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Praxeological analysis of junior secondary students' epistemological obstacles in algebraic operations

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ABSTRACT

This study investigates junior secondary students' epistemological obstacles to learning algebraic operations through a praxeological framework grounded in the Anthropological Theory of the Didactic (ATD), with Didactical Design Research (DDR) as the conceptual orientation. Diagnostic algebra tasks and semi-structured interviews were administered to six seventh-grade students in Indonesia to examine their algebraic techniques and justifications. Students' written and verbal responses were analysed by reconstructing tasks (T), techniques (τ), technologies (θ), and theories (Θ). The findings reveal that students generally exhibit procedural fluency in routine tasks, such as simplification and distributive expansion. However, substantial epistemological obstacles arise in tasks that require justification, relational interpretations of equality, variable generalisation, and contextual transfer. These obstacles are characterised by a misalignment between students' correct techniques and weak or absent justificatory discourse, indicating that procedural correctness does not consistently reflect conceptual understanding. This study contributes to mathematics education by offering a fine-grained praxeological analysis that makes epistemological obstacles often overlooked in error-based analyses visible. By distinguishing students' actions from their justifications, the study clarifies the structural nature of algebraic difficulties and identifies instructional directions that emphasise relational equality, explicit justification, and stable conceptions of variables to support deeper structural and theoretical understanding of algebra.

INTRODUCTION

Algebra plays a central role in secondary school mathematics and functions as a gateway to higher-order mathematical thinking. Proficiency in algebra is widely recognised as essential for students' success in advanced mathematical topics, including functions, equations, and calculus (Kieran, 2016). Despite its importance, a substantial body of research has consistently shown that students encounter persistent difficulties in learning algebraic operations. These difficulties include simplifying expressions, distinguishing between constants and variables, and correctly applying fundamental algebraic properties (Chamundeswari, 2014; Muchoko et al., 2019; Demonty et al., 2018). Such difficulties are often rooted in the overgeneralisation of arithmetic rules, superficial interpretations of symbolic representations, and a reliance on procedural strategies rather than conceptual reasoning (Booth et al., 2016; Welder, 2012). As a result, many students struggle to move beyond arithmetic reasoning and develop a structurally coherent understanding of algebra.

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In mathematics education research, these persistent difficulties are commonly conceptualised as epistemological obstacles, which arise when students' prior knowledge becomes inadequate or misleading in new mathematical contexts (Herscovics, 2018; Subroto & Suryadi, 2018). Epistemological obstacles differ from simple errors because they are embedded in learners' ways of thinking and reasoning, rather than in momentary lapses or miscalculations. Recent studies further indicate that epistemological obstacles in algebra continue to impede students' progression from arithmetic reasoning to algebraic thinking, with long-term consequences for their mathematical development and learning trajectories (Utami & Prabawanto, 2023). These obstacles therefore reflect deeper issues related to how mathematical knowledge is constructed, justified, and applied by learners, particularly in relation to variables, equality, and symbolic manipulation across different algebraic situations and problem contexts.

Although numerous studies have investigated students' algebraic errors and misconceptions and proposed instructional responses to address them (Fauziah et al., 2023; Utami et al., 2023; Wilujeng & Alvarez, 2025), much of the existing research remains largely descriptive. In many cases, students' difficulties are catalogued as incorrect answers or procedural failures, without a systematic analysis of the epistemic structure underlying their mathematical activity. Consequently, the relationship between students' observable techniques and the mathematical rationales that legitimise those techniques often remains insufficiently explored (Kabadaş & Mumcu, 2024; Dassa et al., 2024). This descriptive focus limits the explanatory power of prior studies and constrains efforts to understand why certain difficulties persist across tasks and contexts. These limitations highlight the need for an analytical framework that links students' actions on algebraic tasks to the theoretical foundations of mathematical knowledge.

The Anthropological Theory of the Didactic (ATD) offers such an analytical framework through the concept of praxeology, which conceptualises mathematical activity as an organised system of tasks (T), techniques (τ), technologies (θ), and theories (Θ) (Gascón, 2024). Praxeological analysis enables researchers to examine not only what students do when solving mathematical problems but also how their techniques are justified, stabilised, or remain fragile. Through this lens, students' mathematical activity can be analysed in terms of both practical performance and epistemic justification. While praxeological approaches have been widely applied to the analysis of textbooks, curricula, and institutional mathematical practices (Hochmuth & Peters, 2021; Utami et al., 2022; Panjaitan et al., 2025; Agustito et al., 2025), relatively few studies have employed praxeology to analyse students' empirical responses to algebraic tasks, particularly in relation to epistemological obstacles.

Research within the tradition of Didactical Design Research (DDR) has emphasised the importance of identifying learning obstacles as a foundation for instructional improvement and didactical decision-making (Ruli et al., 2019; Supriadi, 2019; Rohimatunnisa et al., 2025). DDR highlights the pedagogical significance of analysing obstacles prior to designing instructional interventions. In the present study, however, DDR is not adopted as a full methodological framework involving iterative design experiments, classroom enactments, or metapedidactical reflection. Instead, its conceptual orientation serves as a theoretical inspiration, informing the analytical focus on epistemological obstacles as phenomena with didactic relevance. In this sense, DDR provides a background perspective for interpreting the instructional implications of praxeological findings rather than functioning as the primary research methodology (Pauji et al., 2023; Fardian et al., 2025).

Against this background, the present study is a praxeological analysis of junior secondary students' epistemological obstacles in learning algebraic operations, grounded in empirical data from students' written work and interviews. By mapping students' difficulties onto the components of praxeology, this study seeks to provide a systematic account of how epistemological obstacles are manifested in students' algebraic activity. This approach moves beyond surface-level error identification by examining the relationships between tasks, techniques, technologies, and theories within students' reasoning. Through this contribution, the study aims to enrich mathematics education research by offering a theoretically grounded and analytically precise understanding of students' algebra learning difficulties and their underlying epistemic structures.

Literature Review

Algebraic operations in secondary education

Algebra is widely regarded as a foundational domain in mathematics education and has long been positioned as a gateway for students' transition from arithmetic reasoning to more advanced and abstract mathematical thinking (Carraher et al., 2006). Mastery of algebraic operations—including the manipulation of variables, the simplification of expressions, and the application of distributive properties—plays a pivotal role in fostering students' abilities in abstraction, generalisation, and problem solving (Kieran, 2020). Historically, algebra has occupied a central place in school curricula, reflecting its function in structuring mathematical knowledge and shaping students' learning trajectories (Puig & Rojano, 2004). In response to persistent learning difficulties, reform-oriented scholarship has emphasised the need to move beyond a strictly procedural orientation toward instructional approaches that promote conceptual and algebraic thinking (Kaput, 1999). Systematic reviews further confirm that algebraic competence emerges from the integration of conceptual and procedural knowledge with representational fluency, enabling learners to engage more meaningfully with mathematical structures (Sibgatullin et al., 2022). From a theoretical perspective, algebra is not merely a symbolic code but a semiotic and cultural practice through which learners construct meaning about abstract mathematical relations (Radford, 2010). Nevertheless, a substantial body of research consistently indicates that algebra remains one of the most challenging areas of mathematics learning across educational contexts (Hodgen et al., 2018). These challenges are not limited to procedural errors; rather, they reflect enduring conceptual gaps in students' understanding of variables, expressions, and equations (Donevska-Todorova, 2016). Cognitive research has long documented the presence of epistemological obstacles in symbolic manipulation, highlighting the depth and persistence of these difficulties (Sleeman, 1984). Empirical evidence from Indonesia similarly shows that early algebra learning is frequently characterised by misconceptions and weak conceptual foundations (Jupri et al., 2014). Parallel findings from broader Asian contexts further suggest that students' algebraic problem-solving difficulties persist over time, underscoring the need for pedagogical designs that balance procedural fluency with conceptual understanding (Ying et al., 2020; Poon & Leung, 2010).

Learning obstacles and epistemological obstacles

Students' difficulties in learning algebra have frequently been examined through the concept of learning obstacles, understood as barriers that hinder the acquisition and development of mathematical concepts. Prior research indicates that such obstacles may arise from cognitive limitations, instructional practices, or the inherent complexity of mathematical structures themselves (Hendriyanto et al., 2024). Among these categories, epistemological obstacles are particularly salient because they originate from learners' prior knowledge or intuitive reasoning that is productive in certain contexts but becomes inadequate or misleading in algebraic situations (Schneider, 2014). For instance, students often transfer arithmetic rules directly into algebra without recognising the symbolic and relational roles of variables, resulting in persistent errors in symbolic manipulation and algebraic reasoning (Ndemo & Ndemo, 2018; Adnan et al., 2021). Empirical studies further demonstrate that such epistemological obstacles extend beyond procedural difficulties, constraining students' abilities to generalise, justify, and coordinate mathematical ideas across different algebraic contexts (Nansiana et al., 2024). From a historical-epistemological perspective, these difficulties reflect long-standing challenges in the didactics of algebra, rather than isolated instructional shortcomings (Gallardo, 2001).

However, much of the existing research has approached epistemological obstacles primarily through descriptive accounts of errors or misconceptions, offering limited insight into how these obstacles are structurally embedded in students' mathematical activity. In particular, prior studies rarely examine how students' observable strategies are connected to the underlying mathematical rationales that legitimise, or fail to legitimise, their techniques. This limitation points to the need for an analytical framework that can systematically relate students' tasks, techniques, and justifications to uncover the epistemic structure of their algebraic difficulties. Such a requirement motivates the use of a praxeological perspective, as articulated within the Anthropological Theory of the Didactic, to analyse epistemological obstacles not merely as errors but as manifestations of disrupted or incomplete mathematical praxeologies.

The anthropological theory of the didactic and praxeology

The Anthropological Theory of the Didactic (ATD) provides a robust theoretical framework for analysing mathematical knowledge and practice by conceptualising them as praxeologies, that is, organised systems of human activity grounded in epistemic and didactic structures (Chevallard & Bosch, 2020). Within this framework, a praxeology is composed of four interrelated components: tasks, techniques, technologies, and theories, which together coherently structure mathematical activity (Chevallard et al., 2015). Tasks refer to the types of problems to be addressed, techniques denote the procedures or strategies employed to solve those tasks, technologies encompass the explanations and justifications that legitimise the use of particular techniques, and theories represent broader bodies of mathematical knowledge that provide epistemic coherence to technologies (Chevallard, 2007; Chevallard & Sensevy, 2014).

This praxeological framework enables researchers to model mathematical activity not only in terms of observable performance but also in terms of the epistemic justifications underlying students' actions. In this sense, ATD offers a systematic analytical lens for examining how mathematical knowledge is produced, taught, learned, and institutionalised within educational settings (Haspekian et al., 2023; Schmidt, 2016). More recent developments in ATD have further highlighted its relevance to didactic transposition, particularly in mapping the relationships between knowledge to be taught and institutional demands within specific mathematical domains (Strømskag & Chevallard, 2024). In the context of the present study, praxeology offers an appropriate analytical perspective for investigating students' epistemological obstacles in algebra by tracing how learning difficulties manifest through the relationships, or disruptions, among tasks, techniques, technologies, and theories in students' mathematical activity.

Praxeological analysis of student work

Although praxeological analysis has traditionally been applied to the examination of curricular materials, such as textbooks and instructional designs (Utami et al., 2024; Fitriasari et al., 2025), it also offers substantial analytical potential for investigating students' mathematical activity. Students' responses to mathematical tasks can be conceptualised as instances of praxis, in which tasks and techniques are directly observable through written solutions and verbal reasoning (Winsløw, 2011). In contrast, the technological and theoretical components of praxeology are typically less explicit and must be inferred from students' explanations, justifications, or implicit conceptions of mathematical ideas (Hausberger, 2018).

Applying a praxeological lens to student work enables researchers to move beyond surface-level descriptions of errors by systematically relating observable techniques to the justifications that support—or fail to support—their use. In this way, errors and misconceptions can be interpreted as indicators of gaps or disruptions within specific components of praxeology, such as fragile technologies or underdeveloped theoretical understandings (Cosan, 2024). This analytical mapping provides a more coherent and theoretically grounded explanation of epistemological obstacles in algebra, revealing that students' difficulties are structurally embedded in their mathematical activity rather than arising from isolated procedural failures. Consequently, praxeological analysis offers a robust framework for understanding the persistence of students' algebraic difficulties. It aligns closely with the present study's aim of examining epistemological obstacles through students' actual work (Diskin & Hutchinson, 2024).

Didactical design research as an intervention

Didactical Design Research (DDR) is widely recognised as a cyclical process of analysing learning obstacles, designing didactical interventions, and evaluating their effectiveness in fostering conceptual understanding (Suryadi et al., 2017). Within this framework, hypothetical didactical designs are systematically developed to address students' specific difficulties, enabling researchers and teachers to explore alternative pathways of mathematical instruction (Fuadiah et al., 2017). When combined with praxeological analysis, DDR offers a dual lens: while praxeology captures the structural nature of students' obstacles through praxis, techniques, and underlying theories, DDR provides a reflective mechanism for constructing responsive instructional solutions (Jatisunda et al., 2025). This integration not only strengthens the theoretical grounding of mathematics education research but also ensures practical applicability in classroom practice. Empirical studies further demonstrate that DDR supports teacher professionalism and facilitates hybrid didactical approaches to address nonroutine problems, thereby enhancing both instructional design and student learning outcomes (Rudi et al., 2020; Sukarma et al., 2024).

METHODS

Research approach

This study employed a qualitative research approach with a praxeological analytical framework grounded in the Anthropological Theory of the Didactic (ATD). The primary aim was to explore and interpret junior secondary students' epistemological obstacles in learning algebraic operations by analysing their written work and interview responses. Praxeological analysis was used to reconstruct students' mathematical activity in terms of tasks, techniques, technologies, and theories. Didactical Design Research (DDR) was not adopted as a full methodological framework in this study. Instead, DDR served as a theoretical orientation that underscores the didactical importance of diagnosing learning obstacles prior to instructional design, without extending to iterative design experimentation or classroom enactment. Accordingly, the present study focuses on analytical interpretation rather than the development or implementation of didactical interventions.

Subjects of the study

The study was conducted at a state Islamic junior secondary school (MTs) in Majalengka, Indonesia. Six seventh-grade students were selected through purposive sampling to represent a range of mathematical achievement levels (high, medium, and low). The small, context-specific sample was intentionally selected to enable in-depth, case-based analysis of students' algebraic reasoning and epistemological obstacles. As a qualitative exploratory study, the findings are not intended to be generalised to broader populations but to provide rich, interpretive insights into students' mathematical activity..

Research procedure

The study was conducted through three main stages designed to support a praxeological analysis of students' algebraic reasoning.

Preparation of algebraic tasks

A set of algebraic operations tasks was developed in alignment with the Grade 7 junior secondary curriculum, with particular emphasis on simplification, application of the distributive property, and identification of like terms. The tasks were designed as diagnostic instruments to elicit students' algebraic techniques and to reveal potential epistemological obstacles related to variables, expressions, and operations.

Administration of diagnostic tasks

Students individually completed the tasks in a written test format. Their written responses were collected as empirical data to capture observable tasks (T) and techniques (τ), including correct procedures, errors, and non-standard strategies that indicated possible epistemological obstacles.

Follow-up semi-structured interviews

Semi-structured interviews were conducted with each participant to explore their reasoning processes further. Interview questions focused on eliciting students' explanations, justifications, and interpretations of the algebraic tasks. These verbal data were used to analytically infer the technological components (θ), namely the explanations students employed to legitimise their techniques, as well as the theoretical components (Θ) that were implicit, incomplete, or absent in their reasoning. The inference of θ and Θ was grounded in students' articulated reasoning rather than being assumed a priori.

Materials and instruments

The materials and instruments employed in this study were designed to support a praxeological analysis of students' algebraic reasoning and consisted of the following components.

Diagnostic algebra tasks

A set of contextual and symbolic tasks on algebraic operations was developed in alignment with the junior secondary curriculum. The tasks focused on simplification, distributive properties, and the identification of like terms, and were intended to elicit students' algebraic techniques and potential epistemological obstacles.

Interview protocol

A semi-structured interview guide was designed to probe students' reasoning processes, justifications, and interpretations of algebraic rules. The interview questions were explicitly designed to elicit explanations that could be interpreted analytically as technological elements (θ), as well as indications of implicit, incomplete, or absent theoretical understandings (Θ).

Documentation forms

Researcher field notes and classroom records were used to capture contextual information related to task administration and students' responses, providing supplementary data to support interpretation.

Data collection

Data were collected from multiple sources to enable triangulation within the praxeological analysis.

Written student work

Students' written responses to the diagnostic tasks provided empirical evidence of the tasks (T) encountered and the techniques (τ) employed, including standard procedures, non-standard strategies, and errors indicative of epistemological obstacles.

Interview transcripts

Audio-recorded interviews were transcribed verbatim and analysed to capture students' verbal explanations and justifications. These data formed the primary basis for the analytical inference of technological components (θ) and for identifying gaps or fragilities in students' theoretical understandings (Θ).

Curricular and textbook references

Relevant curriculum documents and textbooks were consulted as normative references to identify the expected theoretical structures associated with the algebraic content. These references functioned as analytical benchmarks rather than data sources, enabling comparison between students' actual reasoning and institutionally expected mathematical theories.

Data analysis techniques

Data analysis was conducted using a systematic, two-stage procedure to support a praxeological interpretation of students' algebraic reasoning.

Identification of epistemological obstacles

In the first stage, students' written responses were examined to identify recurring patterns of errors, non-standard strategies, and inconsistencies in algebraic reasoning. These patterns were interpreted analytically as potential epistemological obstacles arising from concepts, procedures, or operational techniques in algebraic operations. This initial analysis was inductive in nature and aimed to characterise areas of difficulty without imposing predefined categories.

Praxeological reconstruction and mapping

In the second stage, each student's written and verbal responses were reconstructed as a praxeology following the framework of the Anthropological Theory of the Didactic. This reconstruction involved the analytical identification of the following components:

- Tasks (T): The algebraic problems presented to the students.
- Techniques (τ): The procedures or strategies employed by students, including both conventional and non-conventional approaches.
- Technologies (θ): The explanations, justifications, or rules articulated by students during interviews to legitimise their techniques.
- Theories (Θ): The implicit, incomplete, or absent mathematical structures inferred from students' reasoning that were expected to underpin the identified technologies.

The identification of θ and Θ was grounded in students' verbal explanations and patterns of reasoning, rather than being assumed a priori. Curriculum documents and textbooks were used as normative references to represent institutionalised praxeologies, enabling analytical comparison between students' reconstructed praxeologies and expected mathematical structures. Through this

Table 1
Praxeological analysis of students' 1 responses

Task	Technique (τ): What the student did	Technology (θ): Student's justification	Theory (Θ): Expected but absent/fragile
Simplify the algebraic expression. $\frac{3}{2}(3x + 2y) + \frac{1}{2}(5x - 4y)$	Expanded each fraction into the parentheses and combined like terms to obtain (7x+y).	"I multiplied the numbers outside the brackets, then combined the same terms."	Distributive property and linear structure of algebraic expressions as general rules (not explicitly conceptualised).
Prove that $2(y + 3) + y = 3y + 6$	Not attempted by the student.	No justification articulated; the student expressed uncertainty about how to begin.	Structural understanding of algebraic identities and the use of properties as tools for proof (absent).
Prove equality $s + s + s + s + s + 4 = 2(s + 2) + 2s$	Substituted (s) with 5, expanded both sides numerically, and compared results.	"I thought it was like a number, so I tried using 5."	Variable as a generalised quantity and relational meaning of equality (absent).
Contextual problem (books on the shelf)	Not attempted by the student.	No explanation provided.	Translation between contextual situations and symbolic algebraic representations (absent).

comparative analysis, the nature and sources of epistemological obstacles in students' algebraic activity were systematically identified.

FINDINGS

The findings of this study are presented through a praxeological analysis of students' responses to four tasks on algebraic operations. Each response was analytically reconstructed into tasks (T), techniques (τ), technologies (θ), and theories (Θ) in accordance with the Anthropological Theory of the Didactic. In this study, techniques refer to the observable procedures used by students, technologies denote the explanations or justifications articulated by students to legitimate their techniques, and theories represent the mathematical norms or structures that are expected to support such justifications but may be implicit, fragile, or absent. This reconstruction enabled a systematic identification of epistemological obstacles by revealing disconnections between what students could do and how they justified their actions. The praxeological profile of Student 1 is presented in Table 1.

The praxeological analysis of Student 1 reveals a clear contrast between stable procedural techniques and fragile or absent epistemic justification. In Problem 1, the student successfully expanded the expression and combined like terms to obtain the correct result $7x + y$. This indicates procedural fluency at the level of technique (τ). However, the student's justification remained procedural. It was not grounded in an explicit articulation of the distributive property as a general mathematical principle, suggesting a limited technological foundation (θ). More pronounced epistemological obstacles emerged in tasks requiring justification and relational reasoning. In Problem 2, the student did not attempt to prove the identity $2(y + 3) + y = 3y + 6$and expressed uncertainty about how to proceed, indicating that algebraic properties were not internalized as tools for validation. In Problem 3, the substitution of $s = 5$Reflects a fragile conception of variables as generalized quantities and an absence of the theoretical norm underlying algebraic equality. Finally, the omission of the contextual task (Problem 4) suggests difficulty in transferring symbolic techniques to contextual modeling. Overall, Student 1's praxeology is characterized by stable techniques in routine manipulation but epistemological obstacles arising from the disconnection between techniques, justifications, and underlying theoretical structures.

The interview data further illuminate the praxeological structure underlying Student 1's written responses as presented in Table 2. In the simplification task (Problem 1), the student articulated a procedural justification focused on "multiplying the numbers outside the brackets" and combining like terms. This explanation reflects a stable technique (τ) supported by a procedural form

Table 2
Students' 1 interview

Problem	Interviewer (I)	Student (S)
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Can you explain how you solved this expression?	*I multiplied the numbers outside the brackets. So,* $\frac{3}{2}\times 3x=\frac{9}{2}x$ *and* $\frac{3}{2}\times 2y=3y$. *Then I did the same with the second part:* $\frac{1}{2}\times 5x=\frac{5}{2}x$, *and* $\frac{1}{2}\times(-4y)=-2y$. *After that, I combined like terms to get* $7x+y$.
Prove that $2(y+3)+y=3y+6$	Did you try to solve this?	*I did not write it. I was not sure how to begin. Perhaps expand the bracket, but I was concerned it might be incorrect.*
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	How did you approach this problem?	*I thought ss was like a number, so I tried with 5. Then I wrote* $2(5+2)+2s=25+25+4$
Determine the number of novels and textbooks remaining on the main shelf after transfer/return.	Can you explain what you wrote for the book's problem?	*I did not answer this question.*

Table 3
Praxeological analysis of students' 2 responses

Task	Technique (τ): What the student did	Technology (θ): Student's justification	Theory (Θ): Expected, emerging, or fragile
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Expanded each fraction into the parentheses and combined like terms to obtain $(7x+y)$.	"I multiplied each fraction into the brackets and then added the same terms together."	Distributive property and linearity of expressions (implicitly recognised, not formalised).
Prove that $2(y+3)+y=3y+6$	Expanded (2(y+3)), added (y), regrouped terms, and rewrote as (3y+6).	"I used the distributive property, then regrouped because of the commutative and associative properties."	Fundamental algebraic properties as tools for justification (explicitly emerging).
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	Simplified both sides separately to (5s+4) and (4s+4) without reconciling the difference.	"I simplified each side, but they did not look the same, so I just wrote the results."	Relational meaning of equality as equivalence between expressions (absent or fragile).
Contextual problem (books)	Modeled the situation algebraically and simplified to (5x) novels and (2y) textbooks.	"I subtracted what was moved and added what was returned."	Algebraic modeling of contextual situations using like terms (functioning).

of technology (θ), but without explicit reference to algebraic properties as general principles. As such, the student's success in routine manipulation was not accompanied by a fully articulated theoretical justification. In contrast, the interview responses to Problems 2 and 3 reveal more pronounced epistemological obstacles. The student's uncertainty about how to begin proving the $(y+3)+y=3y+6$ indicates that algebraic properties were not internalized as tools for justification. In the equality task, the substitution of a variable with a fixed number ("I thought s was like a number") reflects a fragile conception of variables as generalized quantities and an absence of the theoretical norm that underpins algebraic equality. Finally, the lack of response to the contextual problem suggests difficulty in mobilizing symbolic techniques beyond routine procedural contexts. Taken together, the interview data corroborate the praxeological analysis by showing that Student 1's difficulties arise not from a lack of procedural skill, but from weak or absent connections between techniques, justifications, and underlying theoretical structures.

Table 4
Students' 2 interview

Problem	Interviewer (I)	Student (S)
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	I: Can you explain how you solved $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$?	First, I multiplied each fraction into the parentheses. I got $\frac{9x}{2}+\frac{6y}{2}+\frac{5x}{2}-\frac{4y}{2}$. Then I combined the like terms to get 7x + y.
Prove that $2(y+3)+y = 3y+6$	I: How did you prove that $2(y+3)+y = 3y+6$?	I used the distributive property: 2(y + 3) = 2y + 6. Then I added yy so it became 2y + 6 + y. After that, I regrouped to (2 + 1)y + 6 = 3y + 6. I also know this is because of distributive, commutative, and associative properties.
Prove equality $s+s+s+s+s+4 = 2(s+2)+2s$	I: What did you think when solving $s+s+s+s+s+4 = 2(s+2)+2s$	I simplified the left side into 5s + 4 and the right side into 4s + 4. They don't look the same, so I just wrote the results. I wasn't sure how to make them equal.
Determine the number of novels and textbooks remaining on the main shelf after transfer/return.	I: How did you find the number of novels and textbooks left after the transfer?	For novels, I wrote 5x − 2x + 2x = 5x . For textbooks, I did 3y − y = 2y. So the answer is 5x5x novels and 2y textbooks.

The praxeological analysis of Student 2 reveals a more articulated epistemic structure than that of Student 1, particularly in tasks involving algebraic justification as presented in Table 3. In Problems 1 and 2, the student demonstrated stable techniques (τ) supported by explicit technological discourse (θ). Notably, in proving the identity $2(y+3)+y=3y+6$ The student explicitly referred to distributive, commutative, and associative properties, indicating that these properties functioned as tools for justification rather than merely procedural rules. This suggests the emergence of an underlying theoretical awareness (Θ) in routine proof contexts. However, this theoretical activation was not consistently mobilized across tasks. In the equality problem, the student simplified both sides independently without attempting to establish equivalence, revealing a fragile conception of equality as a relational structure. Here, the technique was procedurally correct, but the absence of a relational theoretical norm prevented the student from resolving the discrepancy. By contrast, in the contextual problem, the student successfully translated the situation into algebraic expressions and manipulated them meaningfully, indicating coherence between technique, justification, and context. Overall, Student 2's praxeology is characterized by strong procedural and emerging theoretical resources, with epistemological obstacles localized primarily in the structural interpretation of algebraic equality.

The interview data further clarify the praxeological structure underlying Student 2's written responses as presented in Table 4. In Problems 1 and 2, the student demonstrated stable techniques (τ) supported by explicit technological discourse (θ). Notably, when proving the identity $2(y+3)+y=3y+6$, the student explicitly invoked the distributive, commutative, and associative properties, indicating that these properties functioned as tools for justification rather than as implicit procedural rules. This suggests that, in routine proof contexts, the underlying theoretical norms of elementary algebra (Θ) were partially activated and operationalized. However, this theoretical activation was not consistently mobilized across all tasks. In the equality problem, the student simplified both sides of the equation independently but did not attempt to establish their equivalence, treating the equality as two separate computations. This reveals a localized epistemological obstacle in the relational interpretation of equality, despite otherwise coherent techniques and justifications. By contrast, in the contextual problem, the student successfully translated the situation into algebraic expressions and manipulated them meaningfully, demonstrating alignment between technique, justification, and context. Overall, Student 2's praxeology is characterized by strong procedural techniques and explicit technologies, with epistemological obstacles emerging specifically where relational theoretical norms, such as equality as equivalence, are required but not activated.

Table 5
Praxeological analysis of students' 3 responses

Task	Technique (τ): What the student did	Technology (θ): Student's justification	Theory (Θ): Expected but absent/fragile
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Expanded each term distributively and combined like terms to obtain (7x+y).	"I multiplied each term inside the parentheses and then combined like terms."	Distributive property and linear structure of expressions as general principles (implicitly used, not articulated).
Prove that $2(y+3)+y=3y+6$	Expanded (2(y+3)), added (y), and regrouped terms to obtain (3y+6).	"I expanded it, then added (y) and regrouped."	Algebraic properties as tools for justification in identity proofs (partially activated, procedural).
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	Simplified only the left-hand side to (5s+4); did not continue with the right-hand side.	"I was not sure how to check if they are the same."	Relational meaning of equality and equivalence between expressions (absent).
Contextual problem (books)	Modeled the situation symbolically but misapplied subtraction, yielding (5x+4y).	"I subtracted what was moved and added what was returned."	Consistent algebraic modeling of contextual situations (fragile).

The praxeological analysis of Student 3's work reveals stable procedural techniques accompanied by fragile epistemic support as presented in Table 5. In Problems 1 and 2, the student successfully expanded expressions and regrouped terms to obtain correct results, indicating fluency at the level of technique (τ) in routine symbolic manipulation. However, the accompanying explanations remained procedural. They did not explicitly articulate algebraic properties as general principles for justification, suggesting that the technological level (θ) was limited and the underlying theoretical norms (Θ) were only implicitly activated. More pronounced epistemological obstacles emerged in tasks requiring relational reasoning and contextual consistency. In the equality task, the student simplified only the left-hand side of the expression and explicitly expressed uncertainty about how to determine equivalence, indicating the absence of a relational conception of equality as a theoretical norm. Similarly, in the contextual problem, although the student attempted to model the situation symbolically, an incorrect handling of subtraction led to an inaccurate result. This suggests difficulty in maintaining coherence between symbolic techniques and contextual interpretation. Overall, Student 3's praxeology is characterized by effective techniques in routine tasks but epistemological obstacles arising from weak connections between techniques, justifications, and theoretical structures. To further clarify the nature of these obstacles, a follow-up interview was conducted. The interview data provided insight into the students' procedural reasoning, limited justifications, and expressed uncertainties, thereby substantiating the praxeological interpretation of fragile technologies and absent theoretical norms. Selected excerpts from the interview are presented in Table 6.

The praxeological analysis of Student 3 reveals stable procedural techniques accompanied by fragile epistemic justification. In Problems 1 and 2, the student successfully expanded expressions and regrouped terms to obtain correct results, indicating fluency at the level of technique (τ). However, the accompanying technologies were limited to procedural explanations ("expanding" and "adding") and made no explicit reference to algebraic properties as general principles for justification. As a result, the underlying theoretical norms (Θ) were only partially activated and remained implicit. More pronounced epistemological obstacles emerged in tasks requiring relational reasoning and contextual consistency. In the equality task, the student simplified only one side of the equation and explicitly expressed uncertainty about how to determine whether the two sides were equivalent, indicating the absence of a relational conception of equality. Similarly, in the contextual problem, although the student attempted to model the situation symbolically, an incorrect handling of subtraction led to an inaccurate result. These difficulties suggest that while Student 3's techniques

Table 6
Student 3 interview

Problem	Interviewer (I)	Student (S)
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	I: Can you explain how you solved this expression?	S: I multiplied each term inside the parentheses, then wrote $\frac{9x}{2}+\frac{6y}{2}+\frac{5x}{2}-\frac{4y}{2}$. After that, I combined like terms and got 7x + y.
Prove that $2(y+3)+y=3y+6$	I: How did you prove this identity?	S: I expanded it: 2(y + 3) = 2y + 6. Then I added yyy, so it became 2y + 6 + y. After regrouping, I got (2 + 1)y + 6 = 3y + 6.
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	I: What did you do when solving this equality?	S: I simplified the left side only, so s + s + s + s + s + 4 = 5s + 4. I didn't continue with the right side because I wasn't sure how to check if they are the same.
Determine the number of novels and textbooks remaining on the main shelf after transfer/return.	I: Can you explain what you wrote for the book problem?	S: I started with 5x+3y. Then two novels and one textbook were moved, so I wrote −2x − y. Then the two novels were returned, so I added +2x. My final result was 5x + 4y.

Table 7
Praxeological analysis of students' 3 responses

Task	Technique (τ): What the student did	Technology (θ): Student's justification	Theory (Θ): Expected but absent/fragile
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Expanded each fraction into the parentheses and combined like terms to obtain (7x+y).	"I multiplied each term inside the brackets and then added the same terms."	Distributive property and linear structure of expressions as general rules (implicitly used, not articulated).
Prove that $2(y+3)+y=3y+6$	Not attempted.	No justification provided; student expressed uncertainty.	Algebraic identities and use of properties as tools for proof (absent).
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	Substituted (s=5), expanded numerically, and compared results.	"I thought (s) could be a number, so I tried 5."	Variable as a generalized quantity and relational meaning of equality (absent).
Contextual problem (books)	Modeled the situation symbolically and simplified to (5x+2y).	"I subtracted what was moved and added what was returned."	Algebraic modeling of contextual situations using symbols (functioning).

function effectively in routine symbolic manipulation, they are not consistently supported by robust technologies and theoretical structures when tasks demand justification or transfer across contexts. This disconnection between technique, justification, and theory characterizes the epistemological obstacles identified in Student 3's praxeology. To capture these patterns systematically, Student 4's work was reconstructed into tasks, techniques, technologies, and theories. The details of this praxeological analysis are presented in Table 7.

The praxeological analysis of Student 4's work reveals stable procedural techniques in routine manipulation, alongside significant epistemological obstacles in tasks that require justification and structural reasoning. In the simplification task (Problem 1), the student expanded expressions and combined like terms correctly, indicating fluency at the level of technique (τ). However, the accompanying explanation remained procedural and did not explicitly articulate algebraic properties as general principles, suggesting limited technological support (θ) and only implicit activation of theoretical norms (Θ). More pronounced epistemological obstacles emerged in proof-related tasks. In Problem 2, the student did not attempt to prove the identity $2(y+3)+y=3y+6$, indicating uncertainty in using algebraic properties as tools for justification. In the equality task (Problem 3),

Table 8
Student 4 interview

Problem	Interviewer (I)	Student (S)
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Can you explain how you solved this expression?	I multiplied each term inside the parentheses. So, $\frac{3}{2}\times 3x=\frac{9}{2}x$, $\frac{3}{2}\times 2y=3y$, $\frac{1}{2}\times 5x=\frac{5}{2}x$, and $\frac{1}{2}x(-4y)=-2y$, After that, I combined the terms into $7x+y$.
Prove that $2(y+3)+y=3y+6$	How did you solve this identity?	I didn't try it. I was not sure how to begin, so I left it blank.
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	What did you do when solving this equality?	I thought sss could be a number, so I tried using $s=5$. Then I wrote $2(5+2)+2s=25+25+4$. I wasn't sure how to make both sides the same.
Determine the number of novels and textbooks remaining on the main shelf after transfer/return.	Can you explain what you wrote for the book problem?	I started with $5x+3y$. Then two novels and one textbook were moved, so I subtracted $-2x-y$. Then the two novels were returned, so I added $+2x$. My final result was $5x+2y$.

the student substituted the variable with a fixed number, revealing a fragile conception of variables as generalized quantities and an absence of the relational meaning of equality as a theoretical norm. By contrast, in the contextual problem (Problem 4), the student successfully modeled the situation symbolically and obtained the correct result, indicating coherence between technique and context. Overall, Student 4's praxeology is characterized by effective techniques in routine and contextual tasks, but epistemological obstacles arise from weak or absent theoretical structures in proof-based and relational-reasoning contexts. To further elucidate these obstacles, a follow-up interview was conducted. The interview data provided insight into the students' procedural reasoning, variable interpretation, and expressed uncertainties, thereby substantiating the praxeological interpretation of fragile technologies and the absence of theoretical norms. Selected excerpts from the interview are presented in Table 8.

The interview data provide further insight into the praxeological structure underlying Student 4's written responses. In the simplification task (Problem 1), the student articulated a procedural justification focused on multiplying each term inside the parentheses and combining like terms. This explanation indicates stable techniques (τ) supported by a procedural form of technology (θ), while the underlying algebraic properties remained implicit rather than explicitly articulated as general principles (Θ). More pronounced epistemological obstacles emerged in tasks requiring justification and structural reasoning. In Problem 2, the student did not attempt to prove the identity $2(y+3)+y=3y+6$ and explicitly expressed uncertainty about how to begin, indicating that algebraic properties were not internalized as tools for justification. In the equality task (Problem 3), the student substituted a variable with a fixed number, reflecting a fragile conception of variables as generalized quantities and an absence of equality's relational meaning as a theoretical norm. By contrast, in the contextual problem (Problem 4), the student successfully modeled the situation symbolically and obtained the correct result, demonstrating coherence between technique and context. Overall, the interview corroborates the praxeological analysis by showing that Student 4's epistemological obstacles do not stem from a lack of procedural skill, but from weak or absent connections between techniques, justifications, and underlying theoretical structures. Following this analysis, Student 5's responses were examined to further explore patterns of procedural fluency and epistemological obstacles across cases as presented in Table 9.

The praxeological analysis of Student 5's work reveals a relatively articulated epistemic structure that nonetheless remains fragile across tasks. In Problems 1 and 2, the student demonstrated stable procedural techniques (τ) supported by procedural justifications (θ), successfully simplifying expressions and proving the identity $2(y+3)+y=3y+6$. In these

Table 9
Praxeological analysis of students' 5 responses

Task	Technique (τ): What the student did	Technology (θ): Student's justification	Theory (Θ): Expected, emerging, or fragile
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Expanded each fraction into the parentheses and combined like terms to obtain (7x+y).	"I multiplied each fraction into the brackets and then combined the same terms."	Distributive property and linear structure of expressions (implicitly used, not formalized).
Prove that $2(y+3)+y=3y+6$	Expanded (2(y+3)), added (y), regrouped terms, and rewrote as (3y+6).	"I expanded it and regrouped the terms to show they are the same."	Algebraic properties as tools for justification in identity proofs (emerging).
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	Simplified both sides separately to (5s+4) and (4s+4) without reconciling equivalence.	"I simplified each side, but I did not know how to make them equal."	Relational meaning of equality as equivalence between expressions (absent or fragile).
Contextual problem (books)	Modeled the situation symbolically but misapplied subtraction, yielding (5x+4y).	"I subtracted what was moved and added what was returned."	Consistent algebraic modeling of contextual situations (fragile).

Table 10
Student 5 interview

Problem	Interviewer (I)	Student (S)
Simplify the algebraic expression. $\frac{3}{2}(3x+2y)+\frac{1}{2}(5x-4y)$	Can you explain how you solved this expression?	I multiplied each term inside the parentheses. So, $\frac{3}{2}\times 3x=\frac{9}{2}x$, $\frac{3}{2}\times 2y=3y$, $\frac{1}{2}\times 5x=\frac{5}{2}x$, and $\frac{1}{2}x(-4y)=-2y$, After that, I combined the terms into $7x+y$.
Prove that $2(y+3)+y=3y+6$	How did you prove this identity?	S: I expanded $2(y+3)=2y+6$. Then I added yyy, so it became $2y+6+y$. After regrouping, it was $(2+1)y+6=3y+6$. That's why they are equal.
Prove equality $s+s+s+s+s+4=2(s+2)+2s$	What did you do when solving this equality?	S: I simplified the left side to $5s+4$, and the right side to $4s+4$. They didn't match, so I just wrote both results without showing they are equal.
Determine the number of novels and textbooks remaining on the main shelf after transfer/return.	Can you explain what you wrote for the book problem?	S: I started with $5x+3y$. Then I subtracted 2x and y because two novels and one textbook were moved. After that, I added $+2x$ because the novels were returned. My result was $5x+4y$.

contexts, algebraic properties appeared to function as emerging tools for justification, indicating partial activation of the underlying theoretical norms (Θ). However, this theoretical activation was not consistently mobilized. In the equality task, the student simplified both sides of the expression independently but did not attempt to establish equivalence, revealing a limited relational conception of equality. Similarly, in the contextual problem, although the student represented the situation symbolically, an error in handling subtraction led to an incorrect result. These difficulties suggest that while Student 5 possesses strong procedural skills and emerging theoretical awareness in routine tasks, epistemological obstacles arise when coordination between techniques, justifications, and theoretical structures is required across contexts. Thus, Student 5's praxeology illustrates how partial theoretical activation may coexist with persistent epistemological obstacles in algebraic reasoning. To further examine the student's reasoning and justifications, a follow-up interview was conducted. The interview data provide additional insight into how algebraic techniques were explained, how properties were invoked, and where epistemological obstacles became apparent. Selected excerpts from the interview are presented in Table 10.

The interview with Student 5 provides important insights into the relationship between procedural techniques and epistemic justification in algebraic reasoning. In Problem 1, the student articulated a clear sequence of operations, describing how each fraction was expanded distributively and how like terms were subsequently combined. This explanation reflects stable procedural techniques (τ) supported by operational justifications (θ), indicating fluency in routine symbolic manipulation. In Problem 2, the student explained the proof of the identity $2(y+3)+y=3y+6$by expanding, adding, and regrouping terms. Although algebraic properties such as distributivity and regrouping were effectively used, these properties functioned primarily as procedural resources rather than as explicitly theorized principles. This suggests an emerging but localized activation of the underlying theoretical norms (Θ), limited to familiar identity tasks. However, in Problem 3, the student simplified both sides of the equation independently without establishing equivalence. The student's explanation indicates that equality was interpreted as a comparison between two final expressions rather than as a relational statement that requires justification. This reveals an epistemological obstacle in coordinating techniques and justifications within a relational conception of equality. Similarly, in the contextual problem, although the student constructed an algebraic representation of the situation, an error in handling subtraction led to an incorrect result. This suggests difficulty in consistently transferring symbolic procedures to applied contexts. Overall, the interview corroborates the praxeological analysis by showing that Student 5 possesses strong procedural competence and partial theoretical awareness but encounters epistemological obstacles when coordinating techniques, justifications, and theoretical structures across different types of algebraic tasks.

DISCUSSION

This study examined students' algebraic activity through the praxeological lens of the Anthropological Theory of the Didactic (ATD), distinguishing tasks (T), techniques (τ), technologies (θ), and theories (Θ) across four types of algebraic problems: routine simplification, identity proof, equality between forms, and contextual modeling. While praxeological analysis has been widely applied to textbooks, instructional practices, and curriculum design (Putra & Aljarrah, 2021; Dewi & Juandi, 2025; Llanos & Otero, 2024), fewer studies have employed it to systematically investigate students' discourse as evidence for the presence or absence of technological and theoretical justification. The findings of this study reveal a consistent pattern in which procedural competence coexists with epistemological fragility, particularly in tasks requiring justification, relational reasoning, and generality.

Across students, routine simplification tasks exhibited strong procedural fluency. Most participants successfully expanded expressions and combined like terms to reach the correct simplified form, indicating that the required techniques (τ) were well established. However, students' explanations were predominantly procedural narratives rather than explicit justifications. Statements such as "I multiplied each term inside the brackets" describe what was done but do not articulate why the step is mathematically valid. In praxeological terms, such utterances do not fully function as technologies (θ), because they fail to invoke the underlying theoretical norms (Θ), such as the distributive property as a general algebraic law. This finding suggests that mastery of technique can mask an absence of theoretical grounding, a phenomenon that may remain invisible when analysis focuses solely on final answers.

The epistemological gap becomes more pronounced in identity and equality tasks. Several students either avoided attempting the identity proof or limited their work to simplifying one side of the equation. Even when both sides were simplified, students often failed to establish their equivalence explicitly. This pattern indicates that the equals sign is often interpreted operationally—as an instruction to calculate, rather than relationally, as a statement asserting equivalence between two expressions. The absence of explicit reference to properties such as commutativity, associativity, or distributivity as justificatory tools reflects a missing Θ concerning equality as a relation preserved under transformation. These findings align with Bosch's (2015) observation that epistemological obstacles in school algebra often arise when students are required to move from procedural manipulation to structural reasoning.

Interestingly, performance on contextual problems reveals a different configuration. Some students who struggled with proof and formal equality were nevertheless able to model contextual situations appropriately and carry out relevant calculations. This suggests that contextual sense-making and formal symbolic reasoning may develop as partially independent praxeologies. Prior studies have similarly reported that students can succeed in word-problem modeling despite difficulties with formal algebraic structures (Ningrum et al., 2019; Powell & Fuchs, 2014). From a cognitive perspective, Nathan et al. (1992) argue that contextual problems engage semantic and situational reasoning processes that differ from those required for formal symbolic manipulation. In the present study, however, contextual success was often accompanied by sign errors or inconsistent operations, indicating that semantic understanding alone is insufficient without stable theoretical norms governing algebraic manipulation.

Three interrelated explanations account for these patterns. First, classroom practices may emphasize procedural fluency over justificatory discourse, providing limited opportunities for students to articulate why algebraic steps are valid. This imbalance reflects the longstanding tension between procedural and conceptual knowledge in algebra (Hiebert & Lefevre, 2013; Kieran, 2013) and is consistent with findings that justification does not emerge spontaneously without explicit instructional support (Simon & Blume, 1996; Lannin, 2005). Second, students' conception of variables often collapses into a specific numerical value, undermining the generality required for algebraic proof and identity reasoning. Such misconceptions have been widely documented as barriers to algebraic generalization (Martinez & Castro Superfine, 2012; Žanko et al., 2019). Third, the semantic load of contextual problems may overload working memory, increasing the likelihood of sign errors even when students possess relevant procedural skills (Kieran, 2013; Wladis et al., 2019).

From a didactical design perspective inspired by DDR principles, these findings carry important implications. Tasks that foreground relational equality, requiring symmetric transformations on both sides of an equation, are needed to counter the operational interpretation of the equals sign (Harbour et al., 2016; Jones et al., 2012). Instruction should also explicitly surface technologies (θ) by prompting students to name the properties that justify their actions and to reflect on the validity of each step (Ayala-Altamirano & Molina, 2021). Furthermore, variable-as-general-number conceptions can be strengthened through tasks that contrast "true-for-all" and "true-for-some" statements, helping students distinguish algebraic generality from arithmetic instantiation (Malisani & Spagnolo, 2009). Finally, contextual transfer can be supported by representational tools that explicitly track quantities and operations, reducing cognitive load and improving sign consistency (Booth et al., 2015).

A key strength of the praxeological approach lies in its capacity to analytically separate technique from technology, thereby making visible situations in which students can perform algebraic steps (τ) without accessing the theoretical norms (Θ) that would legitimate those steps. By restricting the identification of Θ to what can reasonably be inferred from student discourse, this study avoids overattributing theoretical understanding and offers a more conservative, discourse-grounded account of epistemological obstacles (Winsløw, 2007; Zakiah et al., 2025). Nevertheless, the study is limited by its small sample size and task set, which constrains generalisability (Marek & Laumann, 2025). In addition, interview prompts may have shaped the forms of justification students provided, introducing potential interpretive bias (Van Dooren, 2025; Lammers et al., 2013). The absence of systematic analysis of classroom interaction further limits ecological triangulation (Abrahamson & Sánchez-García, 2016; Ma & Norwich, 2007). The praxeological analyses indicate that procedural competence in algebra can coexist with significant epistemological obstacles in proof, equality reasoning, and contextual transfer. Addressing these obstacles points to the need for instructional designs that foreground relational equality, explicit justification, and stable conceptions of variables, thereby supporting students' progression from procedural fluency toward structural and theoretical understanding.

Table 11
Cross-case praxeological synthesis of students' algebraic activity

Task Type	Technique (τ): What the student did	Technology (θ): What the student said to justify it	Theory (Θ): Expected but absent theoretical norm
Routine simplification	Expanded brackets and combined like terms to reach a simplified form (e.g., 7x + y)	Described procedural steps ("I multiplied each term, then added them")	Distributive law and linearity as general algebraic properties
Identity proof	Expanded one side of the identity or avoided the task entirely	Asserted sameness without justification or gave no explanation	Equality as an identity holding for all values; laws of operation as justificatory tools
Equality between forms	Simplified only one side or simplified both sides without reconciling them	Treated the equals sign as a signal to compute	Equality as a symmetric and relational statement is preserved under transformation
Contextual algebraic problem	Modeled the situation symbolically and performed calculations	Used contextual cues ("moved," "returned") as informal justification	Consistent sign conventions grounded in quantitative change and invariance
Variable handling across tasks	Substituted a specific number for the variable	Implicitly treated the variable as a fixed value	Variable-as-general-number and generality of algebraic statements

This study contributes to the praxeology literature within ATD by demonstrating that epistemological obstacles in algebra are not limited to the absence of techniques or technologies, but may also take the form of technologies that remain epistemically weak, that is, student explanations that narrate procedures without invoking the theoretical norms that justify them. While previous praxeological studies have primarily examined teachers' praxeologies and agency in instructional design (e.g., Mensah et al., 2024; Mensah, 2025), the present study extends this line of work by providing an empirically grounded account of students' praxeological configurations, highlighting systematic misalignments between τ, θ, and Θ. In particular, the findings reveal a persistent tension between contextual modeling praxeologies and formal algebraic praxeologies, suggesting that success in one domain does not guarantee access to the theoretical structures required in the other. Methodologically, the study advances ATD-based analysis by adopting a conservative inferential stance toward Θ, coding theoretical elements only when supported by student discourse rather than analyst assumptions. This approach strengthens the explanatory power of praxeological analysis and clarifies how epistemological obstacles can be identified with greater precision in studies of student mathematical activity. The praxeological patterns summarised in Table 11 demonstrate how epistemological obstacles in students' algebraic activity can be systematically located in the misalignment between techniques (τ), technologies (θ), and theories (Θ), thereby strengthening the explanatory potential of ATD in empirical studies of student learning.

CONCLUSION

This study addressed its objective of examining students' epistemological obstacles in algebraic operations by applying a praxeological framework that distinguishes techniques (τ), technologies (θ), and theories (Θ). The findings show that students' difficulties are not primarily rooted in a lack of procedural skill, but in persistent misalignments between what students do and how they justify their actions. In particular, the analysis reveals that procedural fluency in routine simplification can coexist with fragile or absent justificatory discourse, especially in tasks involving proof, relational interpretations of equality, variable generalization, and contextual transfer. These results clarify the nature of students' epistemological obstacles by demonstrating that the correctness of procedures alone does not guarantee access to the theoretical norms underlying algebraic reasoning. From a didactical perspective, the study points to several concrete implications for teaching and curriculum design. Algebra instruction should move beyond an emphasis on obtaining correct simplified forms toward systematically engaging students in justifying transformations, establishing equivalence relationally, and articulating the properties that legitimate algebraic steps. Teachers are encouraged to design tasks that require symmetric manipulation of both sides of an equation, explicit use of algebraic properties as warrants for reasoning, and sustained work with variables as generalized quantities rather than as fixed numerical values. In addition, contextual problem-solving should be supported through representational scaffolds that help students track quantities and maintain sign consistency, thereby strengthening the coordination between symbolic manipulation and situational meaning. Conceptually, the articulation between praxeological analysis and principles drawn from Didactical Design Research (DDR) proved valuable for mapping students' epistemological obstacles and identifying directions for potential instructional design. In this study, DDR functions as a didactic horizon rather than a comprehensive methodological framework, supporting the interpretation of praxeological findings without extending to the design of experiments. While the study's scope is limited by a small sample size, a restricted task set, and the absence of broader classroom interaction data, the results offer theoretically grounded insights with clear didactic relevance. Future research should therefore involve larger and more diverse samples, adopt longitudinal designs, and incorporate systematic analyses of classroom discourse to examine further how students' praxeological configurations develop from procedural fluency toward stable structural and theoretical understanding in algebra.

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AUTHOR'S DECLARATION

Author's Contribution	:	MGJ: Responsible for the conception of the study, development of the analytical framework, research design, data collection, data analysis, interpretation of results, and manuscript preparation. TSH: Contributed to conceptual refinement and theoretical validation, and provided a critical review of the manuscript. JJ: Contributed to methodological review, validation of the analysis, and revision of the manuscript. DR: Contributed to data interpretation and manuscript language editing.
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