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Proof in mathematics education: A bibliometric analysis using R biblioshiny and VOSviewer

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ABSTRACT

The modern mathematics curriculum internationally places proof as a central component that should be integrated from primary school to university. Empirical research on proof in mathematics education has grown rapidly in the last two decades; however, few studies have attempted to map this research landscape. This article provides a comprehensive mapping of research on proof through a bibliometric approach using the Web of Science Core Collection (WoSCC) database. Data from 157 articles published between 2016 and 2025 were analyzed using the biblioshiny R-package and VOSviewer. Results show that publication numbers fluctuated, peaking in 2024, with Dawkins et al. being the most cited and ZDM-Mathematics Education the most prolific source. Keyword analysis reveals six distinct clusters, with the USA as the leading contributor. Beyond these trends, the study identifies a significant shift toward the integration of artificial intelligence within the transition-to-proof process. These findings offer a strategic roadmap for researchers to bridge the gap between technological mediation and formal mathematical validation, emphasizing the need for instructional designs that harmonize generative tools with the rigorous construction of deductive proofs. Ultimately, this mapping highlights the urgency of ensuring that digital advancements support the authenticity of proving activities rather than merely facilitating the mechanical production of logical steps.

INTRODUCTION

The modern mathematics curriculum internationally places proof as a central component that should be integrated from elementary through to higher education. This emphasis is reflected in various curriculum documents and recommendations from professional organizations, such as the National Council of Teachers of Mathematics (NCTM), which assert that proof and reasoning are fundamental aspects of mathematics learning at all levels (Knuth, 2002a; Stylianides, 2007; Stylianou et al., 2009). The primary goal of integrating proof into the curriculum is to equip students with essential critical, analytical, and logical thinking skills, which are crucial not only for a deep understanding of mathematical concepts but also for developing transferable thinking skills applicable in everyday life (Hanna, 2000; Hemmi et al., 2013).

Traditionally, mathematical proof has been defined as a series of logical steps connecting assumptions or axioms to a conclusion (Rota, 1997). Beyond formal logic, it serves as a social argument to convince the mathematical community (CadwalladerOlsker, 2011; Hanna, 1990).

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Crucially, proof fulfills multiple pedagogical functions: verification to remove doubt about the truth of a statement, systematization to organize results into a deductive system (Villiers, 1990), and communication of mathematical knowledge, both among mathematicians and within an educational context, so that mathematical ideas can be understood and developed further (Nunokawa, 2010; Risalah & Hodiyanto, 2022; Rocha, 2019).

Despite the importance of proof, various empirical studies indicate that significant weaknesses remain in the understanding and teaching of mathematical proof. Many students struggle to grasp the structure logic and comprehensive meaning of proofs (Mejia-Ramos et al., 2012; Miyazaki et al., 2017). While research suggests studnets often recognize that examples alone are insufficient for verification, they frequently lack the ability to transition from empirical to formal deductive construction (Stylianides, 2007; Stylianides & Stylianides, 2009). Furthermore, the understanding of proof among both teachers and prospective teachers is often limited, which affects the effectiveness of proof instruction in the classroom (Buchbinder & McCrone, 2020; Knuth, 2002b, 2002c; Stylianides & Ball, 2008). Although research-based interventions have been implemented, fundamental challenges persist (Stylianides & Stylianides, 2017).

The challenges in teaching mathematical proof are highly complex. Teachers face limitations in their own knowledge of proof, a lack of experience in guiding the proving process, and difficulties in designing tasks that encourage students to construct proofs independently (Mukuka & Tatira, 2025; Stylianides & Stylianides, 2009). Additionally, a compartmentalized curriculum and pressure for high test scores often lead to proof being taught formally, without emphasizing the underlying meaning and thought processes (Bieda, 2010; Thompson et al., 2012; Varghese, 2017). Further challenges arise from the differing perceptions between teachers and students regarding the purpose and value of proof in mathematics (Dawkins & Weber, 2017; Stylianides et al., 2016).

Empirical research on mathematical proof within the field of mathematics education has experienced rapid growth over the last two decades. Analyzing these research trends is crucial for identifying under-explored areas within the global research framework, ensuring that future investigations are strategically positioned to fill existing knowledge gaps. While previous studies have mapped research in specific contexts—such as Na and Choi (2023) in Korea—or addressed proof as a subset of broader educational themes (Julius et al., 2021; Kartika et al., 2023), they have not specifically examined the nuanced global trends in mathematical proof research. In this regard, bibliometric analysis offers a unique comparative advantage; unlike traditional qualitative reviews, it provides an objective, large-scale mapping of the thematic landscape by utilizing mathematical and statistical methods to track publication patterns and map knowledge domains (Gan et al., 2022; Lazarides et al., 2025). This data-driven approach facilitates a clearer understanding of the field's evolution and its projected future directions.

This article aims to provide a comprehensive mapping of research on proof in mathematics education through a bibliometric approach. The study addresses several key research questions that serve as the foundational pillars for understanding the field's intellectual structure:

1. What are the publication quantity trends related to proof in mathematics education between 2016 and 2025?
2. Which article has achieved the highest citation impact during this period?
3. Which journal stands out as the most productive in this domain?
4. Which country is the most productive contributor to this topic?
5. What are the most important research topics and potential research gap opportunities?

These questions are considered "key" because they systematically map productivity, influence, and thematic evolution, thereby providing a strategic roadmap for researchers to position their studies and for policymakers to develop evidence-based curricula. By revealing emerging topics and under-explored fields, these findings serve as a "crucial reference" for the development of more effective and relevant mathematics proof education (Hanna & Knipping, 2020).

Table 1
Keyword String for Web of Science Database Search

Database	String
Web of Science	TS=('Proof' OR 'Proving' OR 'Prove') AND TS=('Mathematics Education') and 2025 or 2024 or 2023 or 2022 or 2021 or 2020 or 2019 or 2018 or 2017 or 2016 (Publication Years) and Article (Document Types) and English (Languages)

Following this introduction, which establishes the context and pedagogical necessity of proof research, the article details the bibliometric methodology and the specific screening procedures employed to ensure data quality. The results are then presented in detail, followed by a critical discussion of the findings' implications and specific recommendations for future research. The article concludes with a summary and a reflection on its contribution to the global development of proof research in mathematics education.

METHODS

This study utilizes a bibliometric method, a quantitative approach that applies mathematical and statistical techniques to scientific literature to map the intellectual structure and evolution of research trends within a specific field. This method allows researchers to track publication trends, highlight influential authors, map their collaboration networks, and reveal thematic landscapes, all of which contribute to clarifying the global research framework (Gan et al., 2022; Lazarides et al., 2025; McAllister et al., 2022; Sun et al., 2025). The collected articles were analyzed using the Biblioshiny package from R-bibliometrix and VOSviewer version 1.6.20. Biblioshiny operates using R-based algorithms to identify the most cited authors, countries, and journals, while VOSviewer employs distance-based visualization techniques to analyze and map the connections (clusters) between authors and keywords.

The data were retrieved from the Web of Science Core Collection (WoSCC) by Clarivate, a database esteemed for its rigorous bibliographic indexing and comprehensive coverage (Durán-Sánchez et al., 2017; Şanlı, 2022). Using the search string detailed in Table 1, an initial corpus of 474 articles was retrieved on August 26, 2025. To ensure data integrity and relevance, these articles underwent a systematic screening of titles and abstracts. Inclusion criteria were strictly applied to ensure the selected articles aligned with the core research themes, including related constructs such as mathematical argumentation, proof, and proving.

To enhance methodological rigor, this screening process was conducted independently by two reviewers. Any discrepancies regarding article eligibility were resolved through a formal consensus procedure; discussions were maintained until full consensus was attained, thereby ensuring robust inter-rater reliability. This systematic refinement resulted in a final selection of 157 relevant articles for bibliometric analysis.

FINDINGS

The dataset exhibits a compounded annual growth rate (CAGR) of 6.17% and a mean document age of 3.73 years, signaling a nascent and dynamic research landscape. As detailed in Table 2, the authorship structure comprise 369 unique researchers, categorized based on on their participation in the corpus. Spesifically, "Authors of single-authored documents" ($n = 28$) identifies individuals who published independently, while "Authors of multi-authored documents" ($n = 341$) represents those engaged in collaborative works.

This distribution underscores a dominant trend toward scholarly synergy, with a Degree of Collaboration (DC) of 0.81. The disparity between the number of single-authored documents (30) and the respective authors (28) indicates that a small subset of researchers maintains consistent independent productivity. Overall, the Collaboration Index (CI) of 2.69 suggests that research in this

Table 2
Main information about the data

Description	Results
Timespan	2016 - 2025
Sources (Journals)	65
Documents (Articles)	157
Single-authored documents	30
Multi-authored documents	127
Publication Annual Growth Rate	6.17%
Document Average Age	3.73
Average citations per document	5.758
References	6,067
Author's Keywords (DE)	600
Authors	369
Authors of single-authored documents	28
Authors of multi-authored documents	341

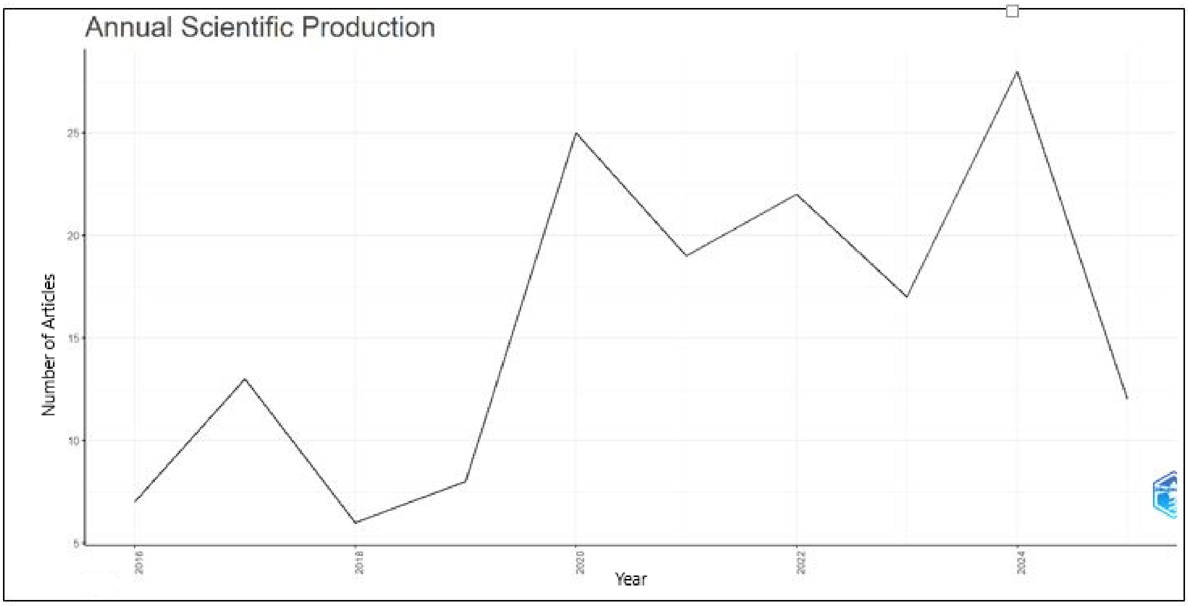

Figure 1. Annual Scientific Production

field is predominantly driven by cohesive teams rather than isolated investigators, facilitating the rapid thematic evolution and integration of novel theoretical frameworks observed in the collection.

As illustrated in Figure 1, the number of articles published annually exhibits a fluctuating yet overall upward trajectory. While the field experienced period-specific volatility between 2016 and 2019, a substantial surge in productivity is observed from 2020 onwards, reaching its peak in 2024 with 28 publications. The observed decline in 2025 is attributed to the incomplete data collection cycle for that year (truncated in August). This general growth pattern, reflected in the 6.17 CAGR, underscores the increasing scholarly attention toward mathematical proof over the last decade.

The scholarly impact and prominence of research within this corpus are detailed in Table 3, which presents the ten most cited articles. Leading the collection, Dawkins and Weber (2017) and Kosiol et al. (2019) exhibit the highest citation counts, with 46 and 38 total citations respectively. These works, along with those Dutilh Novaes (2016) and Fukawa-Connelly et al. (2017), represent foundational contributions that have shaped contemporary discourse on mathematical proof.

Table 3
Top ten most cited articles on proof in mathematics education

Article	DOI	TC	NTC
Dawkins & Weber (2017); Educational Studies in Mathematics	10.1007/s10649-016-9740-5	46	3.71
Kosiol et al. (2019); International Journal of Science and Mathematics Education	10.1007/s10763-018-9925-8	38	3.45
Dutilh Novaes (2016); Philosophical Studies	10.1007/s11098-016-0667-6	35	2.58
Fukawa-Connelly et al. (2017); Journal for Research in Mathematics Education	10.5951/jresematheduc.48.5.0567	30	2.42
Zazkis et al. (2016); Educational Studies in Mathematics	10.1007/s10649-016-9698-3	26	1.92
Melhuish et al. (2020); Journal of Mathematics Teacher Education	10.1007/s10857-018-9408-4	25	2.69
Buchbinder & McCrone (2020); Journal of Mathematical Behavior	10.1016/j.jmathb.2020.100779	24	2.59
Miyazaki et al. (2017); Educational Studies in Mathematics	10.1007/s10649-016-9720-9	23	1.86
Delgado-Rebolledo & Zakaryan (2020); International Journal of Science and Mathematics Education	10.1007/s10763-019-09977-0	21	2.26
David & Zazkis, (2020); International Journal of Mathematical Education in Science and Technology	10.1080/0020739X.2019.1574362	19	2.05

Note: TC = Total Citations; NTC = Normalized Total Citations

The Normalized Total Citations (NTC) further highlight the relative impact of these studies regardless of their publication year. For instance, while Buchbinder and McCrone (2020) and Delgado-Rebolledo and Zakaryan (2020) are more recent, their hight NTC scores (> 2.0). indicate rapid uptake and significant influence within the mathematics education community. Collectively, these highly cited works originate from diverse high-impact journals such as Educational Studies in Mathematics and the Journal for Research in Mathematics Education, reflecting a robust and geographically dispersed intellectual network.

The distribution of the 157 articles across various scholarly outlets provides insight into the primary academic platforms for research on mathematical proof. As detailed in Table 4, the dataset is distributed across 65 journals, with a high concentration of papers in specialized mathematics education periodicals. *ZDM – Mathematics Education* emerges as the most prolific source, contributing 19 articles (12.1% of the corpus), followed by the *International Journal of Mathematical Education in Science and Technology* (IJMEST) (8.92%) and *Educational Studies in Mathematics* (8.28%).

The metrics in Table 4 further underscore the high quality and academic rigor of the field, as the top four sources are predominantly ranked within the Q1 and Q2 quartiles of the Web of Science (WoS). Spesifically, journals such as *International Journal of Science and Mathematics Education* (WoS IF 2.1) and *Education Sciences* (WoS IF 2.6) demonstrate the interdisciplinary reach of proof-related research, extending into broader categories like “Education & Education Research” and “Scientific Disciplines”. Furthermore, the *m-index* values, led by ZDM (0.875) and IJMEST (0.833), indicate sustained impact over time, reflecting these journals’ roles as central hubs for the evolution of theoretical and empirical frameworks in mathematical proof.

Prior to examining the conceptual evolution of the field, it is essential to establish the geographical distribution of the literature to provide context for the subsequent thematic analysis. Table 5 presents the top 15 countries contributing to the research on mathematical proof,

Table 4
Most relevant sources

Source	Total Articles (%)	Impact Factor (2024)	m_index	Subject Categories
ZDM-Mathematics Education	19 (12.1%)	WoS IF 2.4 (Q1)	0.875	Education & Educational Research
International Journal of Mathematical Education in Science and Technology	14 (8.92%)	WoS IF 0.6 (Q3)	0.833	Education & Educational Research
Educational Studies in Mathematics	13 (8.28%)	WoS IF 1.9 (Q2)	0.7	Education & Educational Research
International Journal of Science and Mathematics Education	13 (8.28%)	WoS IF 2.1 (Q1)	0.8	Education & Educational Research
Journal of Mathematical Behavior	10 (6.37%)	WoS IF 1.7 (Q2)	0.667	Education & Educational Research
Mathematics	6 (3.82%)	WoS IF 2.2 (Q1)	0.667	Mathematics
Frontiers in Education	4 (2.55%)	WoS IF 1.9 (Q2)	0.167	Education & Educational Research
International Electronic Journal of Mathematics Education	4 (2.55%)	WoS IF 0.6 (Q3)	0.167	Education & Educational Research; Education; Scientific Disciplines
Journal of Humanistic Mathematics	4 (2.55%)	WoS IF 0.3 (Q4)	0.167	History & Philosophy of Science
Education Sciences	3 (1.91%)	WoS IF 2.6 (Q1)	0.167	Education & Educational Research

Table 5
Top 15 of publications by countries

Country	Freq	%	**Country**	Freq	%
USA	137	28,13%	Japan	14	2,87%
Germany	37	7,60%	Israel	12	2,46%
UK	35	7,19%	Norway	12	2,46%
Spain	34	6,98%	Sweden	10	2,05%
Turkey	22	4,52%	Thailand	9	1,85%
China	19	3,90%	Denmark	8	1,64%
France	16	3,29%	Netherlands	8	1,64%
Italy	16	3,29%			

determined by the primary affiliations of the authors. The data reveals that the United States remains the most dominant contributor, accounting for 137 publications (28.13%). This volume is nearly four times that of the next closest nation, Germany, and represents almost one-third of the total dataset. Such a significant concentration suggests not only a high level of institutional commitment from US-based scholars but also the existence of an extensive and mature collaborative network within the region.

Beyond the United States, a robust research ecosystem is evident across Europe, led by Germany (37 publications; 7.60%), the United Kingdom (35 publications; 7.19%), and Spain (34 publications; 6.98%). These figures underscore the consistent productivity and high-quality research output within the European continent. Furthermore, while their absolute publication counts remain lower than those of Western nations, Asian countries such as China (19 publications; 3.90%) and Japan (14 publications; 2.87%) are emerging as increasingly influential contributors. This shift indicates Asia's growing role in the global research landscape and suggests that the thematic clusters

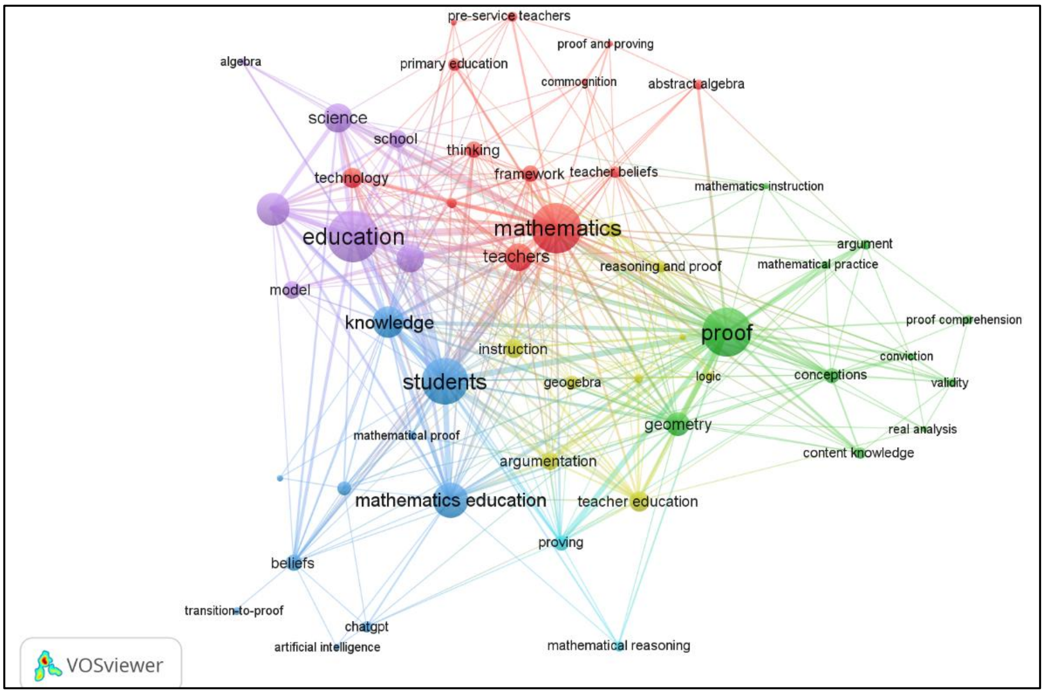

Figure 2. Network keywords analysis

identified in the subsequent co-occurrence analysis are likely influenced by the diverse pedagogical priorities of these various regions.

To identify the core conceptual framework and intellectual pillars of research on mathematical proof, a keyword co-occurrence analysis was conducted using the association strength algorithm within the VOSviewer environment. Figure 2 illustrates the resulting network of 52 keywords that met the minimum threshold of three occurrences. This threshold was strictly applied to ensure that the thematic clusters are based on recurring scholarly patterns rather than isolated instances, thereby providing a statistically significant representation of the field's intellectual structure. In this distance-based visualization, the proximity between two nodes reflects the intensity of their thematic relationship; keywords located closer to one another appear more frequently in the same bibliographic records, indicating a shared research focus.

The network reveals six distinct thematic clusters, labeled based on their highest-degree nodes and their specific pedagogical contexts to ensure the categorization remains theoretically grounded:

1. Cluster 1 (Red): Centered on "mathematics" and "teachers," this cluster explores the systemic side of proof instruction. It includes nodes such as "framework," "teacher beliefs," "pre-service teachers," and "primary/secondary education," indicating research focused on the institutional and foundational preparation for teaching proof.
2. Cluster 2 (Green): Anchored by the node "proof," this cluster focuses on the formal aspects of mathematical reasoning. It connects terms like "logic," "conceptions," "conviction," "validity," and "proof comprehension," reflecting the literature's concern with the rigorousness and internal structure of proof construction.
3. Cluster 3 (Blue): Centered on "students" and "knowledge," this cluster emphasizes cognitive shifts in the digital age. Crucially, it includes the node "transition-to-proof" alongside emerging technologies like "artificial intelligence" and "ChatGPT," representing the frontier of how AI influences undergraduate students' mathematical beliefs and their move toward formal deduction.
4. Cluster 4 (Yellow): This cluster focuses on "mathematical knowledge" and "instruction" through tools like "geogebra" and "argumentation." It highlights how dynamic software and logical heuristics serve as a bridge for developing geometric reasoning.

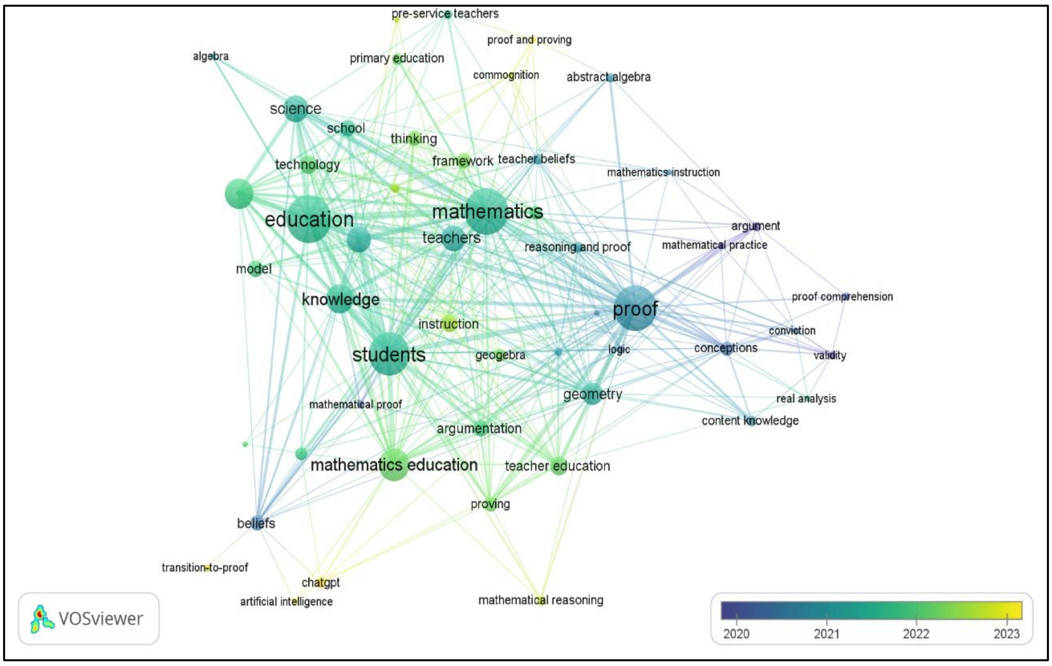

Figure 3. Overlay keywords analysis

5. Cluster 5 (Purple): Anchored by "education," this cluster explores broader models of learning, connecting "achievement" and "performance" with school-level contexts.
6. Cluster 6 (Sian): A specialized niche focusing specifically on the activities of "proving" and "mathematical reasoning" as distinct processes

The temporal progression of the field is captured in the overlay visualization in Figure 3. The color gradient signifies the average publication year (AVP) of the keywords, where dark blue represents established topics and bright yellow indicates emerging research fronts. The visualization indicates that future research trajectories may increasingly involve the integration of generative AI within the transition-to-proof process. Regarding the temporal scale, while the dataset includes publications through August 2025, the timeline in Figure 3 scales primarily to late 2023. This is a deliberate methodological outcome of the VOSviewer algorithm, which calculates the AVP based on the cumulative density of a keyword's appearance across the entire longitudinal period. Consequently, the influence of 2024 and 2025 papers is inherently integrated into the statistical mean; their impact is manifested in the significant shift of keywords toward the bright yellow "2023+" pole, even if the absolute average has not yet moved the legend to a discrete 2025 label.

DISCUSSION

As evidenced by the results presented above, research on mathematical proof has entered a stable development phase, characterized by a Compounded Annual Growth Rate (CAGR) of 6.17%. This upward trajectory, peaking in 2024 with 28 articles, reflects a consolidation of the field in response to the challenges of modern mathematics education. The dominance of multi-authored documents (127 articles) and a Degree of Collaboration (DC) of 0.81 further demonstrate that scientific advancement in this domain relies heavily on scholarly synergy rather than isolated inquiry. This collaborative trend aligns with the findings of Henriksen (2016), which suggest that collective teams in the social and educational sciences tend to produce research with higher citation impact compared to single-authored works.

The intellectual landscape is significantly shaped by several fundamental works that have attained substantial citation counts (Table 3). Studies by Dawkins and Weber (2017) and Kosiol et al. (2019) serve as primary reference points, establishing the contemporary discourse on proof norms and motivations. Furthermore, the high Normalized Total Citation (NTC) scores observed in more recent works, such as Buchbinder and McCrone (2020), indicate a rapid information uptake by the academic community. This concentration of high-impact research in specialized journals like *ZDM – Mathematics Education* and I*JMEST* (Table 4) underscores that quality standards in proof research are increasingly defined by platforms with robust international reputations and high *m-index*.

The intellectual structure mapped through keyword co-occurrence (Figure 2) reveals a thematic distribution divided into six primary clusters. While institutional preparation and formal logical structures (Clusters 1 and 2) remain the pillars of the field, the emergence of keywords in the 2023+ period (Figure 3)—such as "Artificial Intelligence," "ChatGPT," and "transition-to-proof"—signals a shift toward generative technology and digital mediation. This pattern supports the conceptual framework of Cobo et al. (2011), which posits scientific development as a progression from established core knowledge bases toward innovative thematic extensions.

Regarding geographical contributions, the significant presence of the United States (Table 5) highlights a high concentration of research infrastructure, consistent with Glänzel and Schubert (2004) observations on the tendency of international research networks to coalesce around mature institutional ecosystems. Nevertheless, the Collaboration Index (CI) of 2.69 proves that knowledge production is increasingly collective, allowing for scientific validity that transcends a single geographical boundary.

Despite these advancements, the conceptual structure analysis reveals a critical gap where research involving adaptive technology and AI (Cluster 3) remains largely disconnected from studies examining empirical learning outcomes (Cluster 5). This fragmentation reflects the persistent challenge of aligning rapid digital innovation with pedagogical effectiveness—a condition described by Stylianides and Stylianides (2017) as a crucial need to evaluate the long-term impact of proof-related instructional interventions. Consequently, these findings suggest that future research should prioritize instructional designs that harmoniously integrate technology with the cognitive aspects of proving. Robust empirical evaluations of individual learning outcomes are essential to ensure that technology-based learning media effectively enhance students' deductive reasoning in a comprehensive manner.

CONCLUSIONS

This bibliometric study mapping the research on mathematical proof from 2016 to 2025 transitions the focus from traditional logical-formalist approaches to an emerging digital-pedagogical paradigm. The field has reached a state of maturity and consolidation, as evidenced by the peak productivity in 2024 and the central role of high-impact hubs like ZDM – Mathematics Education. While the US remains the dominant contributor, the increasingly collective nature of knowledge production (DC 0.81) suggests that scientific validity now transcends geographical boundaries, providing a stable foundation for the integration of novel theoretical frameworks.

Beyond identifying trends, these findings provide a strategic roadmap for academics to position their work within high-growth areas. For instance, the identified thematic clusters allow researchers to move beyond general descriptions toward designing AI-scaffolded proving tasks that directly address the logic and comprehension gaps prevalent among pre-service teachers. Furthermore, the correlation between institutional frameworks and teacher beliefs highlights a practical need for professional development programs to integrate foundational norms, such as those established by Dawkins and Weber, with contemporary AI validation techniques.

Ultimately, this study serves as a benchmark for institutions to assess productivity while challenging the research community to bridge the critical gap between rapid digital innovation and empirical learning outcomes. Future research must prioritize instructional designs that harmoniously integrate technology with the cognitive aspects of proving. This approach is essential to ensure that generative tools foster deep deductive conviction and authentic mathematical reasoning rather than merely facilitating mechanical proof production.

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AUTHOR'S DECLARATION

Authors' contributions	The study was conceptualized and designed by R and H. R conducted the data collection and drafted the initial manuscript. Both authors performed the bibliometric analysis and revised the paper. H finalized the manuscript and template formatting. Both authors approved the final version.
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Competing interests	The authors declare no competing interests and certify that this work has not been published elsewhere.

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