

Unravelling undergraduate mathematics students' understanding of derivatives

Aniswita^{1*}, Nana Sepriyanti², Gema Hista Medika¹

¹Universitas Islam Negeri Sjech M Djamil Djambek Bukittinggi, Indonesia

²Universitas Islam Negeri Imam Bonjol, Indonesia

Citation: Aniswita, A., Sepriyanti, N., & Medika, G. H. (2024). Unravelling undergraduate mathematics students' understanding of derivatives. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 9(4), 262–272. <https://doi.org/10.23917/jramathedu.v9i4.11267>

ARTICLE HISTORY:

Received 6 Agustus 2024

Revised 9 October 2024

Accepted 20 October 2024

Published 31 October 2024

KEYWORDS:

Calculus differential
Conceptual knowledge
Calculus learning
Derivative

ABSTRACT

Derivatives are a significant concept in calculus; nonetheless, students' conceptual understanding remains inadequate. Therefore, a conceptual understanding of this subject should be a priority. This study aimed to elucidate the conceptual understanding of Mathematics Education students on the topic of Derivatives. A qualitative approach was employed. The participants in this study comprised 62 students enrolled in the Mathematics Education programme who completed the Differential Calculus course. The instrument employed was a conceptual knowledge assessment comprising three questions that examined the definitions of derivatives, derivative theorems, and derivatives of implicit functions. To investigate students' conceptual understanding, the researcher interviewed four students selected to represent each category for every topic. The employed data analysis method was qualitative data analysis as per Miles and Huberman. The findings indicated that the majority of students had not employed the correct notion. Students encounter difficulties in determining the differentiability of a function at a certain point and in applying the rules of multiplication and differentiation to implicit functions. It can be argued that students' conceptual understanding of derivatives was significantly deficient.

INTRODUCTION

Calculus is a very fundamental subject for Mathematics education students (Mahir, 2009; Funny, 2021; Aniswita et al., 2023). According to Dunham (2005), the concepts in Calculus are very important, broad and amazing and are considered the greatest discovery of Modern mathematics (Kidron, 2014). Calculus can solve various problems in various fields such as science, engineering, medicine and economics (Latorre et al., 2007; Alam, 2020; Stevens, 2021). So that the Calculus course is important to study in various study programs that make the concept of Calculus a prerequisite, especially the Mathematics Education study program (Funny, 2021; Hashemi et al., 2015; Dunham, 2005; Mahir, 2009; Rasmussen et al., 2014; Vajravelu et al., 2016). The concepts in Calculus are considered to support students' analytical abilities and bridge basic mathematics to higher-level mathematics (Dunham, 2005; Mahir, 2009).

One of the important concepts in Calculus is the concept of derivatives (Varberg et al., 2016; Hashemi et al., 2014; Kidron, 2014; Mkhathshwa, 2024). The concept of derivatives is the basis for understanding other Calculus concepts, such as the concept of Indefinite Integrals. So that mastery of the concept of derivatives is a must. In fact, this concept is still considered a difficult concept (White et al., 1996; Aniswita, 2016; Deswita et al., 2021). Quite a lot of research reveals the difficulties experienced by students in understanding the concept of derivatives. Among them, research conducted by Orton (1983) which was the first study to report students' difficulties in understanding

*Corresponding author: aniswita@uinbukittinggi.ac.id

the concept of derivatives. There is a significant misunderstanding regarding the definition of derivative and the graphical representation of derivatives. The same thing was also found by Garcia and Flores (2021) that students have difficulty interpreting derivative graphs. Added by Hashemi et al., (2014) and Denbel (2015) that the difficulty is because students focus too much on the symbolic aspect rather than the graphic aspect and connecting the two. This is also reinforced by research Ferrini-Mundy et al., (1994) who found students had difficulty connecting symbolic representation with graphical understanding. In addition, Mkhathshwa (2020) also revealed that students had difficulty using derivative rules in calculating derivatives.

This difficulty stems from students' lack of understanding, which is caused by their low level of conceptual knowledge (Mahir, 2009; Hashemi et al., 2014). Conceptual knowledge can be simply defined as knowledge about a concept. According to Findell et al. (2001), conceptual knowledge refers to an understanding of mathematical concepts, operations, and relationships. This is in line with Schneider et al. (2010), who define conceptual knowledge as abstract understanding of principles and relationships between parts of knowledge within a specific domain. This definition is further reinforced by Khashan (2014), who describes conceptual knowledge as abstract knowledge that addresses the nature of mathematical principles and the relationships among those principles. İşleyen et al., (2003) describe conceptual knowledge in mathematics as knowledge consisting of symbols and demonstrations. Similarly, Baroody et al. (2007) define conceptual knowledge as knowledge of concepts and principles, as well as the relationships between them. Conceptual knowledge is rich in relationships and forms a network of interconnected knowledge (Hiebert et al., 1986). It can be interpreted as an understanding of the relationships among concepts, definitions, and mathematical rules, and the ability to explain them (Zuya, 2017).

A lack of conceptual knowledge can hinder students' ability to transfer and generalize knowledge (Hurrell, 2021). Both of these skills are crucial in learning Calculus. Therefore, it is important to understand students' conceptual understanding of Calculus concepts, particularly the concept of derivatives. This is in line with the opinions of Zuya (2017) and Masduki et al. (2023) regarding the importance of identifying and analyzing how students understand conceptual knowledge as a basis for designing meaningful learning.

Research on students' conceptual understanding of derivatives is still very limited. Most focus on the definition, symbols, graphs, and applications of derivatives, while other important concepts remain unexplored. Therefore, it is necessary to fully uncover students' conceptual understanding of derivatives. It is hoped that this study can serve as a foundation for improving Calculus instruction, enabling students to better understand Calculus concepts, especially derivatives, and making learning more meaningful. As Schoenfeld (1995) stated, "We believe that developing conceptual understanding, not algebraic technique, should be the driving force, and we expect students to engage with mathematics."

METHODS

This study employs a qualitative research approach. According to Bogdan et al. (1992), qualitative research is a procedure that generates descriptive data in the form of spoken or written words, or observable behavior from the research subjects. This aligns with the research objective, which is to examine the conceptual understanding of Mathematics Education students at university on the topic of derivatives.

The research involved 62 students who took the Differential Calculus course. Students were given three questions designed to assess conceptual knowledge on the topic of derivatives. The questions were as follows:

1. Explain whether the function $y = |x|$ is differentiable at $x = 0$
2. Determine the second derivative of the function $y = x^2(2x + 3)$
3. Determine the derivative $\frac{dy}{dx}$ of the implicit function $x^2 + y = x^3y^2$

Students' answers were then categorized into four groups based on the accuracy of the concepts they applied in solving the problems. The categories are shown in Table 1.

To gain more detailed and in-depth insights into students' conceptual knowledge, the researchers conducted interviews with selected students. Four students were chosen to represent each category for every question, labeled with the question number and category code (e.g., 1K1, 1K2,

Table 1

Student answer categories	
Category	Description
Category_1	No answer
Category_2	Incorrect concept
Category_3	Correct concept but with mistakes
Category_4	Correct concept

Table 2

Distribution of student responses		
Category	Number of Students	Percentage
Category_1	29	15.6%
Category_2	92	49.5%
Category_3	37	19.9%
Category_4	28	15%

Table 3

Distribution of student answers on question 1		
Category	Number of Students	Percentage
Category_1	13	21%
Category_2	28	45.2%
Category_3	16	25.8%
Category_4	5	8%

etc.). The collected data were analyzed using qualitative data analysis techniques based on Miles and Huberman (2014), which consist of three stages: data reduction, data display, and conclusion drawing.

FINDINGS

The results showed that most students struggled to solve the given problems. The distribution of answers is shown in Table 2. It shows that 65.1% of students did not understand the concept of derivatives. Only 15% were able to correctly answer the questions using appropriate concepts.

Question 1: Definition of Derivative

Question 1 assesses students' understanding of the definition of a derivative at a specific point. The distribution of students' answers is shown in Table 3. It shows that the majority of students - 41 (66.2%) - fell into Categories 1 and 2. This indicates a lack of conceptual understanding regarding the definition of a derivative. Thirteen students (21%) did not answer the question at all. The excerpt of the interview with student 1K1 are as follows.

- Lecturer : *Why didn't you answer question number 1?*
 1K1 : *I'm sorry, ma'am. I don't know how to answer it.*
 Lecturer : *Try reading it again. What is the question asking?*
 1K1 : *It asks whether the absolute value function is differentiable at $x = 0$.*
 Lecturer : *How can you determine whether a function is differentiable at a point?*
 1K1 : *I don't know, ma'am. I only know how to compute derivatives like in examples.*

$$\begin{aligned} 1) & y = |x| \quad x = 0 \\ & \frac{dy}{dx} = |x| \quad x = 0 \\ & |x| = \frac{dy}{dx} = 0 \end{aligned}$$

Figure 1. Example of a Category 2 Answer for Question 1

$$\begin{aligned} 1) & f'(x) = \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ & = \lim_{x \rightarrow 0} \frac{|(0+h)| - |0|}{h} \\ & = \lim_{x \rightarrow 0} \frac{|h|}{h} = 1 \quad (\text{tidak dapat kerdiferensial}) \\ & \lim_{h \rightarrow 0} \frac{|h|}{h} = \lim_{h \rightarrow 0} 1 \end{aligned}$$

Figure 2. Example of a category_3 answer for question 1

In Category 2, 28 students (45.2%) applied incorrect concepts. Most students gave answers as shown in Figure 1. It is clear the student did not understand the definition of a derivative. They substituted $x = 0$ directly into the absolute value function and concluded the result was 0. Here's an excerpt from an interview with student 1K2.

- Lecturer : Do you understand what the question is asking?
 1K2 : Yes, it asks for the derivative of the absolute value function at zero.
 Lecturer : Are you sure the absolute value function is differentiable at zero?
 1K2 : Hmm, not really sure, ma'am. But if I substitute $x = 0$, I get a result.
 Lecturer : Is that result the function's value or the derivative's value at zero?
 1K2 : Hmm... that might be wrong. Sorry, ma'am. I've never taken the derivative of an absolute value function before.

In Category 3, 16 students (25.8%) attempted to use the definition of the derivative to determine whether the function is differentiable. Figure 2 shows that the student used the limit concept to define the derivative of the function $f(x) = |x|$ at $x = 0$. However, there were errors in formulating the definition. For example, the student incorrectly used Δx instead of $h \rightarrow 0$ and failed to specify the left-hand and right-hand limits, making their reasoning incomplete. This shows fragmented understanding. Excerpt from interview with student 1K3 is as follows.

- Lecturer : Is your answer correct?
 1K3 : Umm, I think so, ma'am.
 Lecturer : Look at your derivative formula. Is it accurate? (pointing)
 1K3 : (Thinking) Sorry, I think it should be h approaching zero.
 Lecturer : Okay. How did you conclude the absolute value function is not differentiable at zero?
 1K3 : I learned it before, but I'm not sure how to conclude it using the formula.

Only 5 students (8%) in category_4 were able to provide correct answers using appropriate conceptual reasoning. Example of student answer of question 1 is shown in Figure 3.

1. $x = 0$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(0) = \lim_{h \rightarrow 0} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \rightarrow 0} \frac{|h|}{h}$$

limit dari kanan $\rightarrow (h \rightarrow 0^+)$

$$= \lim_{h \rightarrow 0^+} \frac{|h|}{h} = \lim_{h \rightarrow 0^+} \frac{h}{h} = 1$$

limit dari kiri $\rightarrow (h \rightarrow 0^-)$

$$= \lim_{h \rightarrow 0^-} \frac{|h|}{h} = \lim_{h \rightarrow 0^-} \frac{-h}{h} = -1$$

Karena limit dari kanan dan kiri tidak sama, maka limit
 $\lim_{h \rightarrow 0} \frac{|h|}{h}$ tidak ada.

dan karena tidak ada maka fungsi $y = |x|$ tidak terdiferensial
 di $x = 0$

Figure 3. Example of a category 4 answer for question 1

Table 4
Distribution of student answers on question 2

Category	Number of Students	Percentage
Category_1	6	9.7%
Category_2	32	51.6%
Category_3	6	9.7%
Category_4	18	29%

From Figure 3, the student correctly defined the function and used left-hand and right-hand limits to verify differentiability. They concluded the function is not differentiable at $x = 0$. Interview excerpt with student 1K4 is as follows.

- Lecturer : Is your answer correct?
 1K4 : Yes, ma'am.
 Lecturer : How did you determine the function is not differentiable at $x = 0$?
 1K4 : Based on the derivative definition. The limit does not exist, so the function is not differentiable.
 Lecturer : How do you check if the limit exists?
 1K4 : By calculating the left-hand and right-hand limits. Since they differ, the limit doesn't exist.

Question 2: Derivative Theorem

Question 2 evaluates students' understanding of derivative theorems, particularly the product rule. There are two ways to solve the question: by first multiplying the functions and then differentiating the result, or by directly applying the product rule. The distribution of student responses is shown in Table 4. It shows that 38 students (61.3%) did not understand the derivative theorem for products of functions. Six students (9.7%) gave no answer due to forgetting the formula or rule for the product of functions, as revealed in the interview with student 2K1.

In Category 2, 32 students (51.6%) used an incorrect concept. Most of them found the derivative of each function separately, then multiplied the results. An example of a Category 2 answer is shown in Figure 4. It indicates that the student misunderstood the product rule. They simply took the derivative of each function individually and then multiplied them $(f(x)g(x))' = f'(x)g'(x)$. The following is an interview excerpt with student 2K2.

b. Turunan ke 2 fungsi $y = x^2 (2x + 3)$

$$y = x^2 (2x + 3)$$
$$y' = 2x \cdot (2 \cdot 0)$$
$$y'' = 2 \cdot 0$$

Figure 4. Example of a category 2 answer for question 2

b) Turunan ke 2

$$y = x^2(2x+3)$$
$$y' = 2x(2x+3) + 2$$
$$y' = 4x^2 + 6x + 2$$
$$y'' = 8x + 6$$

Figure 5. Example of a category 3 answer for question 2

$$u = y = 2x^3 + 3x^2$$
$$\frac{dy}{dx} = 6x^2 + 6x$$
$$\frac{d^2y}{dx^2} = \frac{d}{dx} (6x^2 + 6x)$$
$$= 12x + 6$$

Figure 6. Example of a category_4 answer for question 2

Lecturer : *Is your answer correct?*

2K2 : *I think so, mom.*

Lecturer : *How did you come to that answer?*

2K2 : The derivative of x^2 is $2x$, and the derivative of $2x+3$ is 2. Since the question asks for the second derivative, I differentiated again and got 2 times zero.

In category 3, six students (9.7%) applied the concept incorrectly due to careless errors when using the product rule. An example of student answer of question 2 is shown in Figure 5. It indicates that the student did not fully apply the product rule. For instance, they omitted multiplying by 2 in the proper step. Interview with student 2K3 confirmed this was due to oversight. In Category_4, 18 students (29%) correctly applied the concept. Some students multiplied the functions first, then took the derivative, while others used the product rule directly. In Figure 6, the student correctly applied the product rule or simplified the product before differentiating $((f(x) \cdot g(x))' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$. Here's an interview excerpt with student 2K4.

Lecturer : *Is your answer correct?*

2K4 : *I think so, mom.*

Lecturer : *How did you determine the derivative of the function?*

2K4 : *It's a product of two functions, ma'am. So I used the product rule (pointing at the function).*

Question 3: Implicit Differentiation

Question 3 evaluates students' conceptual understanding of implicit differentiation. The distribution of student responses is shown in Table 5.

Table 5		
Distribution of student answers on question 3		
Category	Number of Students	Percentage
Category_1	10	16.1%
Category_2	32	51.6%
Category_3	15	24.2%
Category_4	5	8.1%

$$C: y = x^2 + y = x^3 y^2$$

$$y' = 2x + 1 = 3x^2 \cdot 2y$$

$$= 8x^2$$

Figure 7. Example of a category 2 answer for question 3

As shown in Table 5, a total of 42 students (67.7%) did not understand the concept of implicit differentiation. Ten students (16.1%) left the question unanswered. Interviews with student 3K1 indicated that this was due to forgetting the method of implicit differentiation. In Category 2, 32 students (51.6%) applied incorrect concepts. An example of a category_2 answer for question 3 is shown in Figure 7. It is clear that the student did not understand the structure of an implicit function. They differentiated each variable with respect to itself, even though the question required differentiation with respect to x . An excerpt from an interview with student 3K2 is as follows.

- Lecturer : *Is your answer correct?*
 3K2 : *I'm not really sure, mom.*
 Lecturer : *How did you find the derivative of this function? (pointing)*
 3K2 : *I was confused, so I treated this function (pointing) as a function of y , then differentiated everything. Since derivative usually involves x , I just chose the terms with variable x , mom.*

In Category 3, 15 students (24.2%) nearly applied the correct concept of implicit differentiation, but made minor errors. An example is shown in Figure 8. It shows the student understood the structure of an implicit function and attempted to differentiate each term accordingly. However, they made a mistake when differentiating a product term and incorrectly continued differentiating the right-hand side of the equation. Interview with student 3K3 revealed is as follows.

- Lecturer : *Is your answer correct?*
 3K3 : *I think it is, but I'm not totally sure.*
 Lecturer : *How did you approach implicit differentiation?*
 3K3 : *I remember we were taught to differentiate each term based on the variable involved*
 Lecturer : *Okay, why does your answer for y^2 look like this? (pointing)*
 3K3 : *I assumed that since $\frac{dy}{dx}$ is the derivative of y then the derivative of y^2 would be $(\frac{dy}{dx})^2$. So I differentiated once more to get $\frac{dy}{dx}$*
 Lecturer : *Why is the y missing in this part?*
 3K3 : *(Smiling) Sorry, mom. I forgot to include it*

In category_4, 5 students (8.1%) successfully solved the problem using the correct concept. An example is shown in Figure 9. It shows the student had a good understanding of implicit functions and applied proper differentiation techniques. Excerpt from the interview with student 3K4 is as follows.

$$\begin{aligned}
 x^2 + y &= x^3 y^2 \\
 2x + \frac{dy}{dx} &= 3x^2 \left(\frac{dy}{dx} \right)^2 \cdot 2y \frac{d}{dx} \\
 2x + \frac{dy}{dx} &= 6x^2 \cdot \frac{dy}{dx} \cdot 2 \frac{dy}{dx} \\
 \frac{dy}{dx} &= \frac{2y}{6x^2 - 2x}
 \end{aligned}$$

Figure 8. Example of a category_3 answer for question 3

$$\begin{aligned}
 \frac{d}{dx} (x^2 + y) &= \frac{d}{dx} (x^3 y^2) \\
 2x + \frac{dy}{dx} &= 3x^2 y^2 + x^3 2y \frac{dy}{dx} \\
 2x - 3x^2 y^2 &= x^3 2y \frac{dy}{dx} - \frac{dy}{dx} \\
 (x^3 2y - 1) \frac{dy}{dx} &= 2x - 3x^2 y^2 \\
 \frac{dy}{dx} &= \frac{2x - 3x^2 y^2}{x^3 2y - 1}
 \end{aligned}$$

Figure 9. Example of a category_4 answer for question 3

Lecturer : *Is your answer correct?*

3K4 : *Yes, ma'am. It follows the rules for implicit differentiation.*

Lecturer : *How did you perform the differentiation?*

3K4 : *I differentiated both sides of the equation with respect to x, then handled each term according to its variable and simplified the result.*

From the above explanation, it is generally evident that students' conceptual understanding is very low. Their grasp of the definition of the derivative and implicit differentiation is weaker compared to their understanding of derivative theorems, particularly the product rule. Only 8% of students were able to correctly solve the problem related to the definition of a derivative, and 8.1% for the implicit function differentiation, using the appropriate conceptual understanding. A slightly higher percentage of students - 29% were able to correctly apply the derivative theorem.

DISCUSSION

This research reveals students' conceptual understanding across three core concepts in derivatives: definition, theorems (rules), and implicit differentiation. Overall, students exhibited weak conceptual understanding. This supports by Hashemi et al. (2014) findings that many students could compute derivatives but lacked a solid grasp of derivative concepts. Gracia and Flores (2021) also found that students struggled to connecting the concept of derivatives with the interpretation of derivative graphs.

The study shows that students have difficulty using the multiplication theorem to calculate the derivative of a function. This is in line with Mkhathswa (2020) research which found that students still have difficulty using the derivative theorem to determine the derivative of a function. However, the definition and derivative of implicit functions are more difficult for students than applying derivative theorems. This reflects findings by Puspita et al., (2023) who noted students often solve derivative problems based on memorized procedures rather than conceptual understanding. Students solve derivative problems from habit, not comprehension.

Students' lack of understanding of limits, ratios, and proportions contributes to the difficulty in grasping the definition of derivatives (Byerley et al., 2012). Conceptual knowledge involves linking mathematical ideas—something many students fail to do. Hashemi et al. (2014) also noted that students struggle to connect mathematical symbols with the quantities they represent. In terms of

the product rule, students often misapplied it by multiplying individual derivatives rather than applying the correct formula. This indicates they could not distinguish between rules for addition and multiplication. Similarly, in implicit differentiation, students struggled due to their familiarity with only explicit functions. They focused on manipulating symbols rather than understanding the varying quantities those symbols represented (Hashemi et al., 2014).

Identifying these difficulties is crucial for designing more meaningful Calculus instruction tailored to students' needs. According to Vygotsky as cited in Santrock (2008), students often possess fragmented conceptual understanding that needs to be developed into structured knowledge. Understanding the needs of students helps teachers design (Masduki et al., 2023) appropriate and meaningful calculus learning. This research is still limited to three important topics and the subjects are still limited. Future researchers could develop other important topics and expand the subjects of research.

CONCLUSION

This study concludes that students' conceptual understanding of derivatives is still low. Most students were unable to apply appropriate concepts to solve the problems. Specifically: For definition of derivatives, many students did not understand that a derivative represents the rate of change or the limit of the average rate of change. Many confused function values with derivative values. For product rule, many students mistakenly believed the derivative of a product is the multiplication of each of its derivatives $(f(x) \cdot g(x))' = f'(x) \cdot g'(x)$. For implicit differentiation, most students did not understand the structure of implicit functions or how to differentiate them. They often ignored the variable y , assuming all functions should be expressed in terms of x only.

ACKNOWLEDGMENT

The researchers would like to thank the Universitas Islam Negeri Bukittinggi for the support and facilities provided.

AUTHOR'S DECLARATION

Authors' contributions	AN: main idea, conceptualization, and writing the manuscript, NS: review and validation, GHM: review and editing
Funding Statement	This research is funded by the research team
Availability of data and materials	All data are available from the authors
Competing interests	This work has not been published or submitted for publication elsewhere, and is entirely original work.

BIBLIOGRAPHY

- Alam, A. (2020). Challenges and possibilities in teaching and learning of calculus: *Journal for the Education of Gifted Young Scientists*, 8(1), 407–433. <https://doi.org/https://doi.org/10.17478/jegys.660201>
- Aniswita, A. (2016). Pengaruh Model Pembelajaran Missauri Mathematic Project Terhadap Nilai Kalkulus Diferensial: Studi Pada Mahasiswa Pendidikan Matematika IAIN Bukittinggi TA 2015-2016. *Tarbiyah*, 23(2), 13–22. <http://dx.doi.org/10.30829/tar.v23i2.105>
- Aniswita, Fauzan, A., & Armia. (2023). Students' Conceptual and Procedural Knowledge on. *AIP Conference Proceedings*. <https://doi.org/10.1063/5.0122392>
- Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). Research commentary: An alternative reconceptualization of procedural and conceptual knowledge. *Journal for research in mathematics education*, 38(2), 115-131.
- Bogdan, R., & Taylor, S. J. (1992). Pengantar Metode Penelitian Kualitatif, Terjemahan oleh Arief Rurchan. *Usaha Nasional*.
- Byerley, C., Hatfield, N., & Thompson, P. W. (2012). Calculus students' understandings of division and rate. *Proceedings of the 15th Annual Conference on Research in Undergraduate Mathematics Education*, 358–363.
- Denbel, D. G. (2015). Some Conceptual Difficulties of Students on Derivation. *Journal of Educational and Management Studies, J. Educ. Manage. Stud.*, 5(4): 211-214.

- Deswita, R., Erita, S., & Ningsih, F. (2021). The Student's Learning Obstacle in Calculus Course. *EDUMATIKA: Jurnal Riset Pendidikan Matematika*, 4(2), 150–162. <https://doi.org/https://doi.org/10.32939/ejrpm.v4i2.1114>
- Dunham, W. (2005). The calculus gallery: Masterpieces from Newton to Lebesgue. *Princeton University Press*. <https://doi.org/https://doi.org/10.5860/choice.43-0368>
- Ferrini-Mundy, J., & Graham, K. (1994). Research in calculus learning: Understanding of limits. *Research Issues in Undergraduate Mathematics Learning, MAA Notes #33*. Washington, DC: Mathematical Association of America.
- Findell, B., Swafford, J., & Kilpatrick, J. (Eds.). (2001). Adding it up: Helping Children Learn Mathematics. *Washington, DC: National Academy Press*.
- Funny, R. A. (2021). Analysis of Engineering Students' Understanding in. *Journal of Physics: Conference Series*. <https://doi.org/https://doi.org/10.1088/1742-6596/1957/1/012002>
- García-García, J., & Dolores-Flores, C. (2021). Preuniversity students' mathematical connections when sketching the graph of derivative and antiderivative functions. *Mathematics Education Research Journal*, 33, 1-22. <https://doi.org/10.1007/s13394-019-00286-x>
- Hashemi, N., Abu, M. S., Kashefi, H., & Rahimi, K. (2014). Undergraduate students' difficulties in. *Procedia - Social and Behavioral Sciences*, 143, 358–366. <https://doi.org/10.1016/j.sbspro.2014.07.495>
- Hashemi, N., Abu, M. S., Kashefi, H., Mokhtar, M., & Rahimi, K. (2015). Designing learning strategy to improve undergraduate students' problem solving in derivatives and integrals: A conceptual framework. *Journal of Mathematics, Science and Technology Education*, 11(2), 227–238. <https://doi.org/https://doi.org/10.12973/eurasia.2015.1318a>
- Hiebert, J., & Lefevre, P. (1986). Conceptual and Procedural Knowledge in Mathematics: An. *Hillsdale, NJ: Erlbaum*, 1–27.
- Hurrell, D. (2021). Conceptual Knowledge OR Procedural Knowledge or Conceptual Knowledge And Procedural Knowledge: Why the Conjunction is Important to Teachers. *Australian Journal of Teacher Education*, 46(2), 57–71. <https://doi.org/10.14221/ajte.2021v46n2.4>
- İşleyen, T., & Isik, A. (2003). Conceptual and procedural learning in mathematics. *Series D: Research in Mathematical Duction*, 7(2), 91–99.
- Khashan, K. H. (2014). Conceptual and procedural knowledge of rational numbers for Riyadh elementary school teachers. *Journal of Education and Human Development*, 3(4), 181–197. <https://doi.org/10.15640/jehd.v3n4a17>
- Kidron, I. (2020). Calculus Teaching and Learning. *Springer Dordrecht*, 69–75. https://doi.org/10.1007/978-94-007-4978-8_18
- Latorre, D. R., Kenelly, J. W., Reed, I. B., Carpenter, L. R., & Harris, C. R. (2007). Calculus Concepts: An Applied Approach to the Mathematics of Change. *An Applied Approach to the Mathematics of Change. In Procedia Manufacturing*.
- Mahir, N. (2009). Conceptual and procedural performance of undergraduate students in integration. *International Journal of Mathematical Education in Science and Technology*, 40(2), 201–211. <https://doi.org/https://doi.org/10.1080/00207390802213591>
- Masduki, M., Suwarsono, S., & Budiarto, M. T. (2023). Relationships between teacher's instructional strategies and their knowledge: A study of seventh-grade mathematics teachers. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 8(3), 164-179. <https://doi.org/10.23917/jramathedu.v8i3.4742>
- Miles, M. B., Huberman, A. M., & Saldana, J. (2014). Qualitative Data Analysis; A Methods Sourcebook. *Arizona State: SAGE*.
- Mkhatshwa, T. P. (2020). Calculus students' quantitative reasoning in the context of solving related rates of. *Mathematical Thinking and Learning*, 22(2), 139–161. <https://doi.org/10.1080/10986065.2019.1658055>
- Mkhatshwa, T. P. (2024). Best practices for teaching the concept of the derivative: Lessons from experienced calculus instructors. *EURASIA Journal of Mathematics, Science and Technology Education*, 20(4). <https://doi.org/10.29333/ejmste/14380>
- Orton, A. (1983). *Student's Understanding of Differentiation*. 14, 235–236. <https://doi.org/10.1007/BF00410540>

- Puspita, E., Suryadi, D., & Rosjanuardi, R. (2023). Learning Obstacles of Prospective Mathematics Teachers: A Case Study on the Topic of Implicit Derivatives. *Kreano: Jurnal Matematika Kreatif-Inovatif*, 14(1), 174–189.
- Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go. *International Journal on Mathematics Education*, 46(4), 507–515. <https://doi.org/10.1007/s11858-014-0615-x>
- Santrock, & . J. (2008). Educational Psychology (6th ed.). McGraw-Hill, 6th ed.
- Schneider, M., and Stern, & E. (2010). The developmental relations between conceptual and procedural. *Developmental Psychology*, 46, 178–192. <https://doi.org/10.1037/a0016701>
- Schoenfeld, A. H. (1995). A Brief Biography of Calculus Reform. *UME Trends: News and Reports on Undergraduate Mathematics Education*, 6(6), 3–5.
- Stevens, & N., B. (2021). Developing the Definite Integral and Accumulation Function Through Adding Up Pieces. *Brigham Young University Follow*. <https://doi.org/https://scholarsarchive.byu.edu/etd/9132%0AThis>
- Vajravelu, K., & Muhs, T. (2016). Integration of digital technology and innovative. *International Journal of Research in Education and Science*, 2(2), 379–395. <https://doi.org/https://doi.org/10.21890/ijres.67867>
- Varberg, D., Purcell, E., and Steve, & R. (2016). Calkulus jilid 1, 9 Edition (9th ed.). Pearson, 9 Edition.
- White, P., and Mitchelmore, & M. (1996). *Conceptual knowledge in introductory Calculus*. 27, 79–95. <https://doi.org/10.2307/749199>
- Zuya, H. E. (2017). Prospective Teachers' Conceptual and Procedural Knowledge in Mathematics: The Case of Algebra. *American Journal of Educational Research*, 5(3), 310–315. <https://doi.org/10.12691/education-5-3-12>