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Students' instrumental understanding in solving spatial mathematical problems across levels of mathematical ability

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ABSTRACT
This study explores students' instrumental understanding of spatial geometry problems, accounting for differences in initial mathematical ability. Many students tend to solve geometry problems procedurally without sufficient conceptual understanding, while research that examines explicitly instrumental understanding in spatial contexts remains limited. A qualitative case study design was employed involving 25 eighth-grade students from a public junior high school in Pagar Alam City, South Sumatra. Data were collected through written tests and in-depth interviews with six students representing high, medium, and low levels of mathematical ability. The analysis was guided by four indicators of instrumental understanding: recalling concepts, identifying concepts, selecting appropriate solution strategies, and representing concepts visually and in written form. The findings indicate apparent differences across ability levels. Students with high initial mathematical ability consistently fulfilled all four indicators across various spatial geometry problems. In contrast, students with medium and low ability demonstrated partial fulfilment of the indicators, particularly in topics such as cylinder volume and triangular prisms. These results suggest that students' initial mathematical ability plays a crucial role in the development of instrumental understanding. Therefore, differentiated instructional strategies aligned with students' ability levels are recommended to support balanced procedural and conceptual learning in spatial geometry.

INTRODUCTION

Mathematics, as a discipline, is inseparable from visual analysis through graphs, diagrams, and coordinate systems, especially in geometry, a spatial discipline that relies on visual representation to solve problems (Masduki et al., 2023). Spatial geometry is one of the essential materials in mathematics taught in junior high schools (Adams et al., 2023). Battista (2007) explained that the concept of spatial geometry involves not only understanding geometric properties but also applying formulas to solve contextual problems (Bakker & van Eerde, 2015). Geometric problems in mathematics often involve non-routine challenges where solution strategies are not immediately apparent, requiring complex problem-solving skills (García-Moya et al., 2024). To optimize problem-solving in spatial geometry, it is necessary to integrate strong instrumental understanding (Sorby et al., 2022).

Instrumental understanding refers to students' ability to use formulas or algorithms correctly without a deep understanding of the underlying concepts (Herheim, 2023). There are four indicators of instrumental understanding applied to analyze students' mathematical understanding based on Skemp's theory (Skemp, 2020), including students' ability to recall concepts learned, ability to

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identify concepts, ability to choose the right concept or strategy to solve problems, and ability to represent concepts in the form of images or writing.

Although this understanding is considered limited, in specific contexts, such as solving routine problems, instrumental understanding can help students reach answers quickly (Buteau et al., 2020). However, the weakness is that students tend to have difficulty with non-routine problems or variations that require relational understanding (Kholid, 2022). In the context of geometric shapes, instrumental understanding is evident when students can apply volume or surface-area formulas correctly—even though they do not fully understand the mathematical reasons for these formulas. As explained by Skemp (1976), instrumental understanding focuses on mastering procedures to achieve results without in-depth conceptual understanding. In line with this, research by Kirkland and McNeil (2021) shows that students who rely on an instrumental understanding are usually quite confident when facing standard problems, but they experience confusion when the problem is modified or placed in a new context. Therefore, students need to develop an instrumental understanding of spatial materials when learning geometry.

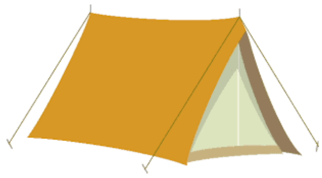
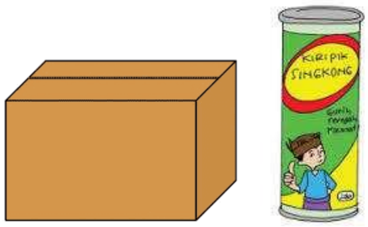
However, in reality, research by Herheim (2023) shows that students in Indonesia tend to have low instrumental understanding skills. Students have difficulty with figural representation and reasoning in geometric spatial concepts. A similar finding was reported by Angraini et al. (2023), who found that junior high school students in Indonesia have low instrumental understanding skills. A conventional, teacher-centered approach to teaching limits students' space to explore, make conjectures, and build their own understanding. Furthermore, according to Amalina and Vidákovich (2023), instrumental understanding abilities are influenced by an individual's initial mathematical ability.

Initial math ability refers to the knowledge, skills, and cognitive readiness that students already have before learning a new concept (Kholid et al., 2021). These early abilities influence how students process information, choose strategies, and connect new ideas to existing knowledge structures (Basir et al., 2022). Students with higher initial math skills tend to have an easier time recognizing relationships, interpreting representations, and executing procedures appropriately, while students with low initial skills often have difficulty constructing meaning from symbolic and figural information (Kahl et al., 2022). Given that instrumental understanding is related to the ability to carry out procedures appropriately and meaningfully, the development of instrumental understanding is greatly influenced by students' initial mathematical abilities (Angraini et al., 2023). Therefore, it is important to explore how instrumental understanding develops in students with different levels of initial math ability, so that learning can be designed more appropriately and responsive to their needs.

Research on the exploration of instrumental understanding in mathematical solving is still limited. Research by Nori et al. (2023) found that students with field-independent (FI) cognitive styles have relational understanding when planning and carrying out problem-solving, but show instrumental understanding—focusing on formulas—when checking their results. Likewise, research conducted by Amir et al (2022) identified strategies and difficulties in solving negative integer problems among fifth-grade students with instrumental understanding. However, no research has specifically examined the role of instrumental understanding in solving spatial geometry problems, especially in relation to students' initial mathematical understanding. Therefore, this study focuses on the instrumental understanding ability of junior high school students with different levels of mathematical ability.

Unlike previous studies that examined instrumental understanding in arithmetic or algebraic contexts, this study provides a qualitative exploration of students' instrumental understanding in spatial geometry by explicitly linking it to initial mathematical ability levels. This study offers a nuanced description of how each indicator of instrumental understanding manifests differently across ability levels, thereby extending Skemp's theory into the context of three-dimensional geometry learning. Based on the above explanation, this study aims to explore students' instrumental understanding abilities in solving spatial geometry problems, with a focus on their initial mathematical understanding. By analyzing the interaction between these two aspects, it is hoped that more effective learning strategies can be obtained to improve students' understanding of spatial geometry material.

Table 1
Questions on instrumental understanding of space structures for grade VIII students

No	Question
1.	 A cube-shaped gift box has a volume of 216 cm^3. Without using a calculator, determine the length of the box's ribs by showing the calculation process. Please draw the cube's shape and mark the lengths of its ribs.
2.	 A camping tent is a triangular prism with a base that is a right triangle (3 cm, 4 cm, 5 cm) and a height of 2 meters. Calculate: a) The surface area of the tent b) Volume of space inside the tent Also, determine how much it costs to buy a tarp to cover the entire surface of the tent, assuming it costs Rp15,000 per square meter!
3.	 A company wants to produce two types of packaging for cassava chips: Packaging A: Blocks with a size of 10 cm × 8 cm × 6 cm Packaging B: Tubes with a diameter of 8 cm and a height of 10 cm Analyze: a. Which packaging has a larger volume? b. Calculate the difference in the area of materials needed to make the two packages! c. If the production cost is Rp500 per cm^2 of material, which packaging is more economical? d. Sketch the two appropriate packages!

METHODS

Research design

This research employs a qualitative, single-case study design to explore students' instrumental understanding in solving geometry problems. The case is bounded within the specific context of a public junior high school in Pagar Alam, with its distinctiveness lying in the participation of students categorized by their initial mathematical ability (high, medium, and low). This design was selected to facilitate an in-depth, holistic investigation of how this complex understanding manifests differently across ability levels in a real-world setting.

Participants

This study involved 25 grade VIII students at one of the State Junior High Schools in Pagar Alam City, South Sumatra. The researcher chose grade VIII subjects because the material used to measure instrumental understanding ability was the spatial numbers taught in grade VIII. Furthermore, after the study was conducted, 6 students were selected, consisting of 2 high-ability, 2 medium-ability, and

2 low-ability students, to be interviewed. The selection of six interview participants was based on maximum variation sampling to capture representative patterns across ability levels. Data saturation was reached when no new indicators of instrumental understanding emerged from additional interviews.

Instrument

The research instruments used in this study include written tests and interviews. In this study, the researcher created 3 test questions on spatial geometry material. Then, the questions were given to 25 junior high school students in grade VIII. The test questions used in this study are included in Table 1. Furthermore, the test instrument was validated by two experts in mathematics education and analyzed using Aiken's CVI. The CVI value obtained was 0.8889, indicating high validity (Aiken, 1980).

Data analysis techniques

Data analysis was conducted using the flow model method. This process encompassed data reduction, data display, and conclusion drawing. The data obtained from students' responses to the instrumental understanding ability test questions were then analyzed using the rubric presented in Table 2.

At this data reduction stage, the researcher obtained the score of each student's instrumental understanding ability on each indicator, namely the indicator score of the student's ability to recall the concepts learned, the ability to identify concepts, the ability to choose the right concept or strategy to solve problems, and the ability to represent concepts in the form of images or writing. Based on the results of the instrumental understanding assessment of three-dimensional geometry, students are grouped into three categories: high, medium, and low in mathematical problem-solving ability (see Table 3). The achievement of mathematical problem-solving ability is measured by students' test scores, with a minimum score of 70.

Based on test results for 25 students, the data are presented in Table 4. Furthermore, to better understand students' instrumental understanding ability, the researcher conducted in-depth interviews to examine students' thought processes in solving problems associated with instrumental understanding indicators. In the next stage, the researcher made conclusions about students' instrumental understanding ability through analyzing answers and interviews.

FINDINGS

This study will present an analysis of instrumental ability for students with high ability (coded K-T), medium ability (coded K-S), and low ability (coded K-R). Indicators of instrumental ability consist of four stages, namely the ability of students to recall the concepts learnt, coded K1, the ability to identify concepts coded K2, the ability to choose the right concept or strategy to solve the problem coded K3, and the ability to represent concepts in the form of drawings or writing coded K4. The logical differences in the instrumental abilities of the three categories are discussed.

High ability student

The group in the high instrumental ability category has, on average, achieved measurable indicators of instrumental ability. This finding is evident in K-T's results, which address completion steps and fulfill each instrumental ability indicator. An example of K-T's response to question number 3 is shown in Figure 1.

Figure 2 shows that K-T understands the problem correctly and can follow the problem-solving plan. Overall, K-T has been able to answer the questions given. On the answer sheet, it can be seen that K-T can remember and use the surface area formulas for a Block ($LP = 2 (pl + pt + lt)$) and a Cylinder ($LP = 2\pi r (r + h)$), and correctly use $\pi = 3.14$ in the problem and solve it well. The following excerpt from the interview supports K-T's answer.

Table 2

Rubric for assessing students' instrumental understanding ability

Assessment Aspects	Score Assessment Criteria	Score
Students' ability to recall concepts learnt	Students can correctly remember mathematical concepts in the form of symbols or mathematical language in the questions.	3
	Students can remember mathematical concepts in the form of symbols or mathematical language in the questions, but only partially.	2
	Students can remember mathematical concepts in the form of symbols or mathematical language in the questions, but not accurately.	1
	Students are unable to remember mathematical concepts in the form of symbols or mathematical language in the questions, or they do not work on them.	0
Ability to identify concepts	Students can correctly represent mathematical concepts in the form of symbols or mathematical language in questions.	3
	Students can represent mathematical concepts in the form of symbols or mathematical language in questions, but partially.	2
	Students can represent mathematical concepts in the form of symbols or mathematical language in questions, but not accurately	1
	Students are unable to represent mathematical concepts in the form of symbols or mathematical language in questions, or do not work on	0
Ability to choose the right concept or strategy to solve problems	Students can correctly complete algorithms or problem-solving tasks sequentially.	3
	Students can complete algorithms or problem-solving sequentially, but not partially.	2
	Students can complete algorithms or problem-solving sequentially, but not accurately.	1
	Students are unable to complete algorithms or problem-solving in a sequential manner.	0
Ability to represent concepts in the form of images or writing	Students can correctly represent concepts in both images and writing.	3
	Students can represent concepts in the form of images or writing, but not fully.	2
	Students can represent concepts in the form of images or writing sequentially, but not accurately.	1
	Students are unable to represent concepts in images or writing, or to do so sequentially.	0

Table 3

Categories of students' math problem-solving ability

Value	**Category**
$x \geq 80$	High
$65 < x < 80$	Medium
$x \leq 65$	Low

(Mangilala & Cajandig, 2025)

Table 4

Student instrumental understanding test result data

Category	**Student**
High	11
Medium	7
Low	7
Sum	25

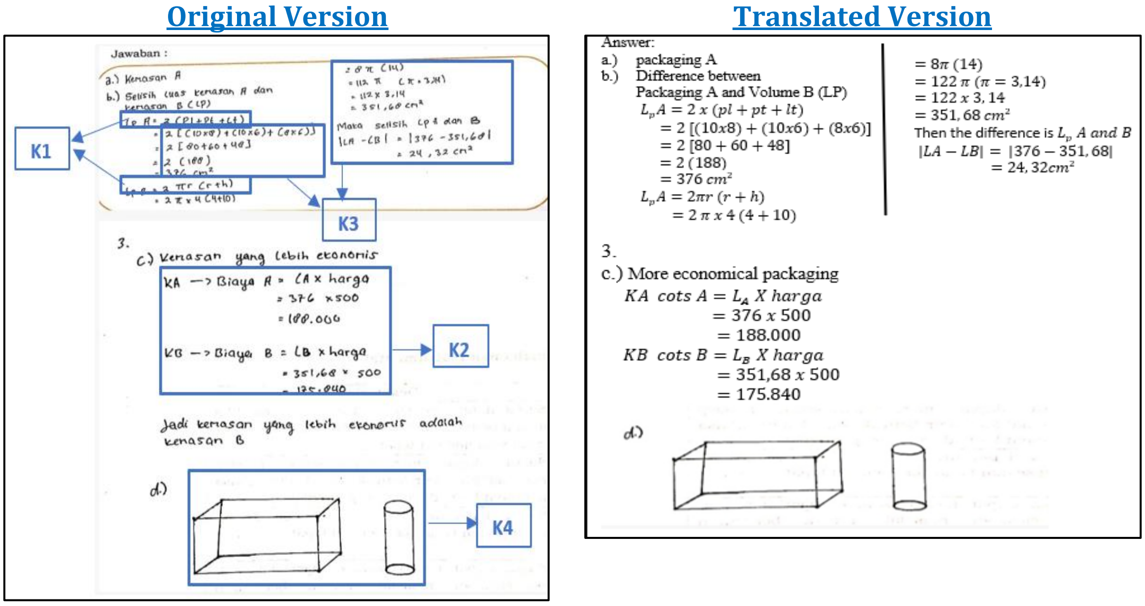


Figure 1. Responses given by K-T

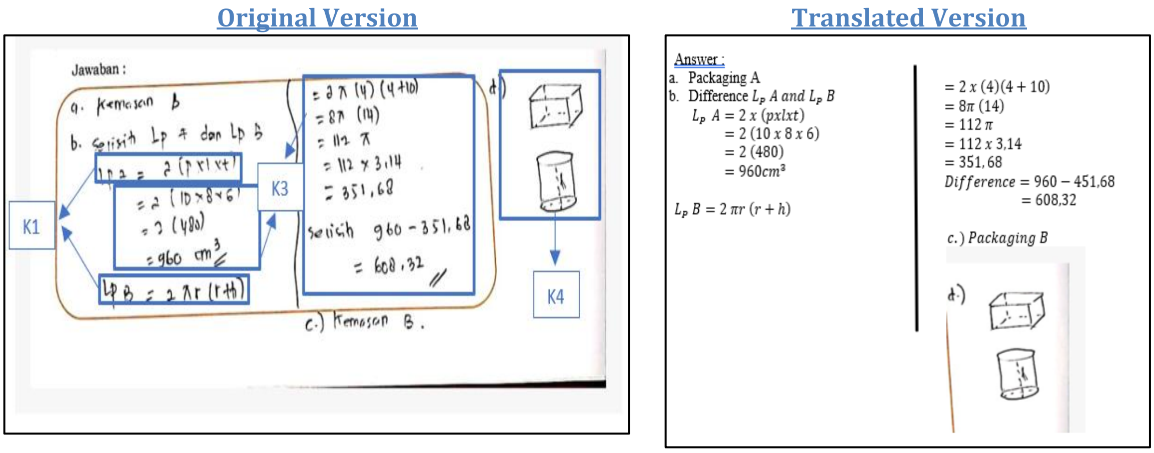


Figure 2. Responses given by K-S

P : *Explain how you can get all these answers?*

K-T : *First, I just guess to answer part a. Then I find the difference between the two packages A and B using the formula that I remember. Next, I look for a more economical package by multiplying the surface area obtained by the price of the package. Finally, I describe the block and cube packaging (K2, K3, and K4)*

P : *Can you explain it? What formula do you use?*

K-T : *The volume of the block v = p×l×t and the volume of the cylinder v = πr(r+h). For the surface area of the block, I use the formula LP=2(pl+pt+lt), while for the surface area of the cylinder LP=2πr(r+h), then the results of the surface areas of A and B I multiply by the production cost, which is 500 (K1)*

Thus, it can be concluded that subject K-T can demonstrate instrumental ability in the young indicators K1, K2, K3, and K4. However, in indicator K1, students are less precise in remembering the formula for the volume of a cylinder, so the answer in point a is less precise.

Medium ability student

The group in the moderate instrumental ability category achieves only three indicators of instrumental ability. This finding is evident in the work results of K-S students, who can solve problems with several errors and meet only a few indicators of instrumental ability. An example of K-S's response to question number 3 is shown in Figure 2.

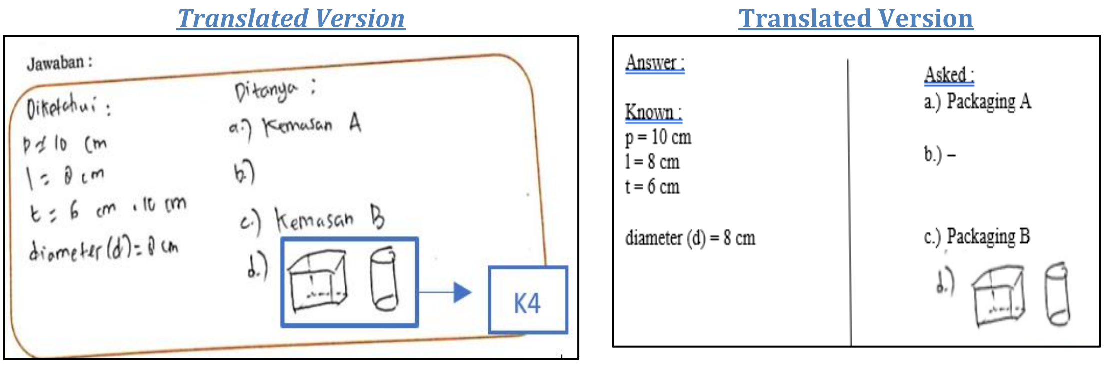


Figure 3. Responses given by K-R

Figure 2 shows that K-S understands the problem correctly, but there is an error in indicator K1. Overall, K-S has been able to work on the problems given. On the answer sheet, it can be seen that the subject can display the formula used and solve the problem well, even though there is an error in recalling the concept, writing the formula for the surface area of a cuboid as LP = 2 (p × l × t). The following excerpt from the interview supports k-S's answer.

P : *Explain how you can get all these answers?*

K-S : *For part a, I calculated the volumes of the cuboid and the cylinder. For part b, I found the difference by calculating the surface area of both packages A and B. I subtracted the two and described the difference (K3 and K4)*

P : *Can you explain it? What formula did you use?*

K-S : *For the volume of the cuboid, I used the formula v = p×l×t, while the volume of the cylinder v = πr(r×h). For the surface area of the cuboid, I used the formula LP=2(p×l×t), while the surface area of the cylinder is LP=2πr(r+h) (K1)*

Thus, it can be concluded that subjects K-S can only show Instrumental ability in indicator K1; students can remember mathematical concepts in the form of symbols or mathematical language in the problem. K3 partially completes algorithms or problem-solving sequentially, and K4 represents concepts in the form of images or writing correctly; however, in indicator K2, subject K-S is unable to represent mathematical concepts in the problem using symbols or mathematical language.

Low ability student

Based on students' work, K-R only meets indicator K4. The subject is still struggling to solve the problems in the questions because he is not used to working on contextual problems and does not understand the concepts he has mastered. An example of K-R's response to question number 3 is shown in Figure 3.

Figure 3. Subject K-R has not been able to solve the given problem correctly. Subject K-R was only able to raise K4 in question no. 3; K-R was not yet able to remember or write down the formula for finding the side of a cube, nor was K-R able to evaluate the requested side of the cube. The following excerpt from the interview supports k-R's answer.

P : *Explain how you can get all these answers?*

K-R : *For the larger volume, of course, the answer is packaging A because cardboard is bigger than cans. Furthermore, for more economical packaging, it is packaging B because the size is smaller than packaging A, and I describe the block and tube packaging (K4)*

P : *Why didn't you answer question (b)?*

K-R : *I do not remember how to find the volume of a cube, sir.*

Table 5

Characteristics of students' instrumental understanding across mathematica ability level

Indicator	High	Medium	Low
Students' ability to recall concepts learned	In this indicator, students can remember mathematical concepts as symbols on the material of cubes, triangular prisms, and blocks, but when presented with cylinders, they are less precise in recalling the formula.	In this indicator, students correctly recall mathematical concepts represented by symbols on cube, triangular prism, block, and cylinder materials in the questions.	In this symbol, students are unable to remember mathematical concepts in the form of symbols in the cube, triangular prism, cuboid, and cylinder materials in the questions.
Ability to identify concepts	In this indicator, students can correctly represent mathematical concepts as symbols on cubes, triangular prisms, blocks, and cylinders in questions.	In this indicator, students can represent mathematical concepts as symbols on the cube and triangular prism material, but they are unable to do so on the block and cylinder material.	In this indicator, students can represent mathematical concepts as cube material symbols, but are unable to do so in triangular prism, cuboid, or cylinder material in the questions.
Ability to choose the right concept or strategy to solve problems	In this indicator, students can correctly complete algorithms or solve problems using cubes, triangular prisms, blocks, and cylinders.	In this indicator, students can complete algorithms or solve problems sequentially on the cube and triangular prism materials, but on the block and cylinder materials, they complete algorithms or solve problems only partially.	In this indicator, students are unable to complete algorithms or solve problems sequentially on the cube, triangular prism, block, and cylinder materials.
Ability to represent concepts in the form of images or writing	In this indicator, students can correctly represent concepts in the form of images or written material on cubes, triangular prisms, blocks, and cylinders.	In this indicator, students can represent concepts as images on block and cylinder material, but cannot do so on cube or triangular prism material.	In this indicator, students can represent concepts as images on cube, block, and cylinder materials, but not on triangular prism materials.

Thus, it can be concluded that the K-R subjects demonstrated instrumental abilities only in the K4 indicator: students correctly represented concepts in the form of pictures or writing on cubes, blocks, and cylinders. Based on the analysis of test and interview data from the three categories, it can be concluded that they share similarities and differences in instrumental ability, as shown in Table 5.

DISCUSSION

Table 5 shows that students with high, medium, and low instrumental understanding abilities differ in their recall of learnt concepts. High- and medium-ability students can remember mathematical concepts as symbols on the surfaces of cubes, triangular prisms, and cuboids, but when presented with cylinders, they are less precise in recalling the formula. This finding states that students with high abilities can remember mathematical concepts in symbolic form effectively for topics such as cubes, triangular prisms, and cuboids, but may have difficulty with the formula for the volume of cylinders. This result aligns with Chiphambo and Mtsi (2021), Choo et al. (2021), and Gargrish et al. (2021). When they have low ability, students are unable to remember mathematical concepts in symbolic form, such as cubes, triangular prisms, cuboids, and cylinders. This result aligns with Lennon-Maslin et al. (2024), who found that some students showed low self-confidence in topics such as cubes and blocks.

The findings on the concept recall indicator (K1) indicate a strong tendency toward Instrumental Understanding among students in the high and middle categories. Although they were able to effectively recall formulas for plane-sided geometric shapes (cubes, cuboids, and triangular prisms), a hallmark of knowing how to use procedures (Skemp's "rules without reasons"), their failure to recall the formula for the volume of a cylinder indicates that this procedural memory was not supported by strong Relational Understanding. This weakness was even more pronounced among students in the low category, who were unable to recall any symbolic concepts, confirming that they had not yet reached the stage of adequate Instrumental Understanding.

Several researchers emphasize that effective procedural proficiency must be preceded by strong conceptual understanding (Barbieri et al., 2023). Difficulties among low-level students in calculating and interpreting context (Handayani & Sutama, 2024) reinforce this. This finding shows that the main problem is that students' Instrumental Understanding memorizes formulas without truly understanding the basic concepts, which results in difficulty solving 3D geometry problems that are rather complicated or require different applications.

The ability to identify concepts: students with high abilities can represent mathematical concepts as symbols on the surfaces of cubes, triangular prisms, cuboids, and cylinders. This result also aligns with Nur and Khotimah (2021) and Wulandari and Ishartono (2022). Students with high abilities can analyze mathematical concepts in questions. Students with moderate abilities can represent mathematical concepts using symbols in cube and triangular prism materials, but they are unable to do so in block and cylinder materials. However, in contrast to the results of Niileksela et al.(2025), which state that moderate ability does not meet the mathematical process indicator in the question. Students with low abilities can represent mathematical concepts in cube material symbols, but cannot do so in triangular prism, cuboid, or cylinder material. Furthermore, based on research by Handayani and Sutama (2024), students with fewer categories have difficulty calculating and interpreting the question context.

An indicator of the ability to choose the right concept or strategy to solve a problem, students with high ability can complete algorithms or problem-solving tasks sequentially using cubes, triangular prisms, blocks, and cylinders. The finding aligns with Foong et al. (2022), which states that high-achieving students demonstrate proficiency in understanding problems, planning solutions, executing them accurately, and verifying answers. Students with moderate ability can complete algorithms or problem-solving sequentially with cubes and triangular prisms, but with blocks and cylinders, they complete only partially. In contrast to the research results, Fernanda and Kholid (2023) state that students with ability can only meet the indicators of problem identification and problem generalization. Students with low ability are unable to complete algorithms or problem-solving tasks in a sequential manner using cubes, triangular prisms, blocks, and cylinders. This result aligns with Fadhilah and Masduki (2023), who state that students with low ability have difficulty in solving geometry problems.

Ability to represent concepts in the form of images or writing. Students with high abilities can represent concepts in the form of images or writing on cubes, triangular prisms, blocks, and cylinders. This finding is in line with Murtiyasa et al (2019) and Nurwijayanti et al (2018), who state that students with high abilities can represent geometric concepts in the form of images. Students with moderate abilities can represent concepts as images on block and cylinder materials, but are unable to do so on cube and triangular prism materials. Furthermore, research by Aziiza and Juandi (2021) indicates that students have difficulty determining the surface area of a prism because they do not understand the concept. Students with low abilities can represent concepts in the form of images on cube, block, and cylinder materials, but are unable to do so on triangular materials, either in image or written form. This finding is also in line with Awalina and Masduki, 2025), who found that students with low abilities have difficulty in solving geometry problems related to spatial reasoning, specifically in visualization. Furthermore, Sibanda (2021) emphasizes that misunderstandings of surface area and volume are primarily due to a lack of mastery of basic facts and concepts.

CONCLUSION

Students' instrumental understanding in solving three-dimensional geometry problems can be classified into high, medium, and low levels, each characterized by distinct patterns of indicator achievement. High-level students demonstrate comprehensive conceptual mastery by consistently fulfilling all instrumental indicators across all tested geometric shapes. Medium-level students show partial and context-dependent mastery, particularly in concept identification and representation, while low-level students exhibit minimal mastery limited to specific aspects. These findings highlight the importance of differentiated instruction that aligns learning strategies, task complexity, and conceptual support with students' levels of understanding, supported by visual media and continuous formative assessment. Although this study is limited to flat-faced solid geometry, the results offer meaningful pedagogical implications and suggest the need for further research in other mathematical domains such as algebra, arithmetic, and statistics.

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Availability of data and materials	All data are available from the authors
Competing interests	The authors declare that the publishing of this paper does not involve any conflicts of interest. This work has never been published or offered for publication elsewhere, and it is completely original.

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