

## Contact stress analysis of a spur gear using Lewis and Hertz theory

Aisha Muhammad<sup>1,2\*</sup>, Ibrahim Haruna Shanono<sup>3</sup>

<sup>1</sup>Department of Mechatronics Engineering, Bayero University, Kano, Nigeria,

<sup>2</sup>Faculty of Manufacturing and Mechatronic Engineering Technology, Universiti Malaysia Pahang, Malaysia.

<sup>3</sup>Department of Electrical Engineering, Bayero University Kano Nigeria.

\*Corresponding author: [ayshermuhd@gmail.com](mailto:ayshermuhd@gmail.com)

Permalink (DOI): <https://doi.org/10.23917/arstech.v2i2.165>

### ARTICLE INFO

#### Article history:

Received 22 July 2020

Revised 17 February 2021

Accepted 21 June 2021

Available online 24 July 2022

Published regularly 30 July 2022

#### Keywords:

Ansys

Contact stress

Finite element analysis

Finite element method

Module

Safety factor

### ABSTRACT

*The rapid development of the automobile and aircraft industries has made applying gear technology necessary. Gears offer the benefits of efficiency, reliability, simplicity and a higher speed ratio with power transmission. However, they cannot transmit power over a long period, are more expensive when compared to belts and chain drives and requires continuous lubrication. They are used for transmitting high load in gear tools. The gear teeth fail when subjected to a high load beyond a certain limit. A deciding factor in gear design is the amount of stress developed on the contact surface of the mating gears. This paper deals with contact stress analysis of spur gear. The theoretical analysis presented in this paper aims to analyse rolling bearing contacts based on the principles of the Hertz and Lewis equations. A 3D domain-based finite element method was used to calculate the stresses between the contact surfaces of the structure. Contact analysis was performed using ANSYS Workbench software to figure out the deformation and optimum stress developed on the teeth of the gear. The simulation results indicated that the stress distribution on the gear contact surface has a maximum value of 144.82 MPa, while the deformation is at a maximum value of 0.01676 mm. The safety factor indicates the maximum level of 15, indicating the safety of the design.*

## 1. INTRODUCTION

A gear is a cylindrical shape machine component designed to transmit force and motion from one mechanical system to another. In order to achieve high-power transmission, gear is used. A spur gear is a fundamental piece of equipment with a tooth parallel to the axis of a shaft on which the gear is positioned, having

about 98.99% functioning efficiency [1]. It can transmit power between two shafts over a specific range [2][3]. It is primarily designed to perform certain functions; as a result, the design and features are carefully chosen and connected. Multiple gears have been created for different functions, with spur gears being the most typical. They can fail and terminate working like any other mechanical component due to failure or other reasons [4] when the

gear material endurance limit is less than the developed stress; after that, failing is bound to occur by the formation of pits which continue to expand triggering the rupture of the tooth surface. This can be prevented by lowering the load on the equipment tooth than its yield strength [2]. Repeated loading of the gear can result in either gear teeth flank pitting or spalling and tooth fracture. Fracture in the tooth or foot is one of the most unwanted damages that can take place in gear as it results in an impossible operation in its unit [4].

The material needed to manufacture gear should have high strength, surface endurance, low friction coefficient and thermal expansion [5]. Both metals and non-metallic materials have been used to manufacture gears, and cast iron materials are used when excellent wearing properties are required. Optimised designs and improved manufacturing processes need to be introduced using non-metallic (wood and pressed paper) and plastic (due to their rust resistance, conductivity, appearance and colours) materials to reduce weight and noise. It is crucial to enhance gear strength to avoid failure, which will trigger the breakdown of the system [6][7][8].

Over the past few years, many researchers have made great efforts in performing contact stress analysis using the Finite Element Method with ANSYS software between two mating gears. ANSYS is a general-purpose finite-element modelling plan for numerically addressing various mechanical problems [9]. The ANSYS program is a practical, multi-purpose evaluation device used in many design techniques [10][11].

The use of cast iron, polycarbonate and nylon material for spur gear was presented using finite element analysis by Shoba et al. [1]. From the obtained result, the seen deflection in gear with cast iron was more considerable than that with polycarbonate and nylon material. They conclude that cast iron materials can be changed with nylon for an efficient, longer span and silent gear. The material mechanical properties of Aluminium Silicon Carbide were experimented with and presented by Neelima et al. [12]. Compared to similar gear but with mild steel and lightweight aluminium material, the ratio of weight to strength was found to have triple and double, respectively.

The contact stress analysis of gears (spur and helical) was also carried out and presented by Seok-Chul et al. [13] to investigate and compare the stress values at different contact points throughout the rotation. Vera et al. [14] assess and confirms the effectiveness of numerical methods using the finite element method to determine the contact stress on mating gear tooth flanks. The modelling and process used to find the flank's maximum stress value was also described and presented. Kristina et al. [15] use FEM to evaluate and analyse a standard gear model for contact stress on the tooth. Hertz equation was used to calculate

and compare with the result obtained from the numerical finite element method. Sushil et al. [16] use Lewis and Hertz's theories have been used to determine the contact and bending stress of a spur gear. Further comparison of each theory with the finite element method was made to determine which approach is the best for calculating the stress of a gear. From the results obtained, Hertz's theory proved to be the best fundamental theory. Likewise, Moya et al. [17] analyse a spur gear to calculate the contact stress distribution. FEM was made used by Bharat et al. [18] to determine the contact stress in gear. The analysis was repeatedly conducted using different values of the module. A conclusive suggestion was made that the module's value could be increased to minimise the developed stress values between the gears.

This paper presents the contact stress analysis of steel material spur gear. Unlike in the previous studies, a detailed analytical calculation was carried out using the Hertz theory. The ANSYS workbench interface was used to analyse the structure to validate the obtained theoretical result. This was achieved using a step-wise method of analysis which is aimed at providing an introduction to static structural analysis for young researchers.

## 2. MODELLING AND SIMULATION

In order to achieve the goal of the study, it was derived as the step in Figure 1.

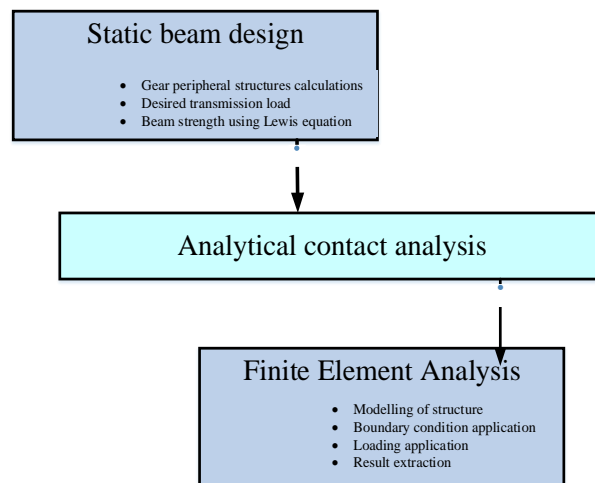


Figure 1. The research flow.

### 2.1. Design for static beam strength

Finding the right gear for a specific application is a complex process. The gear teeth should be able to be within a certain safety margin and make less noise to avoid failing while transmitting force. The two mating gears must work without interference within the right contact length. This paper determines the maximum contact stress

between two mating spur gears of structural steel material with properties as shown in table 1. It is determined using the gear specifications displayed in table 2.

**Table 1. Properties of materials.**

Description	Specification
Material Type/ Property	Structural steel
Young's Modulus	200000 MPa
Density	7850 kg/m <sup>3</sup>
Poisson's ratio	0.30
Bulk modulus	166670 MPa
Shear Modulus	76923 MPa
Tensile strength	250 MPa
Ultimate shear strength	460 MPa

**Table 1. Spur gear parameters.**

Parameter	Value
Number of teeth on both gears	20
Input speed (rpm)	1440
Gear transmission ratio	1
Gear material	Structural steel
Pitch circle diameter (mm)	127
Pressure angle(degrees)	20
Face width (mm)	45

## 2.2. Analytical contact stress analysis

Generally, bending strength, resilience and durability are applied to spur gears for usage in industrial machines. However, bending and contact stress are two basics among the numerous existing stress theories used to identify tooth breakage and wear failure [16].

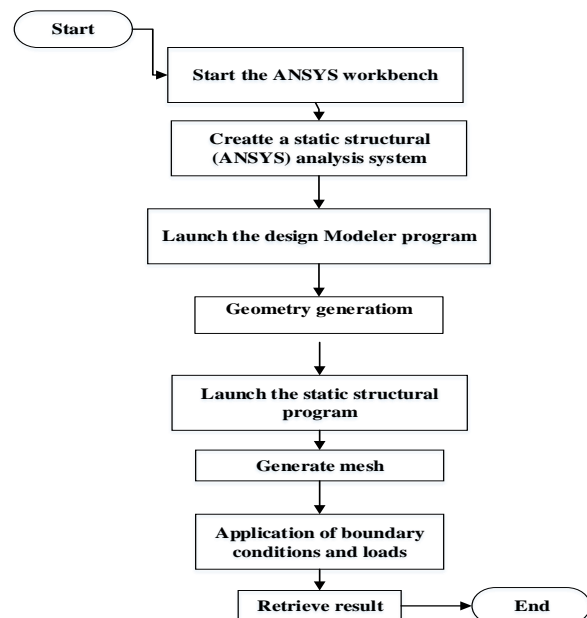
The study used the Hertz equation and Lewis's formula to theoretically calculate the surface contact and bending stress between two mating gear teeth. Lewis and Hertz's theories were used to determine the mathematical expressions of the stress values in gears [19]. A deciding parameter of the proper dimensions needed in gear design was the amount of contact stress developed in the mating gear tooth, hence the need for an in-depth study [18].

## 2.3. Finite element analysis (FEA)

Analysis of an engineering structure has become necessary due to the unexpected occurrence of failure. Such failures will further result in the sudden collapse of the structures when subjected to loads over a specific time [20]. A finite element method is used to represent the geometrical design of systems in terms of a limited number of nodes called "elements". It is an approach used to find the exact solutions with minimal errors. The finite element solution follows an orderly procedure. Fixing design issues, including complicated frameworks, is an excellent feature of the FEM [6], [21]. Contrary to the theoretical method, the Finite Element Method in ANSYS is a straightforward approach used in determining the amount of stress developed in the teeth of mating gears

A desirable feature of FEM is its ability to deal with complex engineering structures and their constraints [10]. Hence, this work assesses a spur gear's deformation, contact stress and safety factor. In this study, the gear was modelled in SolidWorks, which was later imported into the ANSYS workbench software for analysis. The stages involved are shown in Figure 2.

Breaking down a model or structure into more minor elements to analyse each component is known as meshing. It is a discrete design realisation, which helps solve the exact model solutions. For a smaller meshing size, the computational time is higher, and so is the analysis result's accuracy [21].



**Figure 2. FEA Flow.**

The meshing was done using a tetrahedral element with 18097 and 10404 total numbers of generated elements and nodes, respectively. Knowing and declaring the correct value of the material property was very useful for design analysis purposes. A study noted that the results have various variations based on different types of materials having different densities [21]. A torque of 54000 Nm was applied to the rim of the gear. The frictionless support was provided at the inner rim of the pinion in the directions shown in Figure 3.

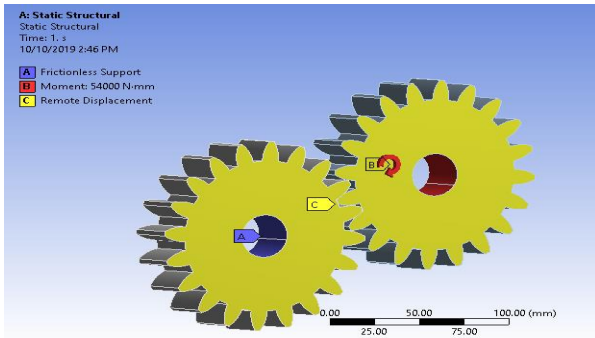


Figure 3. Boundary conditions.

### 3. RESULTS AND DISCUSSION

An essential goal of performing analysis is determining a safer design. Failure of the whole or part of an engineering structure or component might result in loss of life and properties. Like in the human context, when the human body does much work, it becomes stressed and finally breaks down or fails. Similarly, in an engineering structure, failure will occur when a structure is subjected to high stress over time. Von-Mises stress is utilised to assess the failure of structures which is inescapable when the strength of the material is less than the maximum stress value. The safety factor represents the indefinable elements that affect stress values. The safety factor indicates a structure's desired strength-to-stress ratio. It is the ratio of material strength to the maximum stress in a structure. It defines the load-carrying ability of a structure when compared to the actual loads. It is also used to assess the safety of a structure. The calculation of the safety factor involves the yield strength; thus, declaring this value becomes necessary before the start of analysis [22]

Under the constraints, the results were obtained using the structural steel below. The following image, Figure 4, shows the stress distribution level across the spur gear. Maximum stress is at the contact surface of the gears with a value of 144.82MPa. Figure 5 shows the deformation plot when the gear is under the applied torque. The maximum deformation stands at 0.01676mm, which is indicated by the red colour at the rim of the gear. Figure 6 shows the safety factor (SF) level, showing that no material region is overstressed. Most of it

lies within the maximum SF level of 15. The minimum safety factor is given by 1.7263.

The maximum contact stress obtained from the theoretical calculations is 142.05 MPa, while that from the FEA is 144.92 MPa providing a difference of 1.98% (which is within the limit).

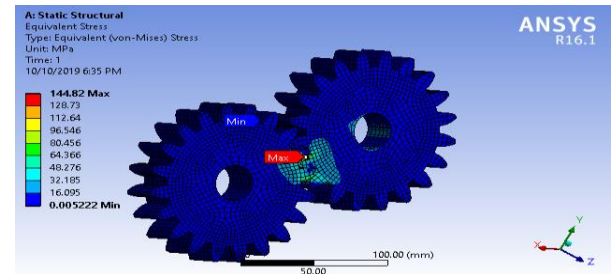


Figure 4. Stress distribution of the gears.

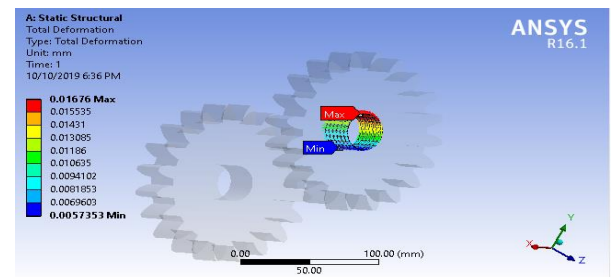


Figure 5. Total deformation in gear.

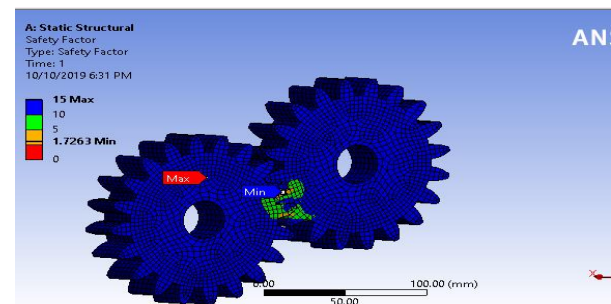


Figure 6. Safety factor.

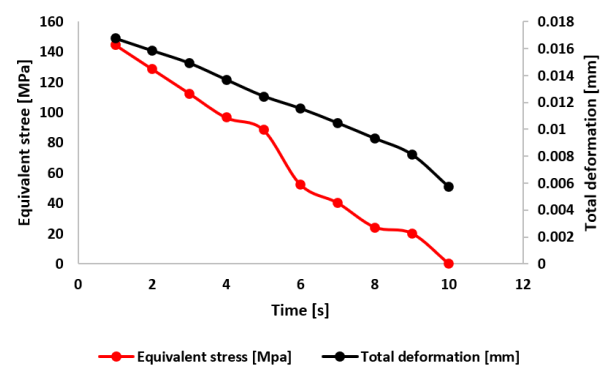


Figure 7. Deformation and stress graph.



Figure 7 shows the graphical representation of the deformation and equivalent stress of 10 iterations at a step value of 1 second.

Further refinement of the FEA structure into much smaller elements results in the convergence of the solution to an exact solution of the problem. Example of these refinement used in FEA includes h-refinement (with h referring to the element size), p-refinement (choosing the polynomial highest order), r-refinement (which reorganise the nodes in the mesh) and hp-refinement (which integrates h and also p-refinement) [23]. This study's convergence standards were based on total deformation with 1% tolerance. Convergence was obtained after ten refinement loops, as shown in Figure 8.

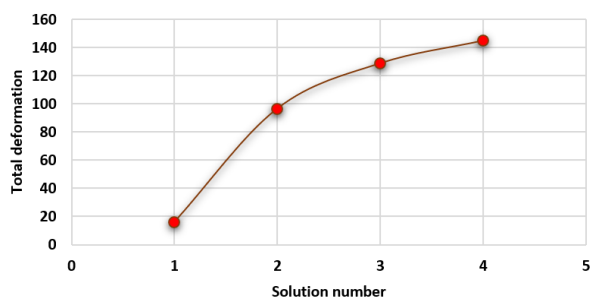


Figure 3. Convergence plot.

#### 4. CONCLUSION

The design and analysis of the contact stress developed in the teeth of a spur gear have been carried out and presented. Theoretical calculation using Hertz and Lewis equation was used to determine the design stress based on the material's ultimate strength (structural steel) with a safety factor of 1.7263. It was found that the allowable bending stress using one-third of the ultimate tensile strength is 87.19. This value is less than the maximum tensile strength, and it implies a safe design.

Analytical calculation using a spur gear with a ratio of 1:1 shows that the maximum contact stress is 142.05 MPa. There is a difference of up to 1.98% when simulated with ANSYS, which is 144.92 MPa.

#### CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

#### ACKNOWLEDGEMENT

The authors would like to thank Bayero University, Kano, Nigeria, for supporting the study.

#### REFERENCES

- [1] T.S. Rani, and T. Dada Khalandar, "Spur gear," *International Journal of Computational Engineering Research*, vol. 3, no. 11, pp. 7-12, 2013.
- [2] V. Wable, and D.M. Mate, "An investigative study of spur gear failure by FEA and photoelastic method", *International Engineering Research Journal*, pp. 746-751, 2016.
- [3] H.P. Rahate, and R.A. Marne, "Contact Stress analysis of composite spur gear using photo-stress method and finite element analysis", *International Research Journal of Engineering and Technology (IRJET)*, vol. 3, no. 7, pp. 540-545, 2016.
- [4] B. Desai, and M.A. Mohite, "Analysis of fatigue crack growth rate and service life of spur gear", *International Engineering Research Journal*, pp. 1685-1689, 2006.
- [5] Sunil, and G. Janiyani, *Machine design*, Rajkot: Department of Mechanical Engineering Darshan Institute of Engineering & Technology, 2017.
- [6] V. S. Prasad, S. A. Hussain, V. Pandurangadu, and K. Palanikumar, "Modeling and Analysis of spur gear for sugarcane juice machine under static load condition by using FEA", *International Journal of Modern Engineering Research (IJMER)*, vol. 2, no. 4, pp. 2862-2866, 2012.
- [7] P.B. Pawar, and A.A. Utpat, "Development of Aluminium based silicon carbide particulate metal matrix composite for spur gear", *The 3rd International Conference on Materials Processing and Characterisation (ICMPC 2014)*, India, 2014.
- [8] A. Muhammad, M.A.H. Ali, and I.H. Shanono, "Finite Element Analysis of a connecting rod in ANSYS: An overview", *The 11th Malaysian Technical Universities Conference on Engineering and Technology (MUCET)*, Malaysia, 2019.
- [9] Klaus-Jurgen, Bathe, *Finite Element Procedures (USA)*, USA: Prentice Hall, Pearson Education, Inc, 2014.
- [10] A. Muhammad, and I.H. Shanono, "Strength analysis and structural optimisation of an l-shaped bracket", *Nigerian Journal of Technological Research*, vol. 13, pp. 14-19, 2018.
- [11] Y. Nakasone, S. Yoshimoto, and T. Stolarski, *Engineering Analysis with ANSYS Software*, UK: MPG Books Ltd., Bodmin, Cornwall, 2006.

- [12] D. Neelima, V. Mahesh, and N. Selvaraj, "Mechanical characterisation of Aluminium silicon carbide composite", *International Journal of Applied Engineering Research*, vol. 1, no. 4, pp. 126-131, 2011.
- [13] S. Hwang, J. Lee, D. Lee, S. Hana, and K. Lee, "Contact stress analysis for a pair of mating gears", *Mathematical and Computer Modelling*, vol. 57, p. 40-49., 2013.  
<https://doi.org/10.1016/j.mcm.2011.06.055>.
- [14] V. Nikolic-Stanojevic, and I. Cvejic, "The Analysis of contact stress on meshed teeth's flanks along the path of contact for a tooth pair", *Mechanics, Automatic Control and Robotics*, vol. 3, pp. 1055-1066, 2003.
- [15] K. Markovic, and M. Franulovic, "Contact stresses in gear teeth due to tip relief profile modification", *Engineering review*, vol. 31, no. 1, pp. 19-26, 2011.  
<https://hrcak.srce.hr/70383>
- [16] S.K. Tiwari, and U.K. Joshi, "Stress analysis of mating involute spur gear teeth", *International Journal of Engineering Research and Technology*, vol. 1, no. 9, pp. 1-12, 2012.
- [17] K. Mao, "new approach for polyamide composite gear design", *Wear*, vol. 262, pp. 432-441, 2007.
- [18] B. Gupta, A. Choubey, and G.V. Varde, "Contact stress analysis of spur gear", *International Journal of Engineering Research and Technology*, vol. 1, no. 4, pp. 1-7, 2012.
- [19] S.A. Quadri, and D.R. Dolas, "Contact stress analysis of involute spur gear under static loading", *International Journal of Scientific Research Engineering & Technology (IJSRET)*, vol. 4, no. 5, pp. 593-596, 2015.
- [20] A. Muhammad, and I.H. Shanono, "Static analysis and optimisation of a connecting rod", *International Journal of Engineering Technology and Sciences*, vol. 6, pp. 24-40, 2019.  
<http://dx.doi.org/10.15282/ijets.6.1.2019.1003>
- [21] A. Muhammad, and I.H. Shanono, "Transient analysis and optimisation of a knuckle joint", *KINETIK*, vol. 4, no. 2, pp. 179-186, 2019  
<https://doi.org/10.22219/kinetik.v4i2.767>
- [22] A. Muhammad, M.A.H. Ali, and I.H. Shanono, "Finite element analysis of a connecting rod in ANSYS: An overview", *IOP Conference Series: Materials Science and Engineering*, vol. 736, No. 2, p. 022119. 2020.  
<https://doi.org/10.1088/1757-899X/736/2/022119>
- [23] A. Muhammad, and I.H. Shanono, "Finite element analysis of the heat transfer in a piston", *Journal of Modern Manufacturing Systems and Technology*, vol. 4, no. 1, pp. 45-51, 2020.  
<https://doi.org/10.15282/jmmst.v4i1.3722>