer & Arıkan, 2002; Sowder & Harel, 1998).

According to the previous research, Almeida (2003) argues that teachers' perceptions and experiences about proof affect their students' gaining proof skills. In order to be able to structure their lessons effectively, mathematics teachers need to know where their core concept derives from and what its underlying mathematical knowledge or principle is. This requires teachers to be equipped with mathematical proof.

Like in proving, teachers have an important role in critical thinking because critical thinking is not an innately given skill, rather, it is acquired later in life. When the studies on critical thinking in the literature are examined, it is seen that there are studies on determining the critical thinking disposition of teachers and teacher candidates (Arısoy, 2017; Arslan, 2020; Balcı, 2021; Bebek, 2022; Cingöz , 2019; Dede, 2021; Ekici, 2022; Farah & Ayoubi , 2020; Günay, 2022; Kahraman, 2022;Kertiyani & Sarjana, 2022; Nas, 2021; Önal, 2020; Öz, 2019; Sıburıan, Corebıma & Saptasarı, 2019; Tezcan, 2020; Tunçer, 2020).

Critical thinking is sown with the self-confidence that the individual gains in the family and then continues in every stage of life. However, at this point, educators have a lot of responsibilities because if the individual can transfer critical thinking to his life at school age, he can live his life more meaningfully, be inquisitive and find solutions to problems after school. These can happen only if the education system is suitable for this and educators have critical thinking skills and are able to transfer this to their students. By teaching critical thinking skills, our country's standard of living will be moved forward (Çıkrıkçı, 1996).

The teaching of this skill falls largely on the teachers. Therefore, it is important not only to determine the skill levels, but also to reveal the factors that will affect the skill and to determine in which direction these factors affect the skill. For this reason, it is important to determine the views of teachers, and therefore prospective teachers who will be teachers in the future, on both critical thinking and proving. In addition, it is very important to reveal the relationship between critical thinking dispositions and their views on making mathematical proofs. In line with this importance, the research problems are as follows:

1. What is the level of pre-service elementary mathematics teachers' critical thinking disposition?
2. How does pre-service elementary mathematics teachers' critical thinking disposition vary by grade level?
3. What is the level of pre-service elementary mathematics teachers' views on doing mathematical proof?
4. How do pre-service elementary mathematics teachers' views on doing mathematical proof vary by grade level?
5. Is there a significant relationship between pre-service elementary mathematics teachers' critical thinking dispositions and their views on doing mathematical proof?
6. Is critical thinking a significant predictor of pre-service elementary mathematics teachers' views on doing mathematical proof?

METHODS

In this study, quantitative research approaches were used. In this study, survey and correlational research methods were used together since it was intended to explore the views of pre-service elementary mathematics teachers regarding critical thinking and doing mathematic proof and to understand whether these two correlates. Survey is a descriptive research model conducted to portrait participants' skills, attitudes, abilities, interests or opinions about a phenomenon or subject. It is also a descriptive research method. Descriptive research is used to answer questions such as "what, where, how, when". On the other hand, the aim of correlational research is to establish the relationship between two or more variables, if any, without making intervention to the variables (Büyüköztürk, Kılıç Çakmak, Akgün, Karadeniz & Demirel, 2014). The present study attempts to describe pre-service elementary mathematics teachers' critical thinking skills and their views on doing proof through survey model in the first place. Secondly, it checks whether there is a relationship between these two variables by using correlational research method.

Study group

The study was carried out on 158 pre-service elementary mathematics teachers enrolled in a state university in the Eastern Black Sea Region of Turkey. The number of students participating from each grade level is shown in Table1. As can be seen in Table 1, the study involved undergraduate students attending all four grades of elementary mathematics teaching department. Of these participants, 34 were enrolled in the 1st grade, 43 in the 2nd grade, 38 in the 3rd grade, and 43 were enrolled in the 4th grade at the time of the study. Proofs are an important part of mathematics, and elementary mathematics teachers also lay the foundation for proofs and therefore for critical thinking. Pre-service elementary mathematics teachers with advanced critical thinking skills can support their students in this regard when they become teachers in the future. Therefore, in this study, pre-service elementary mathematics teachers were studied.

Data collection tools

Two different scales were used in this study. One of these is The California Critical Thinking Disposition Inventory (CCTDI) and the other one is The Questionnaire for Constructing Mathematical Proof (QCMP).

The california critical thinking disposition inventory (CCTDI)

This scale was designed by Facione at the end of the "Delphi Project" organized by the American Philosophical Society in 1990. The Turkish validity and reliability study of the scale was carried out by Kökdemir in 2003. The internal consistency coefficient of the scale was found to be 0.88, but it was calculated as 0.83 in the Turkish version. The California Critical Thinking Disposition Inventory is a six-point Likert-type scale consisting of 51 items. The items in the scale are rated as following: "Totally Disagree", "Disagree", "Partially Disagree", "Partially Agree", "Agree", "Totally Agree", and these statements are given 1, 2, 3, 4, 5, 6 points, respectively. The total score of the scale ranges between 51 and 306 points. As the total score increases, critical thinking disposition becomes

Table 1
Number of Participants for Grade Levels

Grade	n	(%)
1	34	21.5
2	43	27,2
3	38	24.1
4	43	27.2
Total	158	100

higher. Scale items numbered 5, 6, 9, 11, 15, 18, 19, 20, 21, 22, 23, 25, 27, 28, 33, 36, 41, 43, 45, 47, 49, 50 are scored in reverse. The inventory consists of six subscales as truth-seeking (items no 6, 11, 20, 25, 27, 28, 49), open-mindedness (items no 5, 07, 15, 18, 22, 33, 36, 41, 43, 45, 47, 50), analyticity (items no 2, 03, 12, 13, 16, 17, 24, 26, 37, 40), systematicity (items no 4, 09, 10, 19, 21, 23), self-confidence (items no 14, 29, 35, 39, 44, 48, 51), and inquisitiveness (items no 1, 08, 30, 31, 32, 34, 38, 42, 46) (Kökdemir, 2003). Facione, Facione and Giancarlo (1998) state that a score below 40 in any subscale indicates low critical thinking disposition while a score above 50 means high critical thinking disposition. In total, scores below 240 on the entire CCTDI refer to lower levels of critical thinking disposition, 240-300 refer to moderate levels, and scores above 300 refer to higher critical thinking disposition (Kökdemir, 2003).

The questionnaire for constructing mathematical proof (QCMP)

The measurement instrument was developed by Lee (1999) and adapted by İskenderoğlu (2010) to determine pre-service elementary mathematics teachers' proof-related opinions. The internal consistency coefficient of the scale was found to be 0.79. The questionnaire comprises 27 items based on 5-point Likert type. The items are rated with expressions of frequency such as "Always", "Often", "Occasionally", "Seldom" and "Never". Positive items responded as "Always" are given 5 points whereas 1 point is given to the response at the other end, i.e. "Never". In addition, items 1, 2, 5, 7, 8, 10, 13, 23, 25, 26 contain reverse statements and they are analyzed by reversing. There are four factors in the scale. The dimension regarding the individual's mental processes has 7 items (items no 3, 4, 16. 17, 18, 24, 25), the dimension of self-confidence in writing proof has 7 items (items no 1, 2, 6, 7,12, 20, 23), the dimension regarding self-assessment includes 5 items (items no 9, 15, 19, 21, 22), and the last dimension is about attitude-belief regarding proof and includes 8 items (items no 5, 8, 10, 11, 13, 14, 26, 27) (İskenderoğlu, 2010).

Data collection

Two different scales were used in the study. These scales were applied to pre-service elementary mathematics teachers at different times before they started the lesson. Thus, it was thought that pre-service elementary mathematics teachers would fill in the scales more carefully. Because it will take a longer time to fill in the two scales at the same time. Necessary explanations were given to the pre-service elementary mathematics teachers before filling out the scales.

Analysis of data

The data collected in the study were analyzed by using SPSS 21. The statistical methods conducted for analysis included frequency, arithmetic mean and standard deviation. As for the data collected with the CCTDI and QCMP, the total scores were assessed in relation with the variable of grade by using ANOVA test. The relation between the pre-service teachers' critical thinking skills and their views on doing mathematical proof was tested with Pearson Correlation test. Lastly, Simple Linear Regression analysis was applied to calculate the effect of pre-service teachers' critical thinking skills on their views on providing mathematical proof. Decision was made for the applicable tests (parametric, nonparametric) based on the skewness and kurtosis values of the data and the Levene test results (Büyüköztürk, 2010).

FINDINGS

In the following part, findings from the assessment of the pre-service teachers in respect to The California Critical Thinking Disposition Inventory (CCTDI) and the Questionnaire for Constructing Mathematical Proof (QCMP) are presented separately. It is followed by findings about the relationship between the two scales. Table 2 illustrating the pre-service teachers' scores from the CCTDI.

Table 2 illustrates the total score, standard deviation, minimum and maximum values, skewness, and kurtosis values in The California Critical Thinking Disposition Inventory (CCTDI). As can be seen above, the respondents' total score is 46,25 (sd=5,24) in the dimension of analyticity; 43,76 (sd=7,83) in open-mindedness; 39,54 (sd=6,16) in inquisitiveness; 27,70 (sd=5,12) in self-confidence; 22,20 (sd=4,90) in truth-seeking; and the total score in the dimension of systematicity is 22,46 (sd=3,86). The sum of the subscales equals to 201,92 with a standard deviation value of 22,42. Also, the results of the skewness and kurtosis calculations show whether the questionnaire data meet the normality assumption. As can be seen in Table 2, the skewness and kurtosis values are as 0.59 and 0.10, respectively. These figures reveal that the data collected with the CCTDI show a normal distribution. Table 3 displays total CCTDI scores obtained by pre-service teachers in each grade level.

Table 3 shows the CCTDI scores obtained in the subscales by each subgroup of participants attending different grades. For analyticity dimension, the 1st-graders' score was 46,18 (sd=4.99), the 2nd-graders was 46,60 (sd=6.07), the 3rd-graders' was 46,68 (sd=5,64), and the 4th-graders' score was 45,60 (sd=4,17). In the second dimension, open-mindedness, the 1st-graders got 45,26 points (sd=7,38), 2nd-graders got 44,74 (sd=7,36), 3rd-graders got 44,79 (sd=7,36), and the 4th-graders got 40,67 points (sd=9,05). In another dimension, inquisitiveness, the score of the 1st-graders was found to be 39,56 (sd=5,83), 2nd-graders to be 40,72 (sd=6,79), 3rd-graders to be 38,08 (sd=6,34), and the score of the 4th-graders was 39,63 (sd=5,51). Under the dimension of self-confidence, the 1st grade collected 26,62 (sd=6,10), 2nd grade collected 28,37 (sd=4,98), 3rd grade collected 26,84 (sd=5,29), and the 4th grade collected 28,63 points (sd=4,06). The figures for the dimension of truth-seeking were found as 22,47 (sd=4,74) for the 1st grade, 22,77 (sd=5,04) for the 2nd grade, 23,03 (sd=4,31) for the 3rd grade, and 20,70 (sd=5,19) for the final grade. The last dimension, systematicity, the score of the 1st-graders was 22,12 (sd=3,59), 2nd-graders was 22,65 (sd=3,93), 3rd-graders was 23,45 (sd=3,88), and the 4th-graders was 21,67 (sd=3,92). In the following part, Table 4 shows the significance analysis on the differences among the CCTDI scores obtained at grade levels.

A one-way ANOVA test was conducted to check whether the CCTDI scores of the pre-service teachers vary significantly from one grade to another (see Table 4). The results of the analysis showed no significant difference among CCTDI scores of different grade levels [$F(3, 154)=1.188$, $p>0.05$]. In other words, the pre-service teachers' CCTDI results do not change depending on their grade level (see Table 4). In this study, besides critical thinking dispositions, the participants' views on mathematical proofs were exposed. The pre-service teachers' scores in The Questionnaire for Constructing Mathematical Proof (QCMP) are given in Table 5.

Table 5 demonstrates the arithmetic mean, standard deviation, minimum and maximum values, skewness and kurtosis values related to the scores obtained from the Questionnaire for Constructing Mathematical Proof (QCMP). It was found that the arithmetic mean of the pre-service teachers' scores was 3,89 (sd=0,54) for mental process; the same value was 3,11 (sd=0,62) for self-confidence; 3,74 (sd=0,57) for self-assessment; and it was 3,62 (sd=0,46) for attitude-belief. The arithmetic mean of the overall QCMP scores was 3,58 with a standard deviation of 0,43. In addition, the skewness and kurtosis coefficients were calculated as to whether the total scores in the QCMP met the assumption of normality. As can be seen in Table 5, the skewness and kurtosis coefficients were found as -0,34 and 0,38, respectively. It means that the data obtained from this scale satisfied the assumption of normal distribution. Apart from these, the participants' scores in the QCMP were found to differ against the variable of grade level (see Table 6).

Table 2
Pre-service teachers' CCTDI scores

	No of Items	x̄	ss.	Min	Max	Skewness	Kurtosis
Analyticity	10	46,25	5,24	2,10	5,70	-1,01	2,72
Open-mindedness	12	43,76	7,83	1,33	5,33	-0,29	0,58
Inquisitiveness	9	39,54	6,16	2,89	6,00	0,22	-0,52
Self-confidence	7	27,70	5,12	1,86	6,00	-0,09	0,06
Truth-seeking	7	22,20	4,90	1,00	5,43	0,19	0,42
Systematicity	6	22,46	3,86	2,33	5,17	0,02	-0,57
CCTDI TOTAL	51	201,9	22,42	3,08	5,33	0,59	0,10

Table 3
Pre-service teachers' CCTDI scores for grade levels

CCTDI Subscale	Grade	N	x̄	ss
Analyticity	1	34	46.18	4.99
	2	43	46.60	6.07
	3	38	46.68	5.64
	4	43	45.60	4.17
Open-mindedness	1	34	45.26	7.38
	2	43	44.74	7.36
	3	38	44.79	6.47
	4	43	40.67	9.05
Inquisitiveness	1	34	39.56	5.83
	2	43	40.72	6.79
	3	38	38.08	6.34
	4	43	39.63	5.51
Self-confidence	1	34	26.62	6.10
	2	43	28.37	4.98
	3	38	26.84	5.29
	4	43	28.63	4.06
Truth-seeking	1	34	22.47	4.74
	2	43	22.77	5.04
	3	38	23.03	4.31
	4	43	20.70	5.19
Systematicity	1	34	22.12	3.59
	2	43	22.65	3.93
	3	38	23.45	3.88
	4	43	21.67	3.92
CCTDI TOTAL	1	34	202.21	22.33
	2	43	205.86	23.70
	3	38	202.87	22.31
	4	43	196.91	21.09

For the dimension of self-confidence, the arithmetic mean scores were recorded as 2,94 (sd=0,69) for the 1st grade; 3,08 (sd=0,66) for the 2nd grade, 3,16 (sd=0,50) for the 3rd grade, and 3,24 (sd=5,19) for the 4th grade. In the second dimension, self-assessment, the 1st-graders' mean score was 3,66 (sd=0,66), the 2nd-graders' was 3,73 (sd=0,52), the 3rd-graders' was 3,84 (sd=0,55),

Table 4
Distribution of pre-service teachers' CCTDI scores across grade levels

Scale	Grade	n	x̄	sd	sd	F	p	Significant difference (grade)
CCTDI	1	34	3.96	0.44	3-154	1.188	0.32	-
	2	43	4.04	0.46				
	3	38	3.97	0.44				
	4	43	3.86	0.41				

Table 5
Pre-service teachers' QCMP scores

	No of items	x̄	sd.	Min	Max	Skewnes	Kurtosis
Mental process	7	3.89	0.54	1.86	5.00	-0.55	1.30
Self-confidence	7	3.11	0.62	1.43	5.00	-0.10	0.36
Self-assessment	5	3.74	0.57	1.60	5.00	-0.47	0.69
Attitude-belief	8	3.62	0.46	2.25	4.38	-0.43	-0.32
QCMP TOTAL	27	3.58	0.43	2.22	4.59	-0.34	0.38

and the 4th-graders' was 3,72 (sd=0,57). From attitude-belief, the 1st grade obtained the arithmetic means of 3,58 (sd=0,41), the 2nd grade obtained 3,59 (sd=0,43), the 3rd grade obtained 3,72 (ss=0,49), and the final grade obtained 3,58 (sd=0,51). In the last dimension, mental process, the values were found as 3,58 (sd=0,51) for the 1st grade; 3,89 (sd=0,66) for the 2nd grade; 4,00 (sd=0,49) for the 3rd and 3,75 (sd=0,45) for the 4th grade. Below is Table 7, which reveals the results of significance analysis conducted on the participants' QCMP scores at varying grade levels.

A one-way ANOVA test was conducted to see whether the pre-service teachers' QCMP differed significantly according to the variable of grade (see Table 7). The results showed that there was no significant difference across grade levels in this regard [$F(3, 154)=0.848$, $p>0.05$]. In other words, the students' QCMP do not vary depending on the grade at which they are (see Table 7). Table 8 below shows the results regarding the existence or lack of a relationship between the respondents' critical thinking disposition and their views on doing mathematical proof.

Pearson Correlation test was conducted to test the relationship between critical thinking and the view towards mathematical proofs (see Table 8). It was found that there was a moderate and significant positive correlation between CCTDI and QCMP [$r=0.58$, $p=0.00$]. Table 9 below displays results of the analysis conducted to see whether critical thinking is a significant predictor of views regarding doing mathematical proof.

The analysis results showed that the critical thinking dispositions of the pre-service teachers were a significant predictor of their views on doing mathematical proofs ($R=0.582$, $R2=0.34$, $F(1, 156)=79.901$, $p<0.01$) (see Table 9). It can be suggested that the pre-service teachers' views on doing mathematical proof can be accounted for by their critical thinking skills at a rate of 34%.

Table 6
Pre-service teachers' QCMP arithmetic means for grade levels

QCMP subscale	Grade	N	x̄	Sd
Self-confidence	1	34	2.94	0.69
	2	43	3.08	0.66
	3	38	3.16	0.50
	4	43	3.24	0.59
Self-assessment	1	34	3.66	0.66
	2	43	3.73	0.52
	3	38	3.84	0.55
	4	43	3.72	0.57
Attitude-belief	1	34	3.58	0.41
	2	43	3.59	0.43
	3	38	3.72	0.49
	4	43	3.58	0.51
Mental process	1	34	3.89	0.66
	2	43	3.94	0.54
	3	38	4.00	0.49
	4	43	3.75	0.45
QCMP TOTAL	1	34	3.51	0.49
	2	43	3.57	0.43
	3	38	3.67	0.41
	4	43	3.56	0.41

Table 7
Distribution of pre-service teachers' QCMP scores for grade levels

Scale	Grade	n	x̄	sd	sd	F	p	Significant difference (grade)
QCMP	1	34	3.51	0.49	3-154	0.848	0.47	-
	2	43	3.57	0.43				
	3	38	3.67	0.41				
	4	43	3.56	0.41				

Table 8
CCTDI and QCMP correlation analysis results

	QCMP		
	n	r	p
CCTDI	158	0.58	0.00**

**p< 0.01

Table 9
CCTDI and QCMP simple regression analysis results

Variable	B	Standard Error B	β	t	p
MKYYGÖ	1.300	0.257		5.067	0,000
CCTDI	0.576	0.064	0.582	8.939	0,000

R= 0.582 R^2= 0.34
$F_{(1, 156)}$= 79.901 p= 0.00

DISCUSSION AND CONCLUSIONS

This study was carried out to examine pre-service elementary mathematics teachers' critical thinking levels and their views on doing mathematical proofs, as well as to find out if there is a significant relationship between their critical thinking levels and views on doing mathematical proof against the variable of grade level. Moreover, it exposed the relationship between the pre-service teachers' levels of critical thinking and their views on doing mathematical proof.

According to the results, the pre-service teachers in this study have a low level of critical thinking disposition. One possible reason could be the students' lacking real-life and educational experience leading to the development of this skill. Recalling that critical thinking is an acquired skill, not innate, it develops beginning from the family before school and continues through schooling. In this study, the pre-service teachers exhibited a moderate level of disposition to think critically only in two dimensions, that is analyticity and open-mindedness (see Table 2). Although the critical thinking levels of the pre-service elementary mathematics teachers were low in this study, the same skill was found to be at a moderate level by Altuntaş, Yılmaz, & Turan (2017). Similarly, Korkmaz (2009) found out that the students at the faculty of education had an intermediate disposition towards critical thinking, like in the study of Özdemir (2005) noting critical thinking disposition of university students at moderate level. However, Gülveren (2007) found similar results to the present study in that the pre-service teachers' critical thinking dispositions were low. Açışlı (2015), Aliustaoglu and Tuna (2015), and Can and Kaymakçı (2015) studies support the result of this research.

Considering the pre-service teachers' critical thinking dispositions depending on their grade, the lowest score was recorded by the 4th-graders. Apart from this, while the critical thinking disposition level increased from the 1st to the 2nd grade, the level decreased after the 2nd grade (see Table 3). Among other possible reasons, it might be accounted for with the fact that pre-service teachers predominantly take major area courses starting from the 3rd grade. Additionally, as they approach the final year of undergraduate education, pre-service teachers might come under the influence of the upcoming KPSS (Public Personnel Selection Examination) for employment and their anxiety about starting to teach professionally. Such concerns might factor into weakening critical thinking skills among pre-service teachers. Another result of the study revealed that there was no significant difference in the critical thinking levels of the pre-service teachers at different grade levels (see Table 4). In the same vein; Beşoluk and Önder (2010), Ekinci (2009), Küçük and Uzun (2013) concluded that was no significant difference between the critical thinking dispositions of pre-service teachers depending on the grade level at which they were studying. In the study conducted by Karalı (2012), it was emphasized that the students in the upper class had higher levels of critical thinking disposition than those in the lower classes. In the study conducted by Can and Kaymakçı (2015), it is determined that as the grade level increases, the critical thinking dispositions of pre-service teachers improve

When the participating pre-service teachers' opinions on mathematical proof were examined, it was noticed that mental process was the highest while self-confidence was the lowest (see Table 5). Mental process dimension covers the individual's ideas about individual thinking. This dimension relates to the thinking skills, sources of information and motivations that students develop while writing proof. The other dimension, self-confidence, means the individual's confidence in providing proof through his own point of view, and his belief in himself. It implies that pre-service teachers are able to design proof but they are deficient in putting this design into practice. Similar results were reached in Furinghetti and Morselli's (2009), study which suggested that the views on proof affect students' ability to construct proof.

As regards the average of the pre-service teachers' views about doing mathematical proof in connection with grade level, an increase was observed from the 1st through the 3rd grade but the curve turned to the opposite direction during transition to the final year. This could be explained with the students' anxiety about the KPSS to take upon graduation or exam-oriented studying style. When examined in relation with specific dimensions, an increase was seen in the average scores in self-confidence and self-assessment from the 1st through the 4th grade. Likewise, while the dimensions of attitude-belief and mental process increased from the 1st grade until the 3rd grade,

the average scores tended to decrease at transition from the 3rd to the 4th grade. As a conclusion, there was no significant difference in the pre-service teachers' views about providing mathematical proof against grade level variable (see Table 6). In parallel with these results, the study by Moralı, Uğurel, Türnüklü, and Yeşildere (2006) analyzed pre-service mathematics teachers' proof-related opinions and found no significant difference between different class levels. Likewise, in the study of Doruk & Güler (2014), in which prospective teachers' views on proof were examined according to their grade levels, it was determined that pre-service teachers studying in the third grade had more negative views than the candidates in the first and second grades.

Finally, the relationship between critical thinking and views on doing mathematical proof yielded a significant, positive relationship between CCTDI and QCMP at a moderate level. What is more, it is understood that the critical thinking dispositions of the pre-service teachers are a significant predictor of their views on doing mathematical proof. It was seen that pre-service teachers' critical thinking skills were influential at a rate of 34% on their views about doing mathematical proof (see Tables 7-8). To put differently, pre-service elementary mathematics teachers with higher critical thinking tendency held stronger views on constructing mathematical proof. Similarly, the activities organized in the "critical thinking teaching model" proposed by Ivie (2001) encompass a number of processes such as conscious research, active thinking, and evidence-seeking. It is obvious that mathematical proof and critical thinking are interrelated phenomena. Similarly, in Aksu's (2012) study examining the relationship between students' critical thinking dispositions and logical thinking abilities, it was found that there was a positive and significant relationship between critical thinking and logical thinking abilities

Since critical thinking skills contain multiple skills, it is important to develop these skills. To this end, activities should be implemented both at elementary and secondary and tertiary education level. Teachers have an important role for realizing such activities in the classroom environment. For this reason, critical thinking skill can be integrated into some courses or new courses can be offered so that prospective teachers can both develop their critical thinking skills and see good, concrete examples of developing those skills. Thus, proof, which has a place in critical thinking skills, can receive the necessary emphasis as part of these courses. Besides these, future research can address pre-service teachers enrolled at different grade levels for assessing their critical thinking skills, as well as their opinions on mathematical proofs. In this way, it could be understood why this target population possess low critical thinking skills and weak views about mathematical proof.

AUTHOR'S DECLARATION

Authors' contributions	All authors contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.
Funding Statement	The author(s) received no financial support for the research.
Availability of data and materials	All data are available from the authors.
Competing interests	The authors declare that the publishing of this paper does not involve any conflicts of interest. This work has never been published or offered for publication elsewhere, and it is completely original.

BIBLIOGRAPHY

Açışlı, S. (2015). Öğretmen adaylarının öğrenme stilleri ve eleştirel düşünme eğilimlerinin incelenmesi. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi, 9* (1), 23-48. https://doi.org/10.17522/nefefmed.57817

Aksu, G. (2012). *Meslek yüksekokulu öğrencilerinin matematik dersi başarıları ile derse ilişkin tutumları, eleştirel düşünme eğilimleri ve mantıksal düşünme yetenekleri arasındaki ilişkilerin incelenmesi* (Yayınlanmamış yüksek lisans tezi). Adnan Menderes Üniversitesi, Sosyal Bilimler Enstitüsü. http://hdl.handle.net/11607/1308

Aliustaoğlu, F. ve Tuna, A. (2015). Öğretmen adaylarının eleştirel düşünme eğilimlerinin incelenmesi (Kastamonu üniversitesi örneği). *Uluslararası Eğitim Bilimleri Dergisi, 2*(4) , 131-142. https://doi.org/10.16991/INESJOURNAL.56

Almeida, D. (2003). Engendering Proof Attitudes: Can the Genesis of Mathematical Knowledge. *International Journal of Mathematics Education in Science and Technology. 34*(4). 479-488. https://doi.org/10.1080/0020739031000108574

Altuntaş, E. Ç., Yilmaz, M., & Turan, S. L. (2017). Biyoloji öğretmen adaylarının sosyobilimsel bir konudaki eleştirel düşünmelerinin empati açısından incelenmesi. *Bartin Üniversitesi Egitim Fakültesi Dergisi*, *6*(3), 915-931. https://doi.org/10.14686/buefad.311276

Arısoy, B. (2017). *Konu temelli eleştirel düşünme öğretiminin matematik dersinde öğrencilerin eleştirel düşünme becerileri, eleştirel düşünme erdemleri ve matematik dersine ilişkin tutumlarına etkisi* (Yayınlanmamış doktora tezi). Çukurova Üniversitesi, Adana https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=0tTu0ZipVwvaQHb6nzgiEw&no=gXiLbWscMpctRvpwgCJXcA

Arslan, A. (2020). *Okul öncesi öğretmenlerinin öğretme ve öğrenmeye yönelik pedagojik inançlarının öz- yeterlik inançları ve eleştirel düşünme becerileri açısındanincelenmesi: Sakarya ili örneği* (Yayınlanmamış yüksek lisans tezi). İstanbul Aydın Üniversitesi, İstanbul. https://doi.org/10.24315/tred.742927

Balcı, H. (2021). *Robotik eğitiminin öğretmenlerin eleştirel düşünme ve eleştirel düşünmeyi destekleme eğilimlerine etkisi ve öğretmen görüşleri* (Yayınlanmamış yüksek lisans tezi). Amasya Üniversitesi

Bebek, M. (2022). *Bilişim teknolojileri öğretmenlerinin programlamaya ilişkin öz-yeterlilikleri ile eleştirel düşünme eğilimleri arasındaki ilişkinin incelenmesi* (Yayınlanmamış yüksek lisans tezi). Mersin Üniversitesi.https://acikerisim.mersin.edu.tr/?p=17&dil=0&q=&fq[]=F009_strs:%22%C3%B6z+yeterlilik%22&recordid=165429

Beşoluk, Ş., & Önder, İ. (2010). Öğretmen adaylarının öğrenme yaklaşımları, öğrenme stilleri ve eleştirel düşünme eğilimlerinin incelenmesi. *İlköğretim Online*, *9*(2), 679-693. https://dergipark.org.tr/tr/pub/ilkonline/issue/8595/106929

Büyüköztürk, Ş. (2010). *Sosyal bilimler için veri analizi el kitabı. Ankara: Pegem Akademi*. https://dergipark.org.tr/tr/pub/ilkonline/issue/8602/107147

Büyüköztürk, S., Kılıç Çakmak, E., Akgün, O. E., Karadeniz, S. & Demirel, F. (2014). *BİLİMSEL ARAŞTIRMA YÖNTEMLERİ.* Ankara: Pegem Akademi.

Can, Ş. & Kaymakçı, G. (2015). Öğretmen adaylarının eleştirel düşünme eğilimleri. Education Sciences, 10 (2) , 66-83. https://doi.org/10.12739/NWSA.2015.10.2.1C0633

Cingöz, O. (2019). *Din kültürü ve ahlak bilgisi öğretmenlerinin eleştirel düşünme eğitimine ilişkin görüşleri* (Yayınlanmamış doktora tezi). Erciyes Üniversitesi, Kayseri https://doi.org/10.28949/bilimname.553642

Çıkrıkçı-Demirtaşlı N. (1996). *Eleştirel düşünme: Bir ölçme aracı ve bir araştırma*. 3. Ulusal Psikolojik Danışma ve Rehberlik Kongresi, Adana. https://dergipark.org.tr/tr/download/article-file/50181

Dede, I. B. (2021). *Okul öncesi öğretmenlerinin eleştirel düşünme eğilimlerinin belirlenmesi üzerine karma tür bir araştırma*: İstanbul ili Küçükçekmece ilçesi örneği (Yayınlanmamış yüksek lisans tezi) Kafkas Üniversitesi, Kars

Doruk, M., & Güler, G. (2014). İlköğretim matematik öğretmeni adaylarının matematiksel ispata yönelik görüşleri. *Uluslararası Türk Eğitim Bilimleri Dergisi*, *3*, (71-93). https://dergipark.org.tr/tr/pub/goputeb/issue/33496/380151

Ekinci, Ö. (2009). *Öğretmen Adaylarının empatik ve eleştirel düşünme eğilimlerinin incelenmesi*. (Yayımlanmamış yüksek lisans tezi). Çukurova Üniversitesi, Sosyal Bilimler Enstitüsü: Adana. https://dergipark.org.tr/tr/pub/ilkonline/issue/8595/106940

Facione, P. A, Facione, N. C., & Giancarlo, C. A. (1998). *The california critical thinking disposition inventory test manual (revised)*. Millbrae, CA: California Academic Press. http://dx.doi.org/10.22329/il.v20i1.2254

Farah, N. & Ayoubi, Z. (2020). Enhancing the critical thinking skills of grade 8 chemistry students using an ınquiry and reflection teaching method . *Journal of Education in Science Environment and Health , 6* (3) , 207-219 . https://doi.org/10.21891/jeseh.656872

Flores, A., 2002. How Do Children Know That What They Learn in Mathematics is True?, Teaching Children Mathematics, 8, 5, 269–274. https://doi.org/10.5951/TCM.8.5.0269

Furinghetti, F. & Morselli, F. (2009). Every unsuccessful problem solver is unsuccessful in his or her own way: affective and cognitive factors in proving. *Educational Studies in Mathematics, 70*(1), 71-90. http://dx.doi.org/10.1007/s10649-008-9134-4

Gülveren, H. (2007). *Eğitim fakültesi öğrencilerinin eleştirel düşünme becerileri ve bu becerileri etkileyen eleştirel düşünme faktörleri* (Yayınlanmamış Doktora Tezi). Dokuz Eylül Üniversitesi, İzmir.

https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=dexIFnoJealQcUFSvCBWVw&no=vN8YZbjHwSr3zQh2qwopwQ

Günay, Ü. (2022). *Duygusal zekanın eleştirel düşünmeye etkisi: turizm öğretmenlerine yönelik bir araştırma.* (Yayınlanmamış yüksek lisans tezi). Van Yüzüncü yıl Üniversitesi. https://doi.org/10.29228/jatos.73671

Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational studies in mathematics*, *44*(1), 5-23. https://doi.org/10.1023/A:1012737223465

Harel, G. (2008). DNR perspective on mathematics curriculum and instruction, Part I: focus on proving. *ZDM Mathematics Education, 40,* 487–500. https://doi.org/10.1007/s11858-008-0104-1

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *American Mathematical Society, 7*, 234-283. https://doi.org/10.1090/cbmath/007/07

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. *Second handbook of research on mathematics teaching and learning, 2,* 805-842. https://mathweb.ucsd.edu/~harel/TowardComprehensivePerspective.pdf

Ivie, S. D. (2001). Metaphor: A model for teaching critical thinking. *Contemporary Education*, *72*(1), 18. https://www.semanticscholar.org/paper/METAPHOR%3A-A-MODEL-FOR-TEACHING-CRITICAL-THINKING-Ivie/8c24ea24ab850ef078f685eb00d5cbf363f28d8c

İskenderoğlu, T. (2010). İlköğretim matematik öğretmeni adaylarının kanıtlamayla ilgili görüşleri ve kullandıkları kanıt şemaları. Yayınlanmamış doktora tezi. Karadeniz Teknik Üniversitesi, Trabzon, Türkiye.https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=sC6NvdlFF8K68ki1E3ePcw&no=DAbunvcQkrZ17IfsvqPlBg

Jones, K., 2000. The Student Experience of Mathematical Proof at University Level,International *Journal of Mathematical Education in Science and Technology, 31*, 1,53–60. https://doi.org/10.1080/002073900287381

Kahraman, Z. (2022). *Fen bilimleri öğretmenlerinin eleştirel düşünme ve öğretimi hakkındaki görüşlerinin belirlenmesi* (Yayınlanmamış yüksek lisans tezi). Ege Üniversitesi, İzmirs. https://acikbilim.yok.gov.tr/handle/20.500.12812/737218

Karalı, Y. (2012). *Eğitim fakültesi öğrencilerinin eleştirel düşünme eğilimleri (İnönü üniversitesi örneği)* (Yayınlanmamış yüksek lisans tezi). İnönü Üniversitesi. Malatya. http://abakus.inonu.edu.tr/xmlui/handle/11616/1168

Kertiyani, N. M. I., & Sarjana, K. (2022). The critical thinking skill of mathematics education students during pandemic: A Review. *Jurnal Pijar Mipa, 17*(2), 246-251. https://doi.org/10.29303/jpm.v17i2.3425

Knuth, E. J. (2002). Secondary school mathematics teachers' conceptions of proof. *Journal for research in mathematics education, 33(*5), 379-405. https://doi.org/10.2307/4149959

Korkmaz, Ö. (2009). Öğretmenlerin eleştirel düşünme eğilim ve düzeyleri. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi, 10*(1), 1-13. https://dergipark.org.tr/tr/pub/kefad/issue/59520/855964

Kökdemir D (2003). *Belirsizlik Durumlarında Karar Verme ve Problem Çözme* (Yayınlanmamış doktora tezi*).* Ankara Üniversitesi Sosyal Bilimler Enstitüsü Sosyal Psikoloji Anabilim Dalı. https://dspace.ankara.edu.tr/xmlui/handle/20.500.12575/28264

Küçük, D. P., & Uzun, Y. B. (2013). *Müzik Öğretmeni Adaylarının Eleştirel Düşünme Eğilimleri* [*Critical Thinking Tendencies of Music Teacher Candidates*]. Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi, 14(1), 327-345.

Külahçı, A. E. (1995). *Total Quality Management in Higher Education Case Study in Marmara University Engineering Faculty* (Unpublished doctoral dissertation). Marmara Universitesi, Turkey. https://www.proquest.com/openview/9b77e3dbc2d44cf5b3c817657f387c79/1?pq-origsite=gscholar&cbl=2026366&diss=y

Lee, W. (1999). *The relationship between students' proof-writing ability and Van hiele levels of geometric thought in a college geometry course*. Unpublished doctoral dissertation. https://www.proquest.com/openview/7da7446ab2f2be19a324d1ef8f53026a/1?pq-origsite=gscholar&cbl=18750&diss=y

Moralı, S., UğureL, İ., Türnüklü, E., & Yeşildere, S. (2006). Matematik öğretmen adaylarının ispat yapmaya yönelik görüşleri. *Kastamonu Eğitim Dergisi*, *14*(1), 147-160. https://www.acarindex.com/kastamonu-egitim-dergisi/matematik-ogretmen-adaylarinin-ispat-yapmaya-yonelik-gorusleri-922954

Önal, İ. (2020). *Eleştirel düşünme becerilerine yönelik bir program geliştirme çalışması.* (Yayınlanmamış doktora tezi). Yıldız Teknik Üniversitesi, İstanbul. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=eDufnSBxIC_gh-tIC2l-yQ&no=MAQ6aijWRQi_uASb2uY8sQ

Öz, C. (2019). *Okul yöneticilerinin ve öğretmenlerin eleştirel düşünme becerilerine verdiği önemin kişisel ve mesleki açıdan incelenmesi* (Yayınlanmamış yüksek lisans tezi). İstanbul Aydın Üniversitesi , İstanbul.

https://www.academia.edu/42249635/Okul_Y%C3%B6neticilerinin_ve_%C3%96%C4%9Fretmenlerin_Ele%C5%9Ftirel_D%C3%BC%C5%9F%C3%BCnme_Becerilerine_Verdi%C4%9Fi_%C3%96nemin_Ki%C5%9Fisel_ve_Mesleki_A%C3%A7%C4%B1dan_%C4%B0ncelenmesi

Özdemir, S. M. (2005). Üniversite öğrencilerinin eleştirel düşünme becerilerinin çeşitli değişkenler açısından değerlendirilmesi. *Türk Eğitim Bilimleri Dergisi*, 3(3), 297-316. https://dergipark.org.tr/tr/pub/tebd/issue/26123/275185

Özer, Ö. & Arıkan, A., (2002). *Lise Matematik Derslerinde Öğrencilerin İspat Yapabilme Düzeyleri*, V. Fen Bilimleri ve Matematik Eğitimi Kongresi, Eylül, c: 1083-1089.

Schoenfeld, A. (1994). *Reflections on doing and teaching mathematics.* In A. Schoenfeld (Ed.), Mathematical Thinking and Problem Solving (pp. 53-70). Lawrence Erlbaum Associates. https://www.researchgate.net/publication/247374625_Reflection_on_Doing_and_Teaching_Mathematics

Selden, A., & Selden, J. (2003). Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem?. *Journal for research in mathematics education*, *34*(1), 4-36. https://doi.org/10.2307/30034698

Sıburıan, J., Corebıma, A. D., ., İ. & Saptasarı, M. (2019). The Correlation Between Critical and Creative Thinking Skills on Cognitive Learning Results. *Eurasian Journal of Educational Research, 19 (81),* 99-114. https://doi.org/10.14689/ejer.2019.81.6

Sowder, L. & Harel, G., 1998. Types of Students' Justifications, *The Mathematics Teacher*, 91, 8, 670–675. https://doi.org/10.5951/MT.91.8.0670

Tall, D., & Mejia-Ramos, J. P. (2006). *The long-term cognitive development of different types of reasoning and proof*. In *Conference on Explanation and Proof in Mathematics: Philosophical and Educational Perspectives* (pp. 1-11). https://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2006g-mejia-tall.pdf

Tezcan, H. (2020). *Din öğretiminde eleştirel düşünme: DKAB dersinde öğrencilerin eleştirel düşünme becerilerini geliştirmeye yönelik bir eylem araştırması* (Yayınlanmamış yüksek lisans tezi). İstanbul Üniversitesi. http://nek.istanbul.edu.tr:4444/ekos/TEZ/ET002795.pdf

Tunçer, E. (2020). *Ortaokul matematik öğretmenlerinin eleştirel düşünme eğilim ve uygulama algıları arasındaki ilişki*. (Yayınlanmamış yüksek lisans tezi). Düzce Üniversitesi. https://doi.org/10.47215/aji.1000040